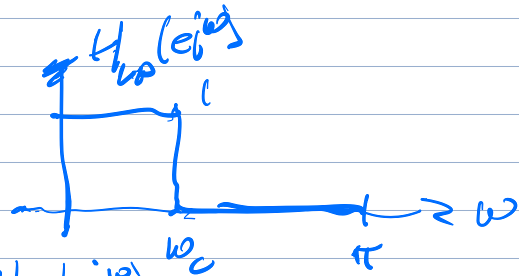
9/13/24

Recitation 3... Efficient Upsampling & Downsampling

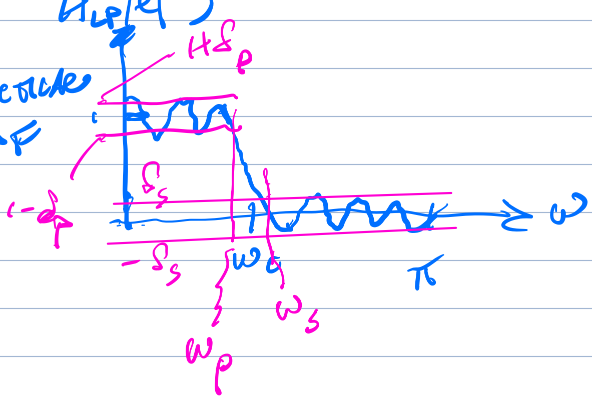
Practical Filter Design

$$h_{LP}[n] = \frac{\sin(\omega_c n)}{\pi n}, \quad \forall n$$

Ideal LPF



Practical LPF

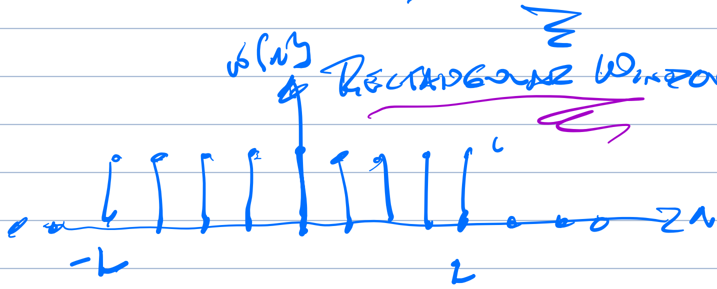


FIR Design Techniques

★ Window Design

$$h[n] = h_d[n] \cdot w[n]$$

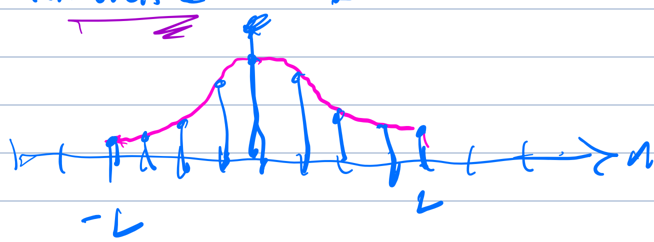
Rectangular Window



$$N = 2L+1$$

$$W(e^{j\omega}) = \frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

Hamming Window



OPTIMUM EQUIRIPPLE FILTER

(PARKS–McCLELLAN ALGORITHM)

[FILTER DESIGN DEMO USING filterDesigner IN MATLAB]

EFFICIENT DECIMATION + INTERPOLATION

DECIMATION $\quad x[n] \to \boxed{LPF} \xrightarrow{w[n]} \boxed{\downarrow M} \to y_d = w[mM] = x[mM]$ IF NO ALIASING

GAIN 1

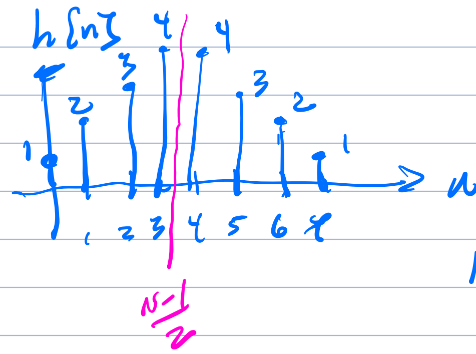
π/M

INTERPOLATION $\quad x[n] \to \boxed{\uparrow L} \to \boxed{LPF} \to y[m] = x_i[m] = x[m/L]$ m/L INTEGER

