10|25|24 PREPARATION FOR PROBLEM SET 7

CLASSICAL PSD EST -

ASSUME OBSERVE $x[n]$, $0 \le n \le N-1$

UNBIASED A.C EST

$$c'_{xx}[m] = \frac{1}{N-|m|-1} \sum_{n=0}^{N-|m|-1} x[n]\,x[n+m]$$

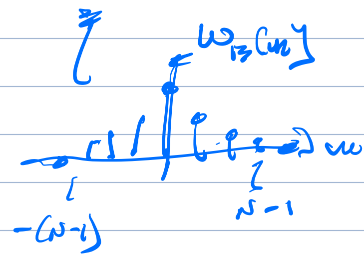
$$E\left[c'_{xx}[m]\right] = \phi_{xx}[m], \quad 0 \le |m| \le N-1$$

BIASED A.C EST.

$$c_{xx}[m] = \frac{1}{N} \sum_{n=0}^{N-|m|-1} x[n]\,x[n+m] = \frac{N-|m|-1}{N} \cdot c'_{xx}[m]$$

$$E\left[c_{xx}[m]\right] = \frac{N-|m|-1}{N} E\left[c'_{xx}[m]\right]$$

$$= \frac{N-|m|-1}{N} \cdot \phi_{xx}[m]$$



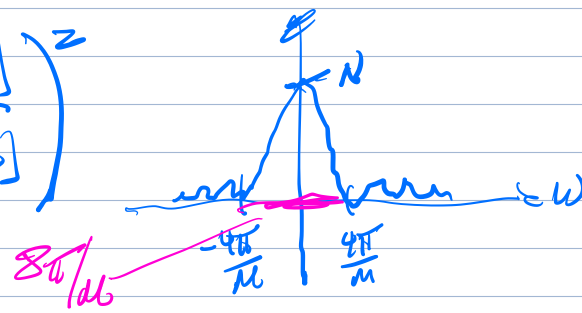
PSD FUNCTIONS

PERIODOGRAM $$I_N(\omega) = \sum_{m=-(N-1)}^{N-1} c_{xx}[m]\, e^{-j\omega m}$$

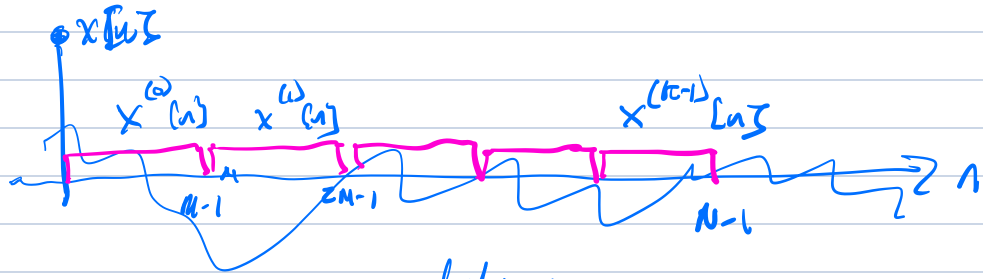
$$E\left[I_N(\omega)\right] = E\left[FT\{c_{xx}[m]\}\right] = FT\{E[c_{xx}[m]]\}$$

$$= \frac{1}{2\pi} W_B(e^{j\omega}) \circledast P_{xx}(\omega)$$

$$W_B(e^{j\omega}) = \frac{1}{N}\left(\frac{\sin(\omega N/2)}{\sin(\omega/2)}\right)^2$$



BARTLETT METHOD FOR REDUCING VARIANCE



Let $N = KM$

COMPUTE AVERAGE of PERIODOGRAMS FOR SEGMENTS

$$B_{xx}(\omega) = \frac{1}{K} \sum_{i=1}^{K} I_M^{(i)}(\omega)$$

$$I_M^{(i)}(\omega) = \frac{1}{M} \left| \sum_{n=0}^{N-1} x^{(i)}[n] e^{-j\omega n} \right|^2$$

$$E\left[B_{xx}(\omega)\right] = \frac{1}{2\pi} P_{xx}(\omega) \circledast \underset{\substack{\uparrow \\ \text{LENGTH } M}}{W_B(e^{j\omega})}$$

