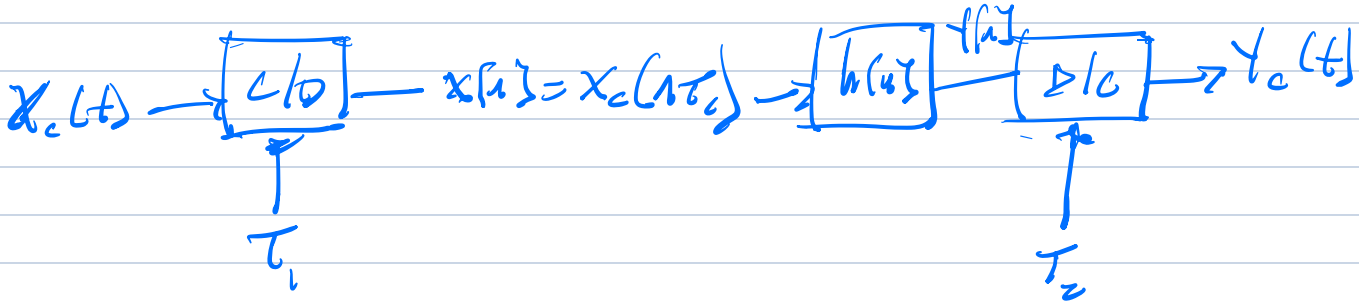
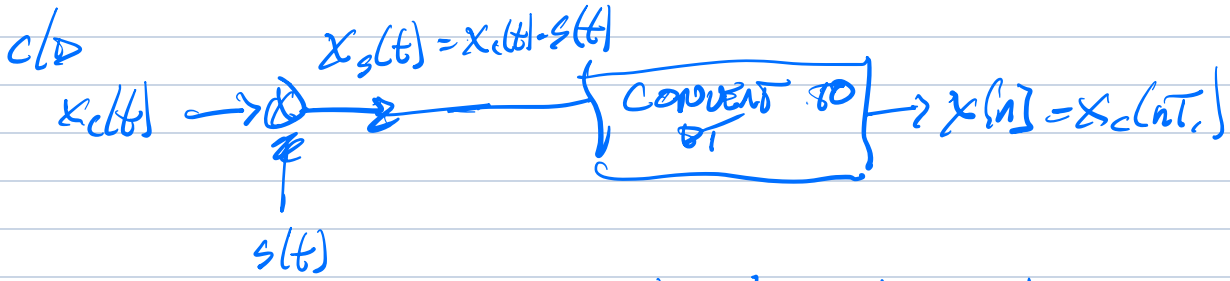
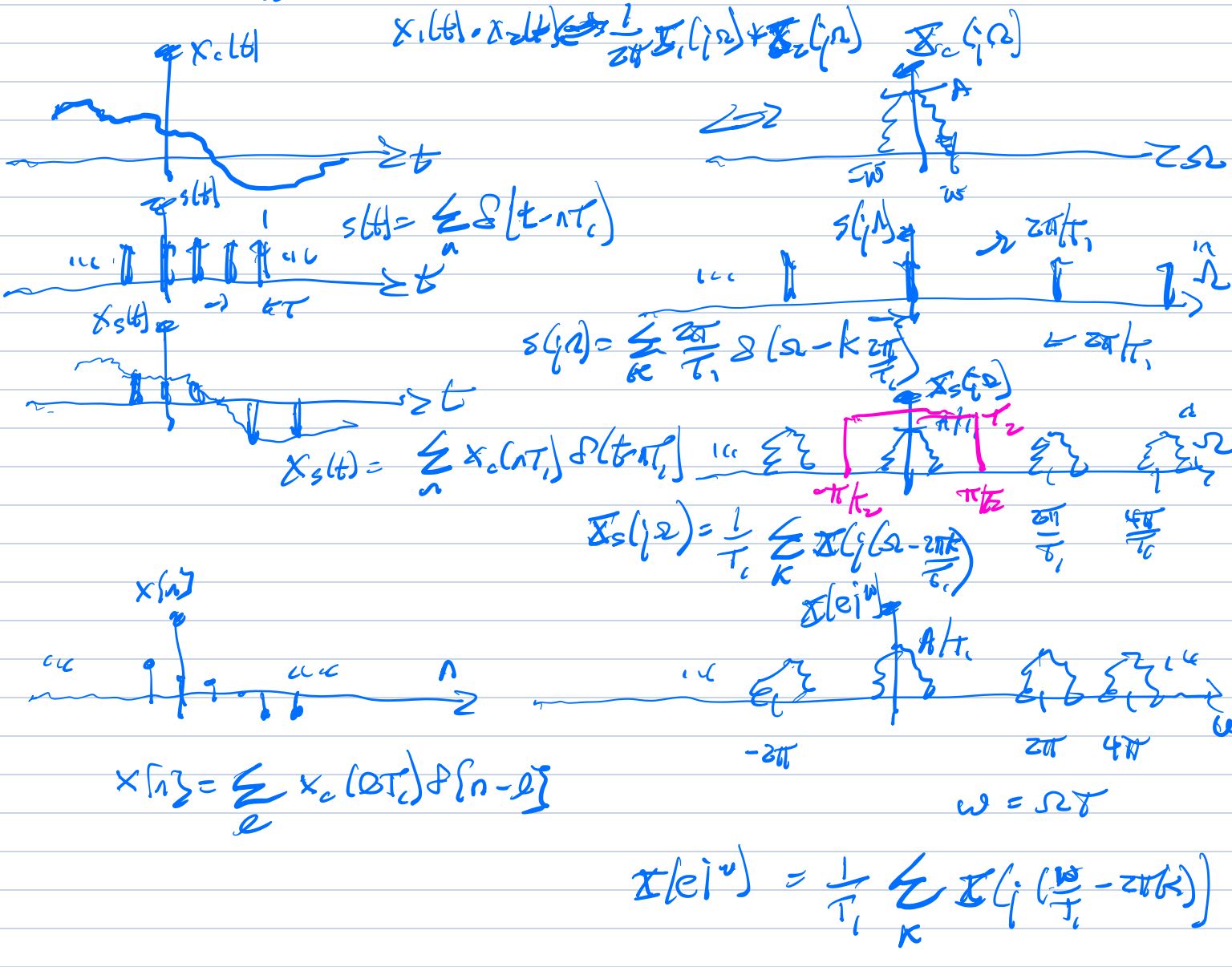
8/30/24 RECITATION 1