INSERT L−1 ZERO SAMPLES $\quad$ GAIN L $\quad \pi/L$

CONSIDER $L=4$

LET $N=8$

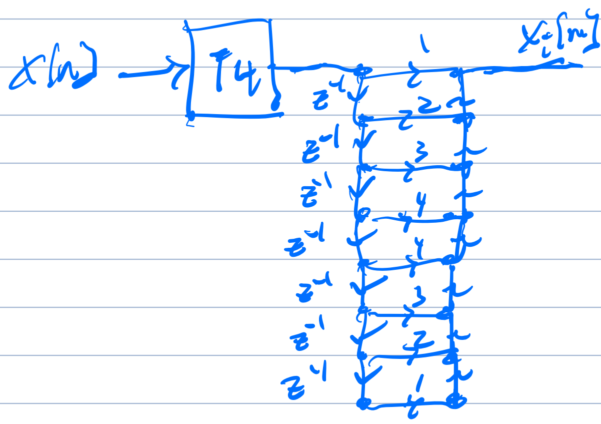


$h[n] = h[N-1-n]$

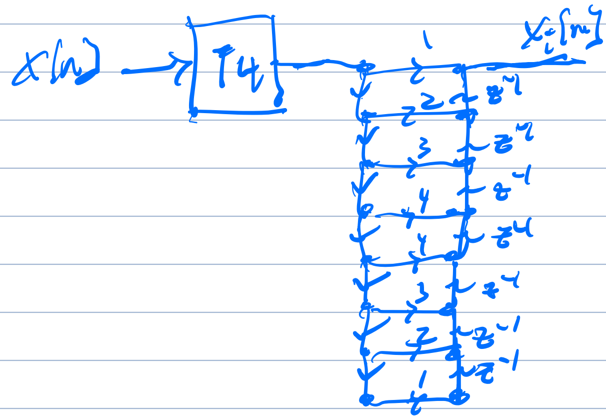
$H(e^{j\omega}) = |H(e^{j\omega})| e^{-j\omega 3.5}$