$$\text{VAR}\left[B_{xx}(\omega)\right] = \text{VAR}\left[\frac{1}{K} \sum_{i=1}^{K} \underbrace{I_M^{(i)}(\omega)}_{\text{SOME VAR}}\right]$$

$$\approx \frac{1}{K^2} \sum \text{VAR of } I_M(\omega)$$

$$= \frac{1}{K^{\cancel{2}}} \cdot \cancel{K} \cdot \text{VAR of } I_M(\omega)$$

SMOOTHING IN FREQUENCY

$$S_{xx}(\omega) = \frac{1}{2\pi} P_{xx}(\omega) \circledast W_s(e^{j\omega})$$

WANT REAL, NON-NEGATIVE

$\Updownarrow$

IDTFT $\quad s_{xx}[m] = \sum_{m=-(N-1)}^{N-1} w_s[m]\,\phi_{xx}[m]$

FOR $W_s(e^{j\omega})$ TO BE REAL, NON-NEG

REQUIRE $w_s[m]$ BE AN AUTO-CORRELATION FUNCTION

NEED $w_s[m] = w_s'[m] * w_s'[-m]$

but $\quad S_{xx}(\omega) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x[n]\,w_s'[n]\,e^{-j\omega n}\right|^2$

PSD ESTIMATION BY MODELING

(PARAMETRIC PSD ESTIMATION)

$$w[n] \longrightarrow \boxed{h[n]} \longrightarrow x[n]$$

$H(e^{j\omega}), H(z)$

$$\phi_{xx}[m] = \phi_{ww}[m] * \phi_{hh}[m]$$
$$= \phi_{ww}[m] * h[m] * h[-m]$$

$$P_{xx}(\omega) = P_{ww}(\omega) \cdot H(e^{j\omega}) \cdot H^*(e^{j\omega})$$

LSI FILTER

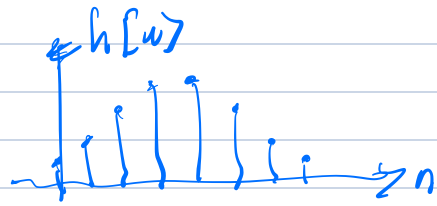
$$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1-\sum_{k=1}^{N} a_k z^{-k}} = \frac{B(z)}{A(z)}$$

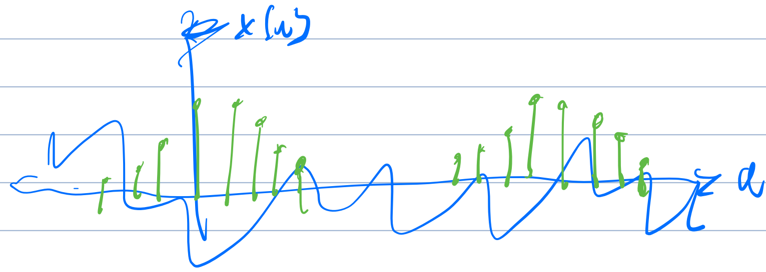
THREE MODELS

1. $H(z)$ ZEROS, NO POLES

MOVING-AVERAGE MODEL (MA)

$$H(z) = \sum_{\ell=0}^{M} b_\ell z^{-\ell}$$





2. $H(z)$ POLES, NO ZEROS

$$H(z) = \frac{G}{1-\sum_{k=1}^{N} a_k z^{-k}} = \frac{G}{A(z)}$$

AUTO REGRESSIVE MODEL (AR)

3. $H(z)$ POLES & ZEROS

AUTO-REGRESSIVE MOVING AVERAGE (ARMA)

$$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1-\sum_{k=1}^{N} a_k z^{-k}} = \frac{B(z)}{A(z)}$$

$$H(z) = \frac{\sum_{l=0}^{M} b_l z^{-l}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{X(z)}{W(z)}$$

AR MODEL:

$$\sum_{k=1}^{N} a_k \phi_{xx}[|m-k|] = \phi_{xx}[m], \quad m = 1, 2, 3, \ldots N$$

YULE-WALKER EQ.