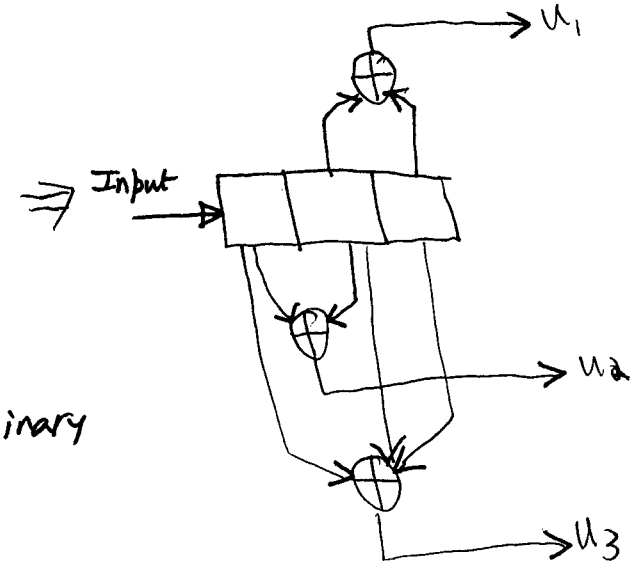
18-550 HW#6 Solutions

Problem 1 (6.1 in Sklar)

