Carnegie Mellon DSP-I Electrical & Computer ENGINEERING

Digital Signal Processing I (18-791)
Fall Semester, 2002

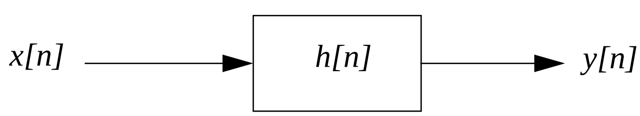
QUIZ 1

Two hours — **Closed book**
October 10, 2002 — **One sheet of notes**

Note: The questions on this exam are not very difficult. Be sure to think about ways of answering the questions using reasoning, transform properties, etc. before resorting to "brute-force" solutions. Although it may seem that there are many questions, most of them are very brief and easy.

The multiple parts of the longer of the questions can be solved separately and will be graded independently.

Question 1 (33 %):



An LSI system has input $x[n]$, output $y[n]$, and unit sample response

$$h[n] = 2^{n+2}u[n+2]$$

(a) Obtain an analytical expression for the $H(z)$, the Z-transform of $h[n]$ and its region of convergence.

(b) Determine whether the system is causal and/or stable. Be sure to explain your reasoning.

(c) Using any method, obtain the output of the system when the input is

$$x_1[n] = \left(\frac{1}{3}\right)^n u[n]$$

(d) Obtain the response of the system when the input is the complex periodic function $x_2[n] = e^{j\pi n/2}$ for all n. You may use any method to obtain your answer, but transform techniques are easiest here.

(e) Write the linear constant coefficient difference equation that can implement the system. Provide the conditions that apply when the input is the time function $x_1[n]$ in part (c).

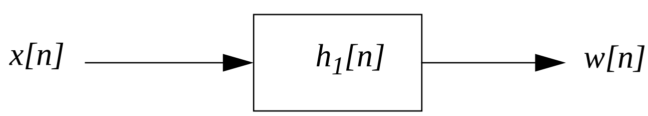
(f) Obtain an analytical expression for a second unit sample response $h_2[n]$ which has the same Z-transform as $h[n]$ but that is a left-sided time function.

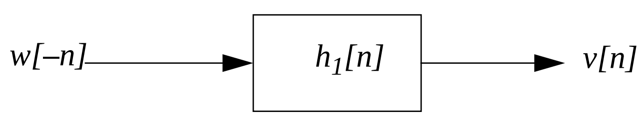
Question 2 (20%): (Modified from the actual question that Al Oppenheim asked me during my Ph.D. oral qualifying exam in the spring of 1973!)

The help file for the MATLAB function **`FILTFILT`** reads in part:

```
Y = FILTFILT(B, A, X) filters the data in vector X with the filter
described by vectors A and B to create the filtered data Y.  .... After
filtering in the forward direction, the filtered sequence is then
reversed and run back through the filter; Y is the time reverse of the
output of the second filtering operation.
```

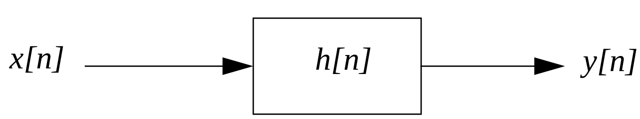
This can be expressed as the following sequence of operations for system input $x[n]$ and output $y[n]$:





$$y[n] = v[-n]$$

(a) It can be shown that the sequence of operations above is LSI and hence equivalent to the block diagram



If the unit sample response of the filter in `filtfilt` is $h_1[n] = \left(\frac{1}{2}\right)^n u[n]$, obtain an analytical expression for $h[n]$, the overall unit sample response of the system.