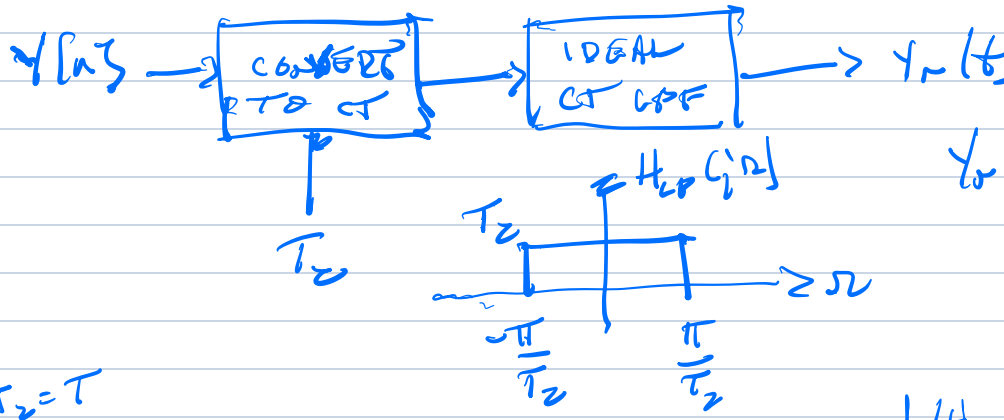
REVIEW OF CT/DT SAMPLING







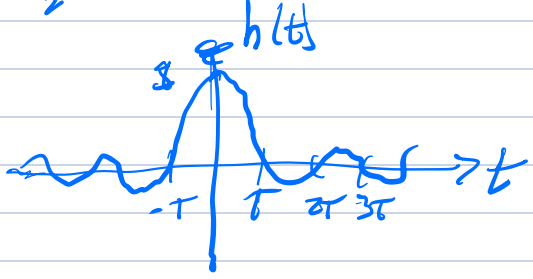
D/C CONVERSION $\quad y_s(t) = \sum_n y[n]\,\delta(t - nT_2)$



