

Recitation 9

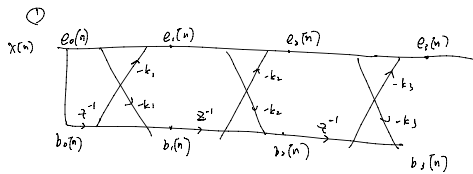
Friday, November 8, 2024 9:58 AM

① Lattice Filter (all-zero
all-pole)

② Estimate LPC Gain

③ Hint on Computer Homework

④ why $b \rightarrow$ backward ~~error~~ signal



$$e_i(n) = e_{i-1}(n) - k_i b_{i-1}(n-1) \quad \checkmark \quad \text{--- (1)}$$

$$b_i(n) = -k_i e_{i-1}(n) + b_{i-1}(n-1) \quad \checkmark \quad \text{--- (2)}$$

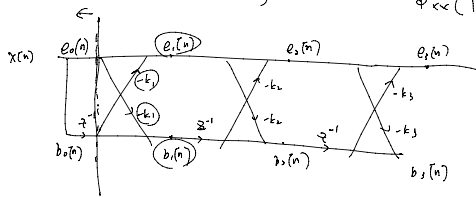
PARCOR

$$\hat{k}_i = E[e_i(n)] = E[(e_{i-1}(n) - k_i b_{i-1}(n-1))] = 0$$

$$\frac{\partial \hat{k}_i}{\partial k_i} = E[-2(e_{i-1}(n) - k_i b_{i-1}(n-1)) b_{i-1}(n-1)] = 0$$

$$E[e_{i-1}(n) b_{i-1}(n)] = k_i E[b_{i-1}^2(n-1)]$$

$$k_i = \frac{E[e_{i-1}(n) b_{i-1}(n)]}{E[b_{i-1}^2(n-1)]} \quad \checkmark = \frac{\phi_{ex}(0,1)}{\phi_{xx}(1,1)} = \frac{\phi_{ex}(1)}{\phi_{xx}(0)} \quad \text{--- (3)}$$

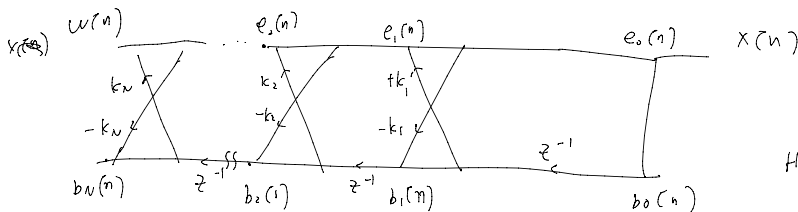
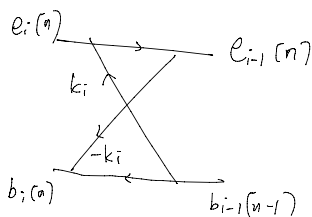


$$H(z) = A(z) = 1 - \sum_{k=1}^P a_k z^{-k}$$

All-pole

$$e_{i-1}(n) = e_i(n) + k_i b_{i-1}(n-1)$$

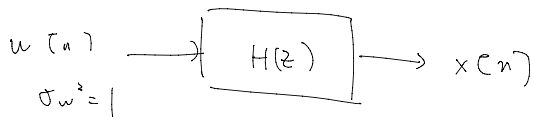
$$b_i(n) = -k_i e_{i-1}(n) + b_{i-1}(n-1)$$



$$H(z) = \frac{1}{A(z)}$$

stable if $|k_i| < 1 \quad \forall i$

Estimate the LPC Gain



$$H(z) = \frac{G}{1 - \sum_{k=1}^P \alpha_k z^{-k}}$$

YOLE-WALKER EQ , hw 7.4

$$\phi_{xx}(m) = \begin{cases} \sum_{k=1}^P \alpha_k \phi_{xx}(m-k) + G^2 \cdot \frac{1}{\sigma_w^2} & \text{for } m=0 \\ \sum_{k=1}^P \alpha_k \phi_{xx}(m-k) & \text{for } m > 1 \end{cases}$$