(b) Determine whether or not the overall system specified by $h[n]$ is

1. Causal
2. Stable

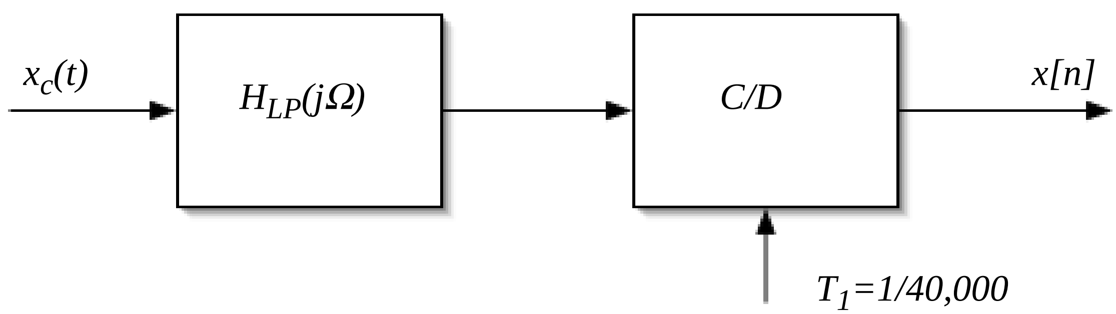
3. Zero phase
4. Linear phase
5. Maximum phase
6. Minimum phase
7. Allpass

Be sure to state the reasons for your answers. (You will not be given full credit unless you do so.)

Question 3 (30 %):

The 2002 Federal Free Music Act has just been passed by the FCC and authorizes a new broadcast service in which high quality audio signals are passed through narrowband low-frequency radio channels and then decoded using digital signal processing. You are planning to open a new commercial service in this broadcast band along with a few friends from 18-791.

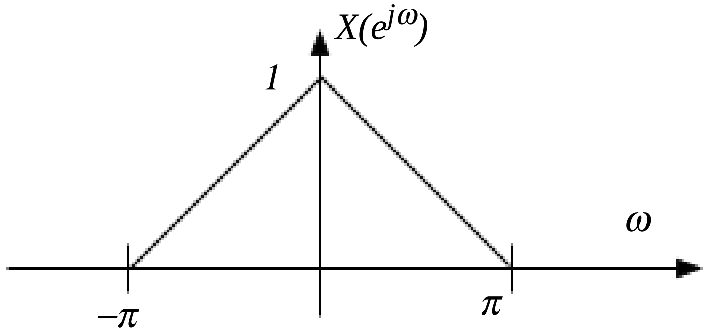
First, we can (crudely) model the process by which audio signals are recorded onto audio CDs with the following block diagram, This isan approximation (for easy hand calculations) in that the actual sampling frequency is 44.1 kHz, not 40 kHz.)



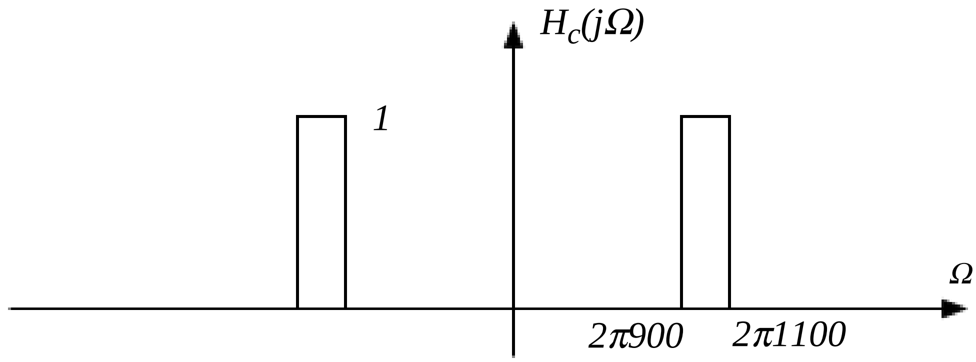
The continuous-time lowpass filter $H_{LP}(j\Omega)$ has the transfer function

$$H_{LP}(j\Omega) = \begin{cases} 1, & |\Omega| < 2\pi 20000 \\ 0, & \text{otherwise} \end{cases}$$

You may assume that the resulting discrete-time function $x[n]$ has the following DTFT.