$y_r(t) = y_s(t) * h_{LP}(t)$

LET $T_1 = T_2 = T$

$$h_{LP}(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}}$$



$$y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t - nT) = y_s(t) * h_{LP}(t)$$

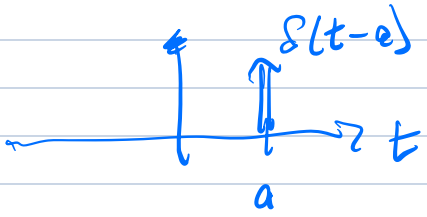
$$= \int_{-\infty}^{\infty} y_s(\tau)\, h_{LP}(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} y[n]\,\delta(\tau - nT)\, h_{LP}(t-\tau)\, d\tau$$

$$= \sum_{n=-\infty}^{\infty} y[n] \underbrace{\int_{-\infty}^{\infty} \delta(\tau - nT)\, h_{LP}(t-\tau)\, d\tau}_{h_{LP}(t - nT)}$$

$$= \sum_{n=-\infty}^{\infty} y[n]\, h_{LP}(t - nT)$$

PROPERTIES OF DELTA FUNCTIONS



$$\int_{-\infty}^{\infty} \delta(t-a)\,\phi(t)\,dt = \phi(a)$$

CONSIDER $\int_{-\infty}^{\infty} \delta(t)\,\phi(t)\,dt = \phi(0)$

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$$\int_{-\infty}^{\infty} \delta(2t)\,\phi(t)\,dt$$

let $2t = t'$; $t = \frac{t'}{2}$

$dt = \frac{dt'}{2}$

$$= \int_{-\infty}^{\infty} \delta(t')\,\phi\left(\frac{t'}{2}\right)\frac{dt'}{2} = \frac{1}{2}\int_{-\infty}^{\infty} \delta(t')\,\phi\left(\frac{t'}{2}\right)dt'$$