$g_1(X) = X + X^2$

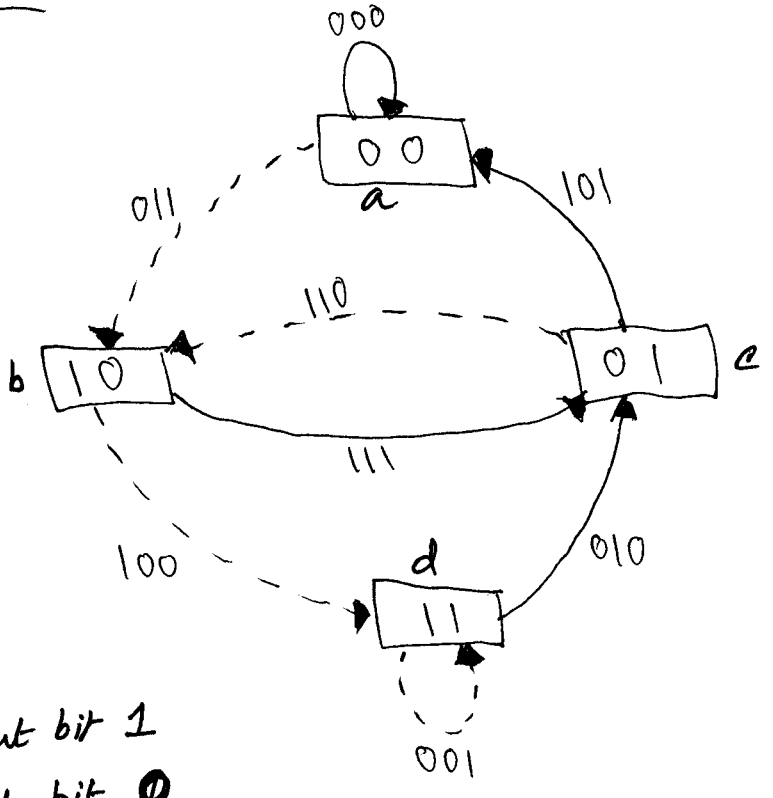
$g_2(X) = 1 + X$

$g_3(X) = 1 + X + X^2$



All adders are binary adders

State Diagram



– – – input bit 1

——— input bit 0

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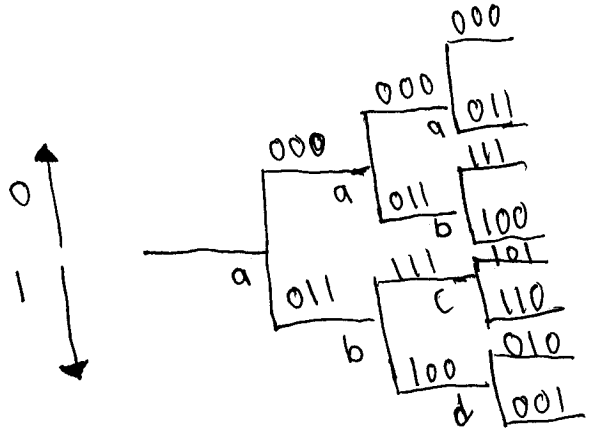
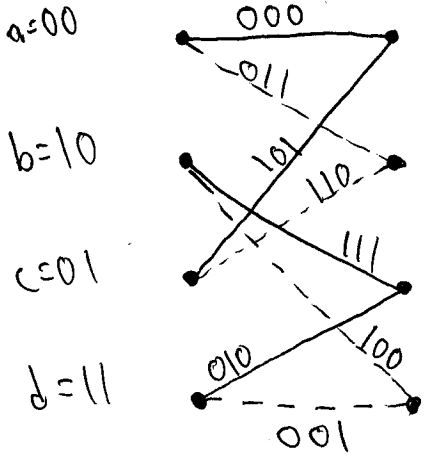
(6.1 cnt'd)

Tree Diagram



Trellis Diagram



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Problem 2 (6.3 in Sklar)