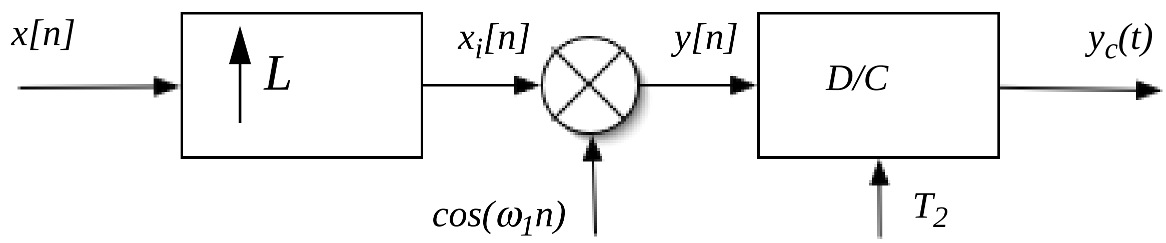
You have licensed the rights to use a channel in the radio spectrum with center frequency 1000 Hz (yes, I said low frequencies!) and bandwidth 200 Hz. This looks like the following in the frequency domain:



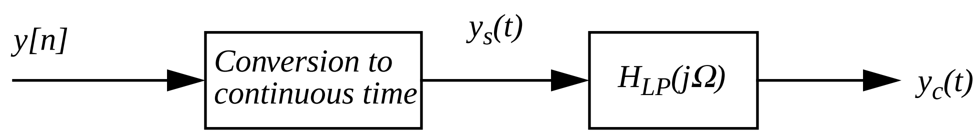
Specifically,

$$H_c(j\Omega) = \begin{cases} 1, & 2\pi 900 < |\Omega| < 2\pi 1100 \\ 0, & \text{otherwise} \end{cases}$$

Your service will be based on the following type of signal processing:



As we discussed in class, the D/C conversion is accomplished by converting the output signal $y[n]$ into continuous time according to the processing shown below.



In the figure above, $y[n]$ is first converted into a continuous-time sequence of impulses according to the relation

$$y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\delta(t - nT_2)$$

The continuous-time lowpass filter used in the D/C conversion process has the transfer function

$$H_{LP}(j\Omega) = \begin{cases} T_2, & |\Omega| \le \pi/T_2 \\ 0, & \text{otherwise} \end{cases}$$

The upsampling operation is also performed as discussed in class. Specifically, $L-1$ samples of value zero are inserted in between successive samples of $x[n]$. This “expanded” signal is then passed through an ideal discrete-time lowpass filter with frequency response for $0 \le |\omega| < \pi$

$$H_i(e^{j\omega}) = \begin{cases} L & |\omega| \le \pi/L \\ 0, & \text{otherwise} \end{cases}$$

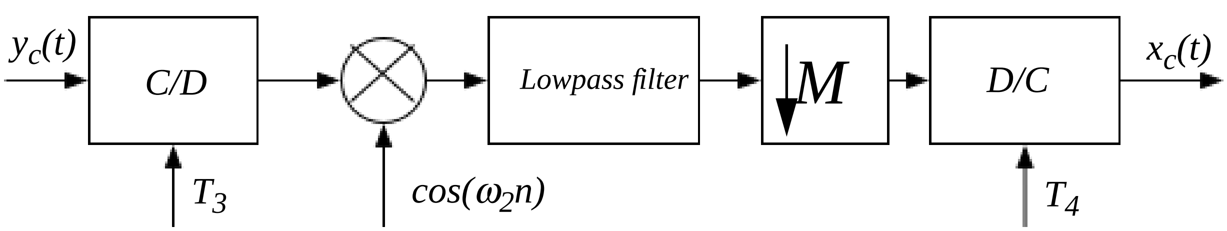
* * * * * *

(a) Sketch and dimension $X_i(e^{j\omega})$, $Y(e^{j\omega})$, and $Y_c(j\Omega)$, the DTFTs of $x_i[n]$, $y[n]$, and the CTFT of $y_c(t)$ for the parameter values $L = 4$, $\omega_1 = \pi/2$, and $T_2 = \frac{1}{4000}$.