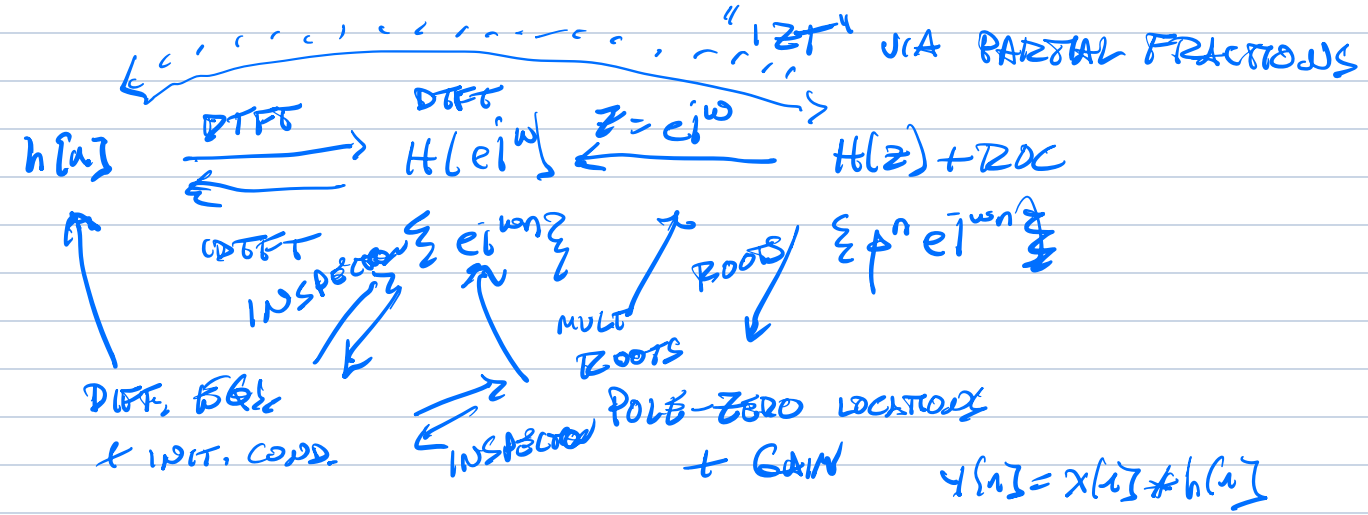
$$= \frac{1}{2}\phi(0)$$

$$\int_{-\infty}^{\infty} e^{-3t}\,\delta(4t-5)\,dt$$

let $t' = 4t$; $t = \frac{t'}{4}$, $dt = \frac{dt'}{4}$

$$= \frac{1}{4}\int_{-\infty}^{\infty} e^{-3t'/4}\,\delta(t'-5)\,dt' = \frac{1}{4}e^{-15/4}$$

REPRESENTATION OF DT SIGNALS + SYSTEMS

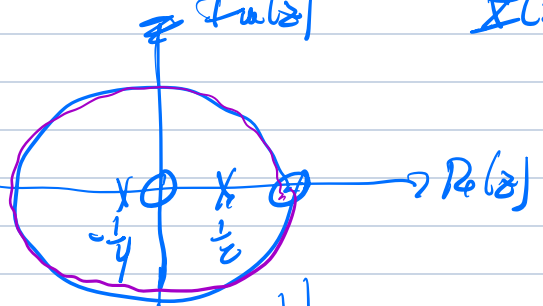


$$y[n] = x[n] * h[n]$$

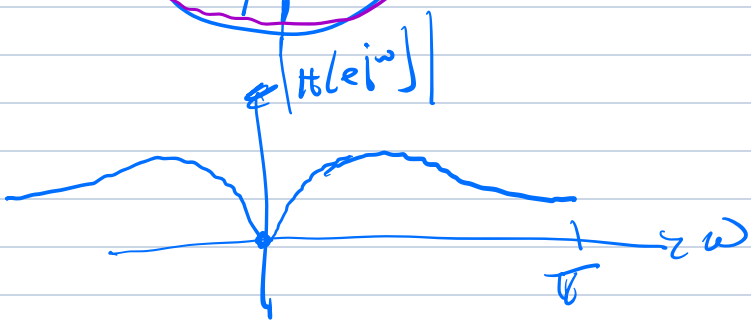
$$Y(z) = X(z) \cdot H(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

EX $H(z) = \dfrac{z(z-1)}{\left(z+\frac{1}{4}\right)\left(z-\frac{1}{2}\right)} = \dfrac{z^2 - z}{z^2 - \frac{1}{4}z - \frac{1}{8}}$



$$H(z) = \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}} = \frac{Y(z)}{X(z)}$$



$$Y(z) - z^{-1}Y(z) = X(z) - z^{-1}X(z) - \frac{1}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z)$$

$$y[n] - y[n-1] = x[n] - x[n-1] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2]$$