$K = 3$ $g_1(X) = 1 + X + X^2$

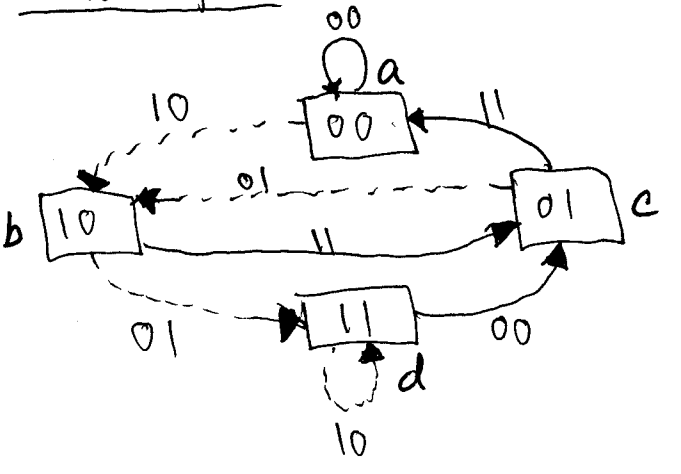
$g_2(X) = X + X^2$

rate $= \frac{1}{2}$

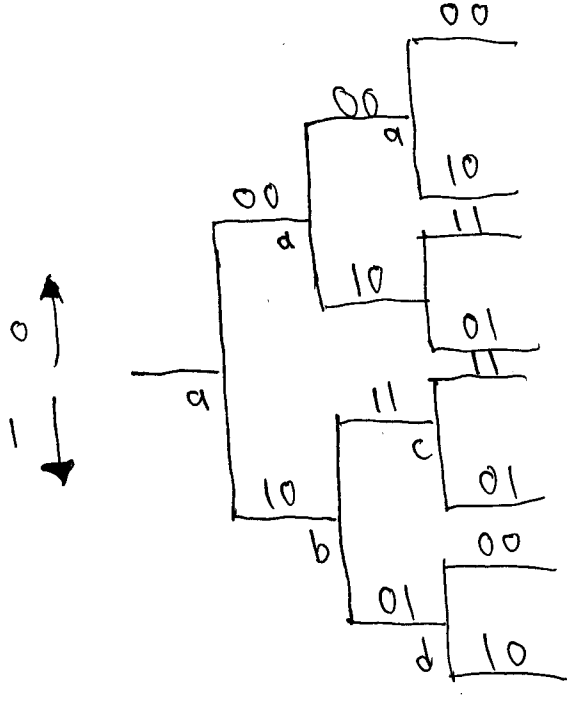
——— input bit 0

— — — input bit 1

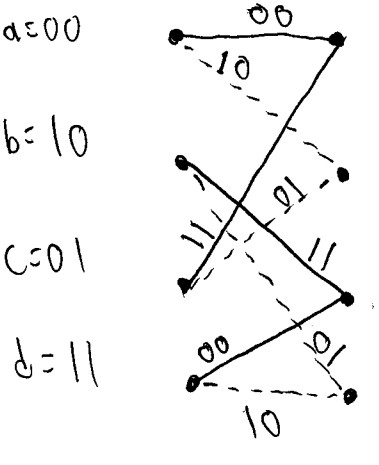
State Diagram



Tree Diagram



Trellis Diagram



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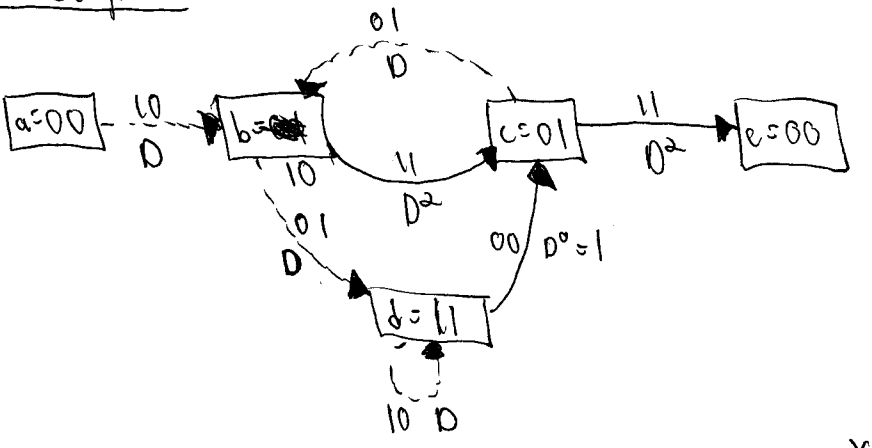
Problem 3 (6.8 in Sklar)

Using the results from Problem 3:

modified state diagram



state equations

$$X_b = DX_a + DX_c$$

$$X_c = D^0 X_b + X_d$$

$$X_d = DX_b + DX_d$$

$$X_e = D^2 X_c$$

$X_d = DX_b + DX_d$

$\Rightarrow X_d = \frac{D}{1-D} X_b$

$X_b = DX_a + DX_c$

$\Rightarrow X_a = \frac{1}{D} X_b - X_c$

$= \frac{1}{D} X_b - \left[D^2 X_b + \frac{D}{1-D} X_b\right]$

transfer function $T(D) = \frac{X_e}{X_a} = \frac{D^2 X_c}{X_a} = \frac{D^2\left[D^2 X_b + X_d\right]}{X_a}$

$= D^4 X_b + \frac{D^3}{1-D} X_b$

$= \frac{D^4 X_b + \frac{D^3}{1-D} X_b}{X_a}$

$= \frac{D^4 X_b + \frac{D^3}{1-D} X_b}{\frac{1}{D} X_b - D^2 X_b - \frac{D}{1-D} X_b}$

$= \frac{D^4 - D^5 + D^3}{\frac{1-D}{D} - (D^2 - D^3) - D}$

$= \frac{D^5 - D^6 + D^4}{1 - D - D^3 + D^4 - D^2}$

$T(D) = \frac{D^4 + D^5 - D^6}{1 - (D + D^2 + D^3 - D^4)}$

$= D^4 + 2D^5 + 2D^6 + \ldots$

So, $\boxed{df = 4.}$

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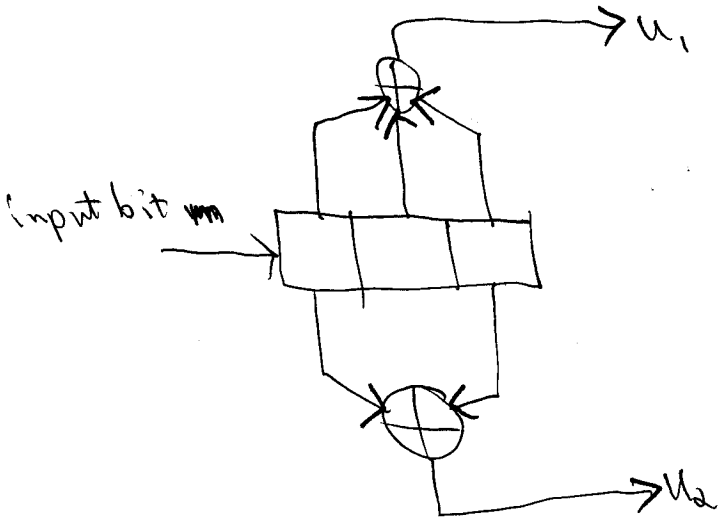
Problem 4 (6.10 in Sklar)

