Problem 1 (20 points) Please evaluate the shape-adaptive DCT (SA-DCT) of the following 8×8 image block

\[
\begin{array}{cccccc}
56 & 50 & & & & \\
80 & 70 & 40 & & & \\
40 & 50 & 77 & 74 & 43 & \\
21 & 60 & 70 & 40 & 50 & 30 \\
8 & 30 & 28 & 60 & 85 & 40 \\
15 & 80 & 90 & & & 30 \\
\end{array}
\]

where empty entries represent pels that are outside the object contour. Show the DCT coefficients as an 8×8 block. Note that not all the 64 coefficients will be present.

Note The \( N \)-point DCT of \( x(0), x(1), \ldots, x(N-1) \) is defined as

\[
y(m) = k_m \sum_{n=0}^{N-1} \cos \left[ \frac{(2n+1)m\pi}{2N} \right] x(n) \quad \text{where} \quad k_m = \begin{cases} \frac{\sqrt{1/N}}{\sqrt{2/N}} & \text{when } m = 0 \\ \frac{\sqrt{2/N}}{\sqrt{2/N}} & \text{otherwise} \end{cases}
\]

Problem 2 (80 points) Quantize (and dequantize) the DCT coefficients you get in Problem 1 by the stepsize \( Q \) without the dead zone, i.e., round each coefficient to the nearest multiple of \( Q \). Apply the Inverse SA-DCT to the quantized DCT coefficients to reconstruct the pel values (with rounding to the nearest integer). For \( Q = 4, 8, 16, \) and 32, show the reconstructed pel values as an 8×8 block similar to the one above in Problem 1, and compute the resulting PSNR (only for pels inside the object contour).

Note The \( N \)-point Inverse DCT of \( y(0), y(1), \ldots, y(N-1) \) is defined as

\[
x(n) = \sum_{m=0}^{N-1} k_m \cos \left[ \frac{(2n+1)m\pi}{2N} \right] y(m) \quad \text{where} \quad k_m = \begin{cases} \frac{\sqrt{1/N}}{\sqrt{2/N}} & \text{when } m = 0 \\ \frac{\sqrt{2/N}}{\sqrt{2/N}} & \text{otherwise} \end{cases}
\]