$$G^2 = (\phi_{xx}(0,0) - \sum_{k=1}^P \alpha_k \phi_{xx}(0,k)) / \sigma_w^2 \quad (\text{COVARIANCE})$$

$$= (\phi_{xx}(0) - \sum_{k=1}^P \alpha_k \phi(k)) / \sigma_w^2 \quad (\text{Autocorrelation})$$

$$H(z) = \frac{G}{A(z)} = \frac{X(z)}{W(z)}$$

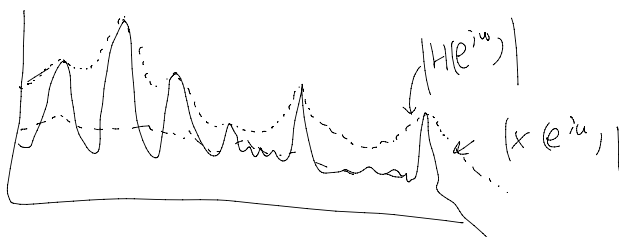
$$e[n] = x[n] - \sum_{k=1}^P \alpha_k x[n-k]$$

$$\underline{E(z)} = X(z) A(z) = G W(z)$$

$$A(z) = \frac{G}{H(z)}$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} |E(e^{j\omega})|^2 d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 |A(e^{j\omega})|^2 d\omega$$

$$= \frac{G^2}{2\pi} \int_{-\pi}^{\pi} \frac{|X(e^{j\omega})|^2}{|H(e^{j\omega})|^2} d\omega$$



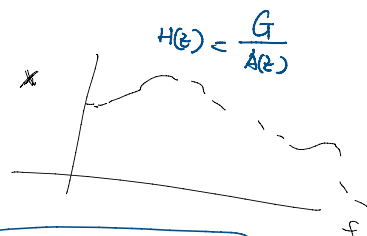
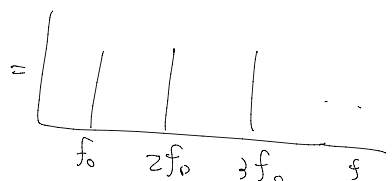
O.S. Fig 11.2

Problemset C9.2

$S(z)$

$w(t)$: excitation signal

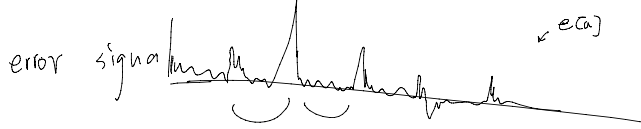
Voiced



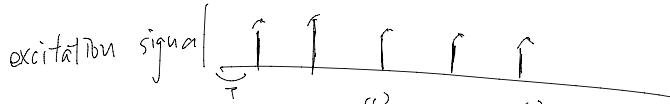
f_0 $2f_0$ $3f_0$

f_0 $2f_0$ $3f_0$ f

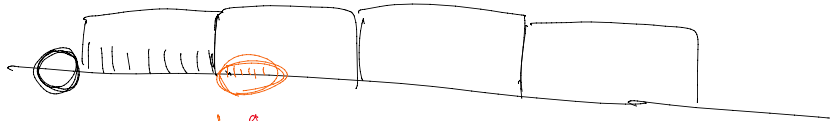
How to determine f_0



Use $f_{ee}(n)$ to calculate T T T
note for males voice $90 \leq f_0 \leq 160$ Hz



$W^{(1)}(z)$ $W^{(2)}(z)$
 $A^{(1)}$ $A^{(2)}$
 $\downarrow G^{(1)}$ $\downarrow G^{(2)}$



z_f , used as z_i for next frame, FIR version of OLA

impulses
 $\downarrow$
Voiced signal: $S(n) = w(z) * \frac{G}{A(z)}$
Unvoiced signal: $S(n) = w(z) * \frac{G}{A(z)}$
 $\uparrow$
white noise