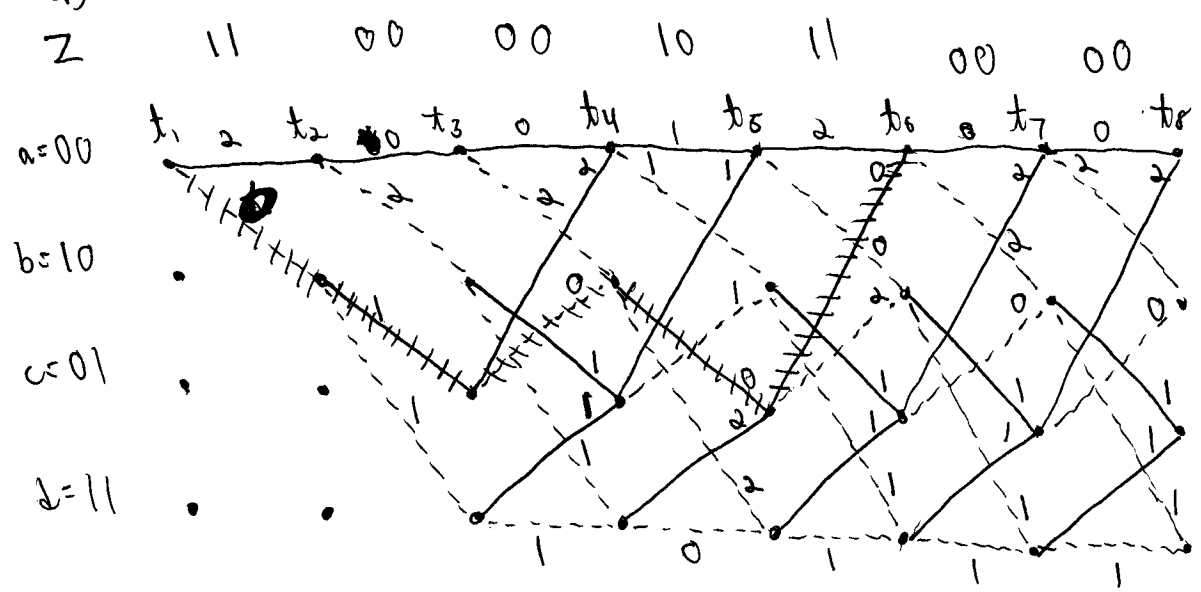
Figure 6.3 encoder ($K=3$, rate $=\frac{1}{2}$)



receive $Z=(11\,00\,00\,10\,11$ rest "0")

Use the Trellis diagram in Fig 6.7

a)



+++ = minimum difference path

b) if $m = 10100$ was the message, then the encoded sequence $U = 11\ 10\ 00\ 10\ 11$.

Since $Z = 11\ 00\ 00\ 10\ 11$,

↑ This bit is in error.

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Problem 5 (6.12 in Sklar)

$\frac{E_b}{N_0} = 6\,dB$ coherent BPSK rate $= \frac{1}{2}$ $\frac{E_c}{N_0} = r\frac{E_b}{N_0}$

From Section 6.4.4, for the Figure 6.3 encoder we have

$$\left.\frac{\partial T(D,N)}{\partial N}\right|_{N=1} = \frac{D^5}{(1-2D)^2}$$

By equation 6.21,

$$P_B \le Q\left(\sqrt{2d_f \frac{E_c}{N_0}}\right) e^{\left(d_f \frac{E_c}{N_0}\right)} \left.\frac{\partial T(D,N)}{\partial N}\right|_{N=1,\,D=e^{-\left(\frac{E_c}{N_0}\right)}}$$

From Section 6.4.1, $d_f = 5$.

$$\frac{E_c}{N_0} = \frac{1}{2}(6\,dB) = 3\,dB \approx 2.$$

$$P_B \le Q\left(\sqrt{2\times 5\times 2}\right) e^{(5\times 2)} \cdot \frac{e^{(-2)^5}}{(1-2\cdot e^{-2})^2}$$

$$= Q(\sqrt{20})\, e^{10} \cdot (8.535\times 10^{-5})$$

$$= Q(4.47)\times 1.88$$

$$\approx \frac{1}{4.47\sqrt{2\pi}} e^{-10} \times 1.88 = \boxed{7.6\times 10^{-6}}$$

b) Uncoded:

BPSK: $P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2\times 3.98}\right) = Q(2.82)$.