$\measuredangle H(e^{j\omega}) = -3.5\omega$

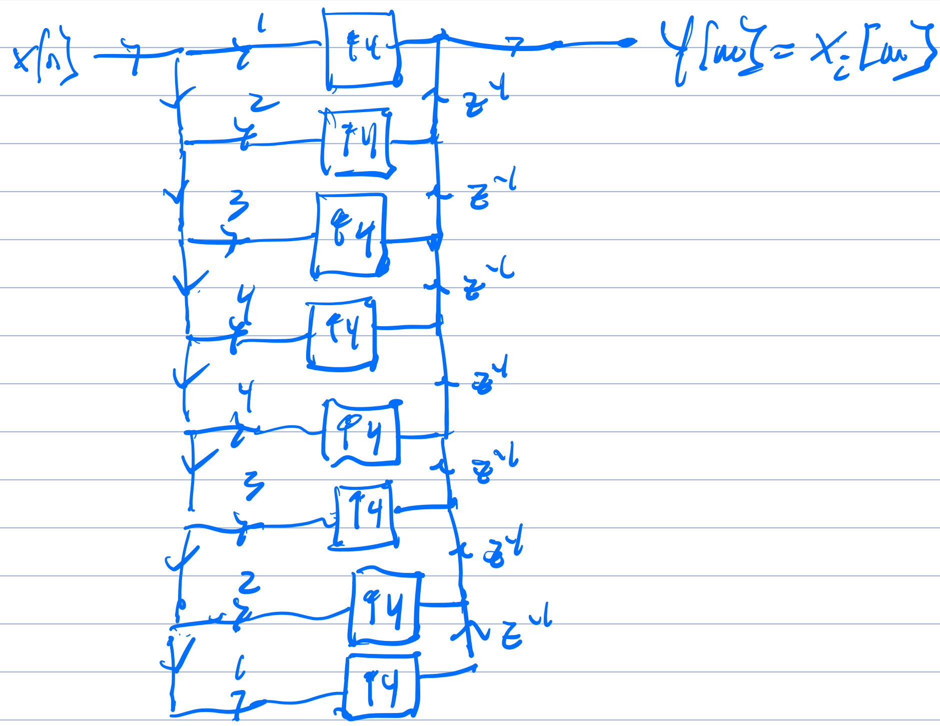
DIRECT-FORM IMPLEMENTATION



TRANSPOSED-FORM IMPLEMENTATION

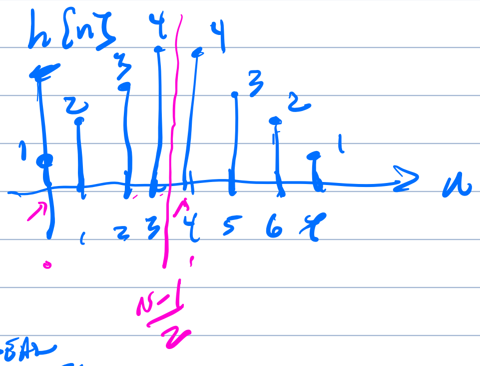


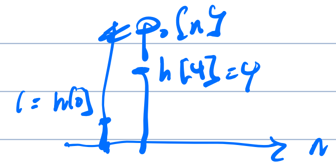
EFFICIENT IMPLEMENTATION

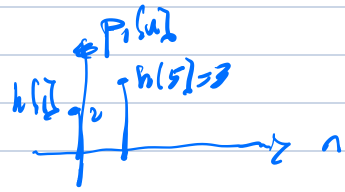


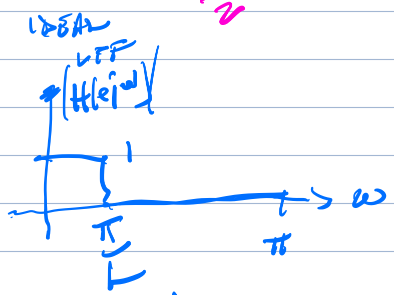
POLYPHASE IMPLEMENTATION

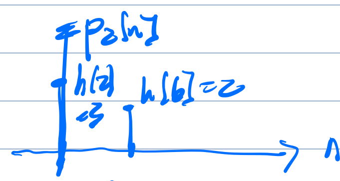
$$p_\ell[n] = h[nL + \ell], \quad 0 \le \ell \le L-1$$

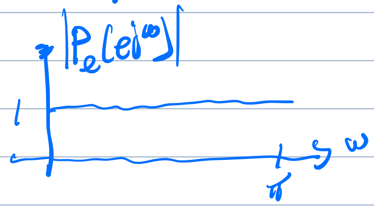


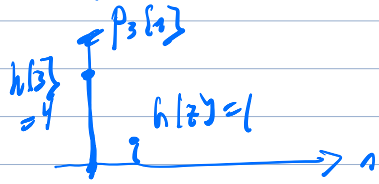


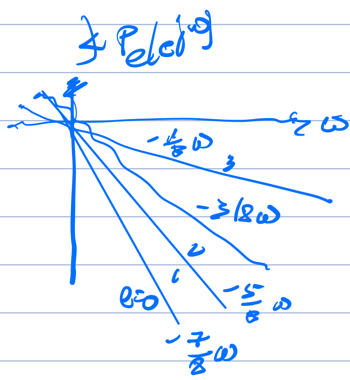










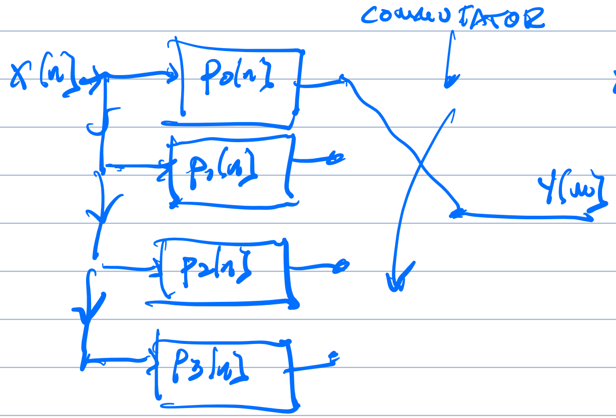


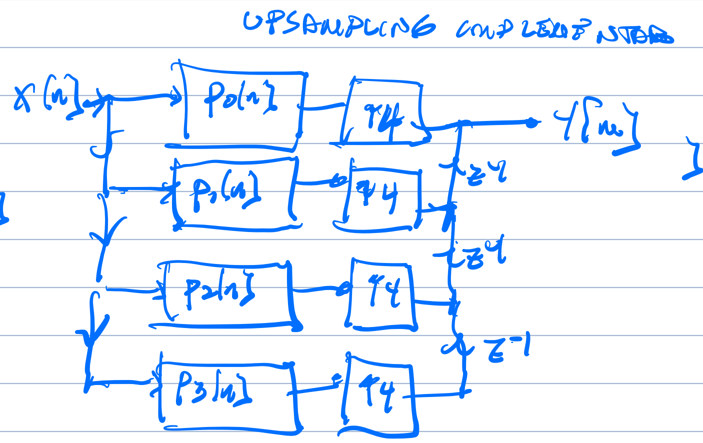
$$\angle P_\ell(e^{j\omega}) = -\frac{\omega}{L}\left(\frac{N-1}{2} - \ell\right)$$

$$\angle P_0(e^{j\omega}) = -\frac{\omega}{4} \cdot \frac{7}{2} = -\frac{7}{8}\omega \qquad \angle P_2(e^{j\omega}) = -\frac{\omega}{4}\left(\frac{3}{2}\right) = -\frac{3}{8}\omega$$