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n] - x[n-1]$$

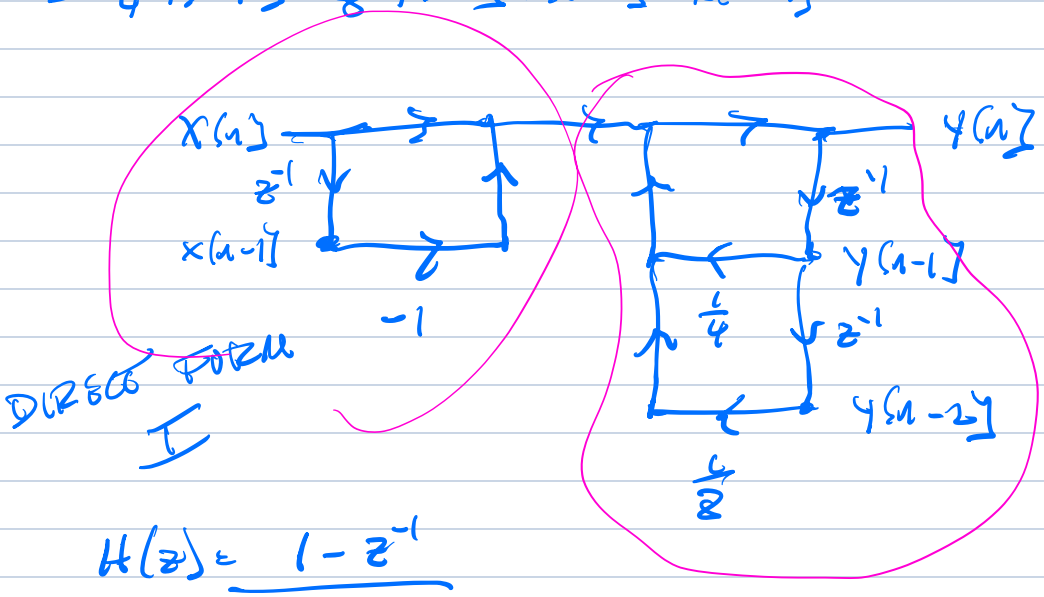
SIGNAL-FLOW DIAGRAMS

ADDITION

MULT. BY CONST.

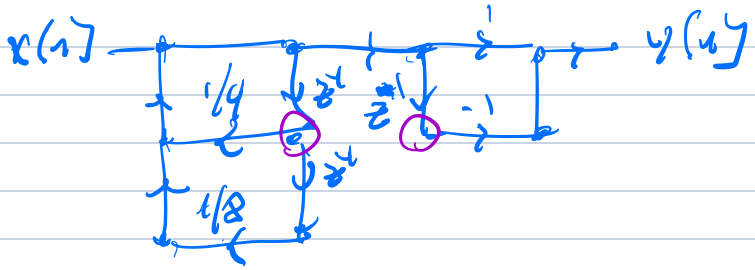
DELAY

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n] - x[n-1]$$

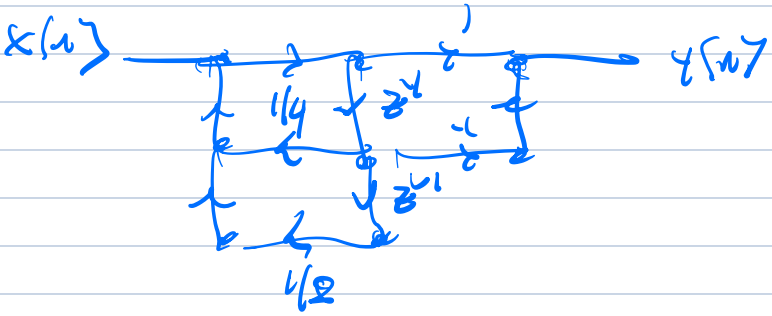


DIRECT FORM I

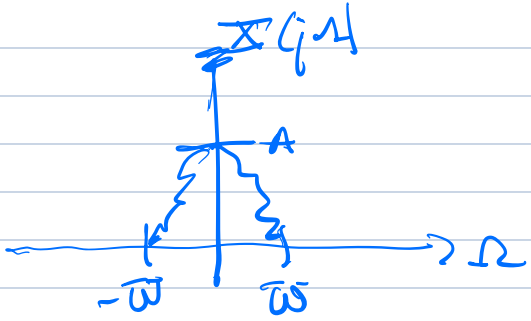
$$H(z) = \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$



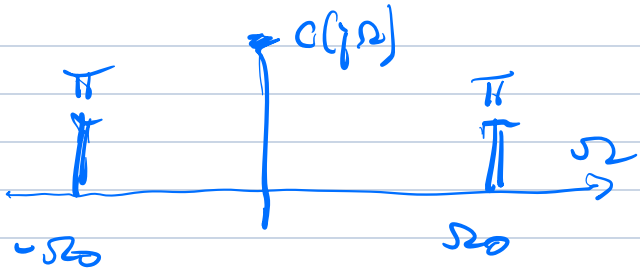
DIRECT FORM II CANONICAL FORM



BASIC DSB/SC



$$c(t) = \cos(\Omega_0 t) = \frac{e^{j\Omega_0 t} + e^{-j\Omega_0 t}}{2}$$



$$x_c(t) = x(t)\, c(t)$$

$$\Downarrow$$

$$X_c(j\Omega) = \frac{1}{2\pi} X(j\Omega) * C(j\Omega)$$