From Table B.1, $Q(2.82) = \boxed{2.4\times 10^{-3}}$

Performance Improvement $= \frac{2.4\times 10^{-3}}{7.6\times 10^{-6}} = \boxed{315.8}$

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Problem 6 (6.17 in Sklar)

rate = $\frac{2}{3}$ state contents are rightmost

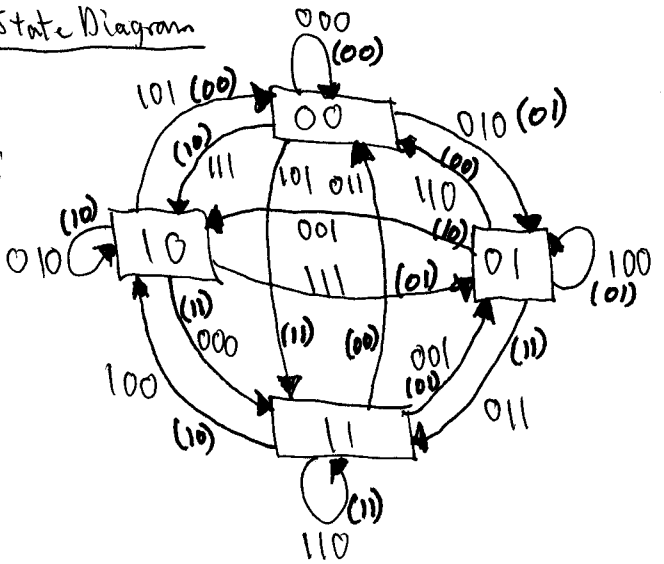
k = 2 K-1 = 1 2-bit registers

K = 2

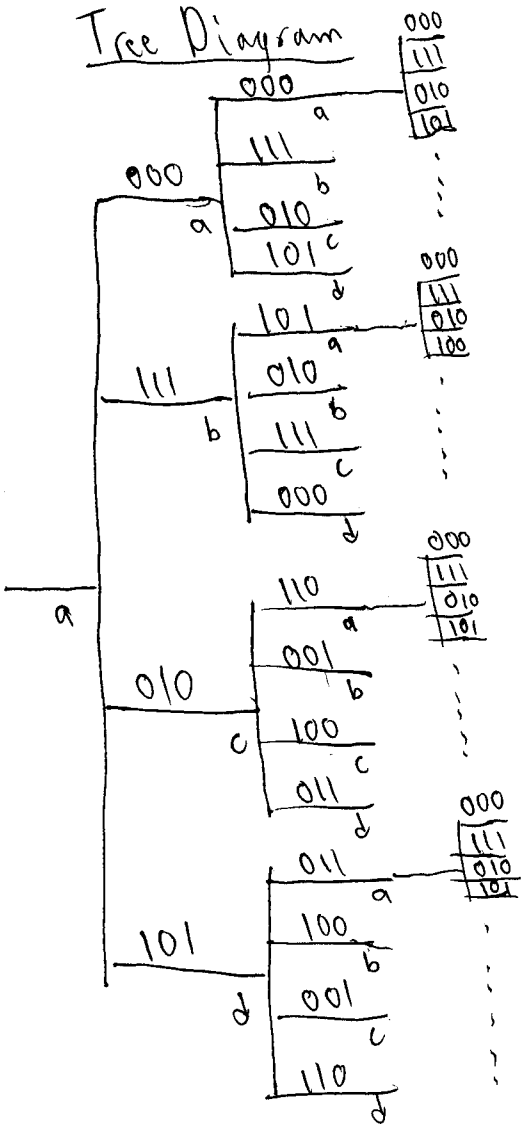
Since 2 input bits come in at a time, there are 4 outgoing branches from each state.

We show the input in parentheses

State Diagram



Tree Diagram



Trellis Diagram