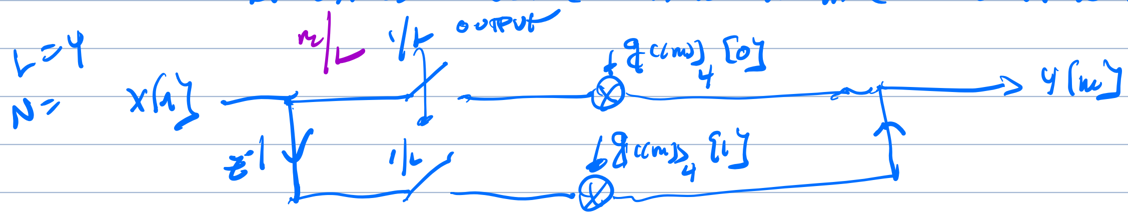
$$\angle P_1(e^{j\omega}) = -\frac{\omega}{4}\,\frac{5}{2} = -\frac{5}{8}\omega \qquad \angle P_3(e^{j\omega}) = -\frac{1}{8}\omega$$

POLY PHASE INTERPOLATOR



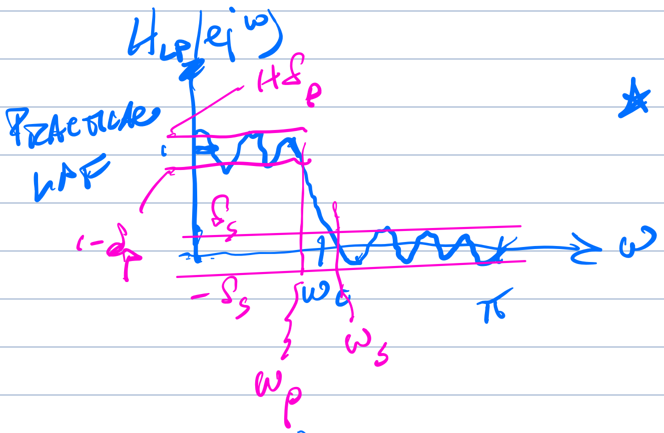


INTERPOLATION USING TIME-VARYING COEFFICIENTS



KAISER ESTIMATE FOR FILTER SIZE

$$N = \frac{-20\log_{10}\sqrt{\delta_p \delta_s} - 13}{14.6\,(\Delta\omega)/2\pi}$$



CONSIDER

★ PASSBAND RIPPLE ~1 dB

STOP BAND RIPPLE −80 dB

$\omega_p = .4\pi, \quad \omega_s = .45\pi$

$\Delta\omega = \omega_s - \omega_p = .05\pi$

$20\log_{10}(1-\delta_p) = -1$

$\log_{10}(1-\delta_p) = -1/20$

$1-\delta_p = 10^{-1/20}$

$\delta_p = 1 - 10^{-1/20} = .1087$

$20\log_{10}\delta_s = -80$

$\log_{10}\delta_s = -4$

$\delta_s = 10^{-4} = .0001$

$$N = \frac{-20\log_{10}\sqrt{\delta_p \delta_s} - 13}{(14.6)(\Delta\omega)/2\pi} = \frac{-20\log_{10}\left(\sqrt{(.0087)(.0001)}\right) - 13}{(14.6)(.05\cancel{\pi})/2\cancel{\pi}}$$

$$N \approx 100$$

MULTI RATE FILTER DESIGN

ORIGINAL

$$\longrightarrow \boxed{H(e^{j\omega})} \longrightarrow$$

MULT-RATE

$$\longrightarrow \boxed{H_{LP}(e^{j\omega})} \longrightarrow \boxed{\downarrow M} \longrightarrow \boxed{H'(e^{j\omega})} \longrightarrow \boxed{\uparrow M} \longrightarrow \boxed{H_{LP}(e^{j\omega})} \longrightarrow$$