(b) If you worked the problem correctly, you should have obtained a spectrum $Y(j\Omega)$ that is centered around $2\pi 1000$ but that is too “wide” for the channel specified by the function $H_c(j\Omega)$. It is claimed that this problem can be remedied by changing the value of **one** of the parameters L, ω_1, and T_2. Determine the new parameter value that would enable the signal with spectrum $Y(j\Omega)$ to be passed through $H_c(j\Omega)$ without alteration of the signal.

(c) With the series of operations above you have actually passed a 20,000-Hz signal through a 100-Hz bandpass channel! But this would not be very valuable if the original signal could not be recovered and played.

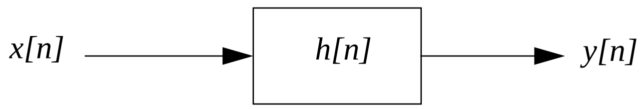
It is claimed that the following sequence of operations could recover the original signal $x_c(t)$. This is essentially the inverse of the combination of operations described in part (a) and in the definition of the original sampling process.



Considering this system as the inverse of the system you analyzed previously as well as the inverse of the original sampling process, obtain values of T_3, T_4 ω_2, the gain and cutoff frequency of the discrete-time lowpass filter, and the passband magnitude and cutoff frequency of the discrete-time lowpass filter.

If you did not complete part (b), work this part using the parameter values the parameter values $L = 4$, $\omega_1 = \pi/2$, and $T_2 = \frac{1}{4000}$ specified in part (a) and ignore the effect of $H_c(j\Omega)$. If you completed part (b), use the modified parameter values that you developed in part (b).

Question 4 (17%):



A stable LSI system has an input $x[n]$ and an output $y[n]$. The transfer function of the system is

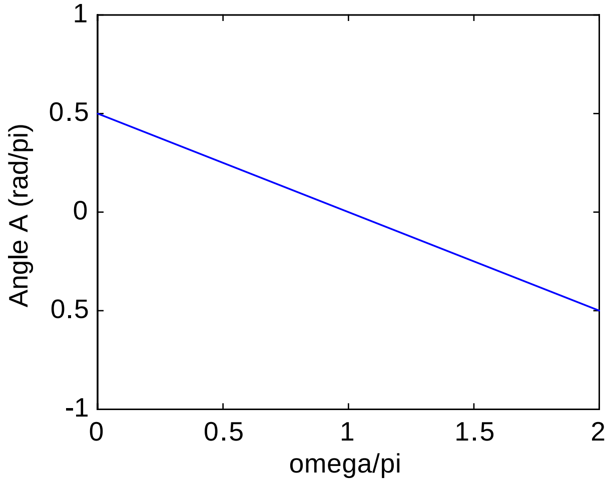
$$H(z) = \frac{z(z-1)}{\left(z-\frac{1}{2}\right)(z-2)}$$

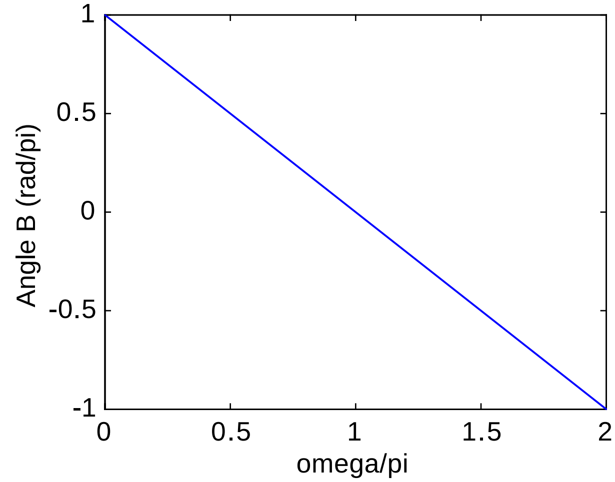
(a) What is the region of convergence of $H(z)$?

