Problem 5.1:

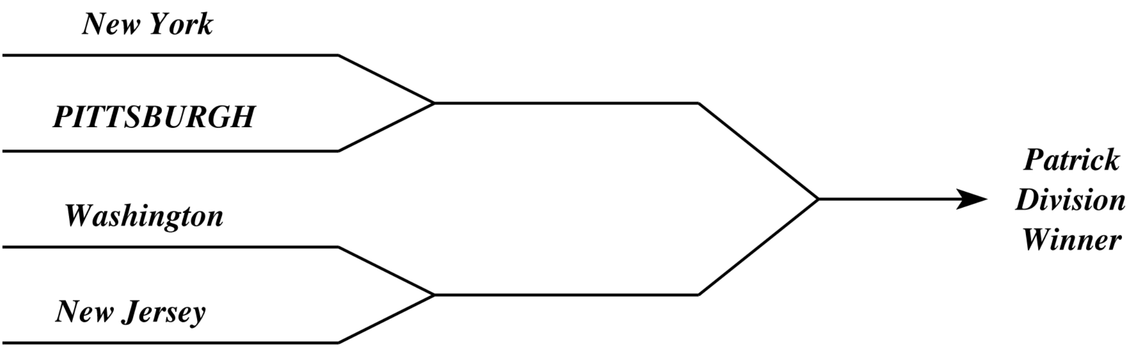
[**Note:** This problem originally appeared on last year's Quiz 2, right before the NHL strike began. It's a good thing that play actually resumed, as it's unlikely that we would have won the Stanley Cup with this approach.]

In the wake of the coming players' strike the board of directors of the National Hockey League (NHL) has commisioned the CMU ECE and statistics departments to propose a scheme by which the all-important Patrick Division Championship could be determined.

The blue-ribbon CMU commission has proposed a probabilistic algorithm based on the Patrick Division standings as of the end of play on March 30 (the last day for which statistics were available to the commission). These standings are:

1. New York Rangers	103
2. Washington Capitals	95
3. New Jersey Devils	85
4. **Pittsburgh Penguins**	84

The numbers in the right column refer to the number of points amassed by each team during the season. (Two points are awarded for each game won by a given team, and one point is awarded for each tie.)



The simulated "playoffs" are to be organized as according to the diagram above. Specifically, the Penguins will "play" a single "game" with the Rangers and the Capitals will "play" a single "game" with the Devils in the first round. The winners of each of these games will "play" each other for a single "game" to determine the overall Patrick Division Champion for 1992.

The winner of each "game" is a random event with the probabilities of the outcome determined solely by the number of points each time has on March 30. Let the probability that Team *i* beats Team *j* in a single "game" between the two deams be represented by the symbol P_{ij}. The commission has defined P_{ij} by the equation

$$P_{ij} = \frac{\text{Number of points amassed by Team } i}{\text{Number of points amassed by Team } i + \text{Number of points amassed by Team } j}$$

For example, the probability that the Penguins beat the Rangers in their first game is

$$P_{41} = \frac{84}{103+84} = .449$$

(There's no need to calculate numerical probabilities at this point, though.)

(a) What is the probability that the Pittsburgh Penguins win the Patrick Division? Express your answers in terms of the various symbols P_{ij}.

(b) You are told that the reigning Stanley Cup champion Pittsburgh Penguins have indeed won the Patrick Division. What is the probability that they played New Jersey in the finals? Express your answers in terms of the various symbols P_{ij}.

(c) It is now proposed that the initial rounds be played as a "best out of three" series of "games". In other words, the first team to win two "games" wins the series. (If either team wins both of the first two "games" the third "game" is simply not played.)

What is the probability that the Penguins beat the Rangers in a "best out of three" series of games? Express your answers in terms of the various symbols P_{ij}.

(d) [Optional for extra credit] Are the Penguins more likely to beat the Rangers if they play a single game or "best out of three"? Explain why you think the answer comes out the way it does.

Problem 5.4:

The probability density function for continuous random variable x is

$$f_x(\alpha) = \begin{cases} K, & a < x \le b \\ 0, & \text{otherwise} \end{cases}$$

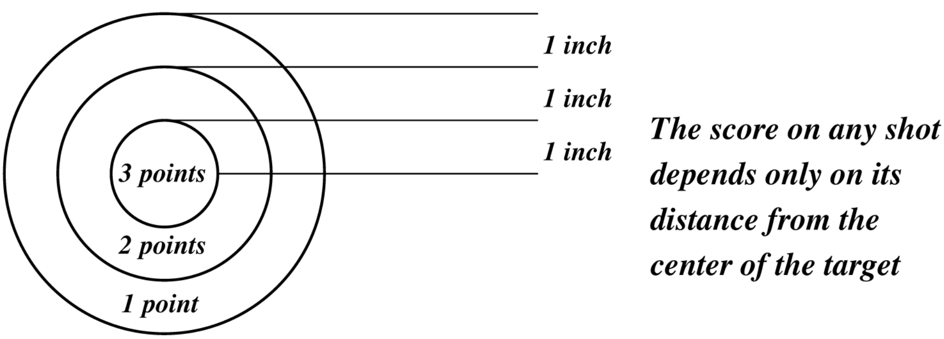
(a) Determine σ_x, the standard deviation of random variable x.

(b) Determine the conditional standard deviation of x given that $|x - E[x]| > \sigma_x$. (In other words, obtain the standard deviation of a new random variable that has the properties of the original variable x except that it is further restricted in range.)

(c) If $y = cx + d$, where c and d are constants, determine $E[y]$ and σ_y in terms of $E[x]$ and σ_x. Do your results depend on the form of the PDF for the random variable x?

Problem 5.5:

Each day Wyatt Uyrp shoots one "game" by firing at a target with the following dimensions and scores for each shot:



His pellet supply isn't too predictable, and the number of shopts for any day's game is equally likely to be one, two, or three. Furthermore, Wyatt tires rapidly with each shot. Given that it is the k^{th} pellet in a particular game, the value of R (the distance from target center to point of impact) for a pellet is a random variable with probability density function

$$f_{R|k}(\alpha | k = k_0) = \frac{1}{k_0} \text{ if } 0 \le \alpha \le k_0; \quad 0 \text{ otherwise}$$

(a) Determine and plot the PDF for random variable s_3, where s_3 is Mr. Uyrp's score on a three-shot game.

(b) Given only that a particular pellet was used during a three-shot game, determine and sketch the probability density function for R, the distance from the target center to where it hit.

(c) Given that Wyatt scored a total of exactly six points on a game, determine the probability that this was a two-shot game.

(d) If, in a randomly chosen game, we know only that at least one pellet hit exactly 1.5 inches from the center, determine the expected value of his score on this game.

(e) A particular pellet, marked at the factory, was used eventually by Wyatt. Determine the PDF for the number of points he scored on the shot which consumed this pellet.