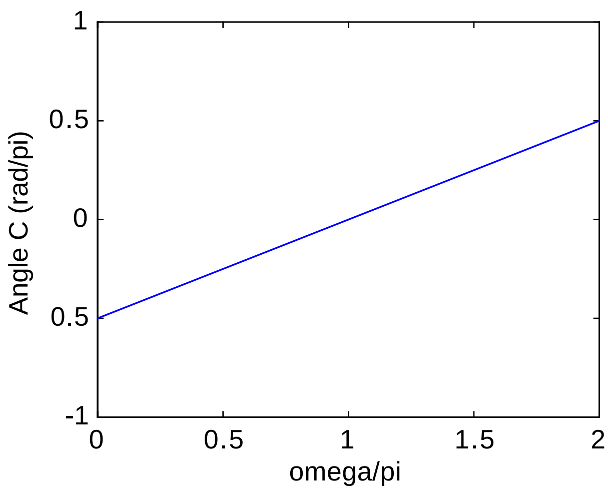
(b) Sketch and dimension the magnitude of the DTFT of $h[n]$, $H(e^{j\omega})$.

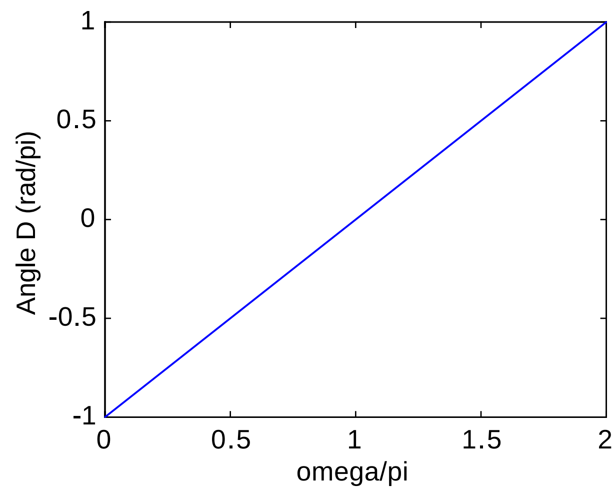
(c) Consider the four functions in the figure on the next page, which are potential plots of the phase of $H(e^{j\omega})$, the DTFT of $h[n]$.

Which of these functions do you consider to be closest to the actual phase of $H(e^{j\omega})$? (Please note that numerical values for both the horizontal and vertical axes in these figures are plotted in normalized fashion, divided by π. For example, the actual value of Angle A as ω approaches 2π radians is.$-\pi/2$ You must explain your reasoning to obtain full credit.


Angle A (rad/pi)
omega/pi
1
0.5
0
0.5
-1
0
0.5
1
1.5
2


Angle B (rad/pi)
omega/pi
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2


Angle C (rad/pi)
omega/pi
1
0.5
0
0.5
-1
0
0.5
1
1.5
2


Angle D (rad/pi)
omega/pi
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2

TRANSFORM FORMULAE

Continuous-time Fourier transform (CTFT):

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t}d\Omega \qquad X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$

Continuous-time Fourier series (CTFS):

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\Omega_0 t};\ T_0 = 2\pi/\Omega_0 \qquad X[k] = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-jk\Omega_0 t}dt$$

Discrete-time Fourier Transform (DTFT):

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega \qquad X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Z-transform:

$$x[n] = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz \qquad X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Discrete Fourier Series/Discrete Fourier transform (DFS/DFT):

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N} \qquad X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$