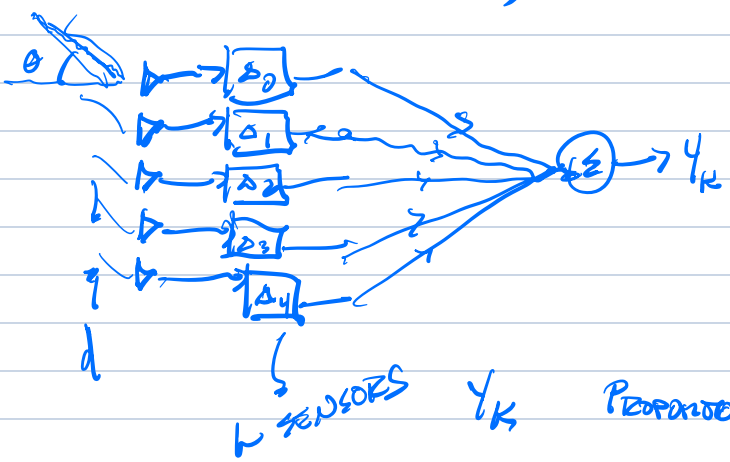
11/25/24 INTRO TO BROADBAND ADAPTIVE ARRAYS

(VAN CHAP 14, NOTES CHAP 9)



DIFFERENCE IN ARRIVAL TIME

$$\Delta T = \frac{d}{c}\sin\theta$$

$c \cong 340$ m/s

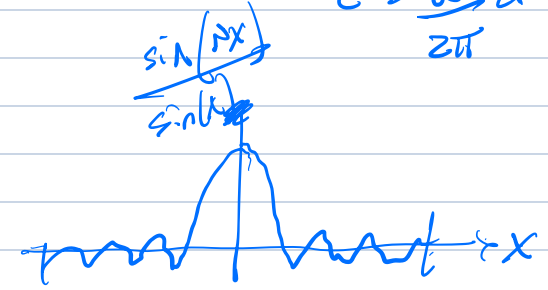
y_k PROPORTIONAL TO

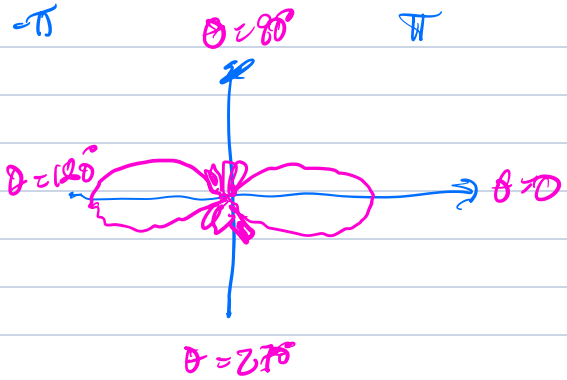
$$A(\omega) = \frac{\sin\left(\frac{k\omega d \sin\theta}{2c}\right)}{\sin\left(\frac{\omega d \sin\theta}{2c}\right)}$$

$A(\omega, \theta)$

$$c = \frac{\omega}{2\pi} d$$

$$= \frac{\sin\left(k\pi \frac{d}{\lambda}\sin\theta\right)}{\sin\left(\pi \frac{d}{\lambda}\sin\theta\right)}$$



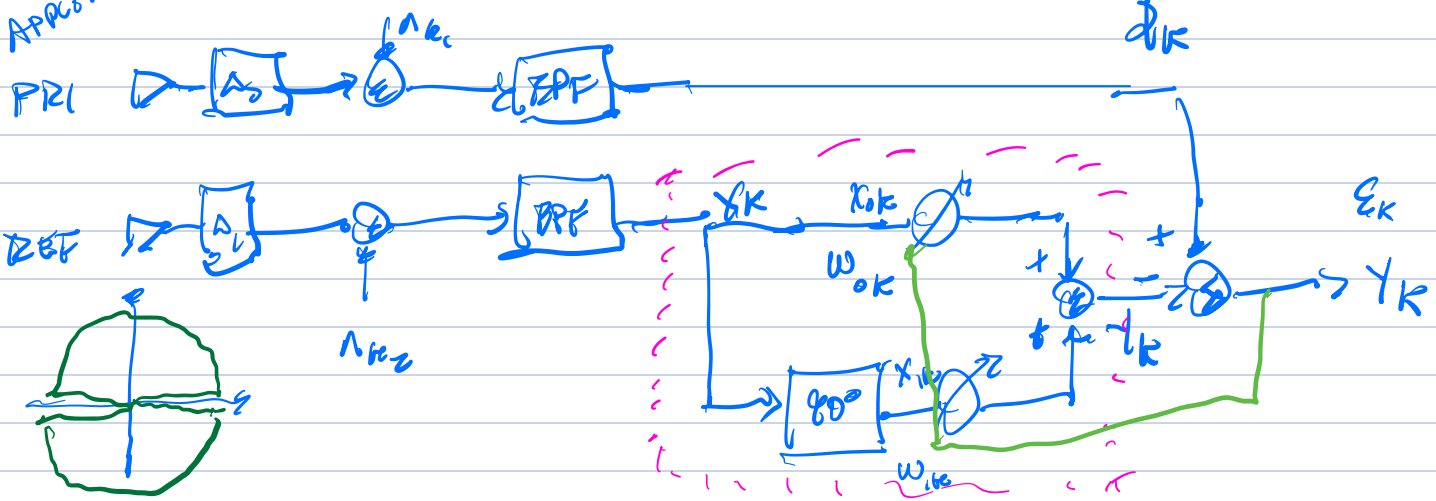


TO AVOID SPATIAL ALIASING

REQUIRE $d \le \frac{c}{2f \sin\theta}$

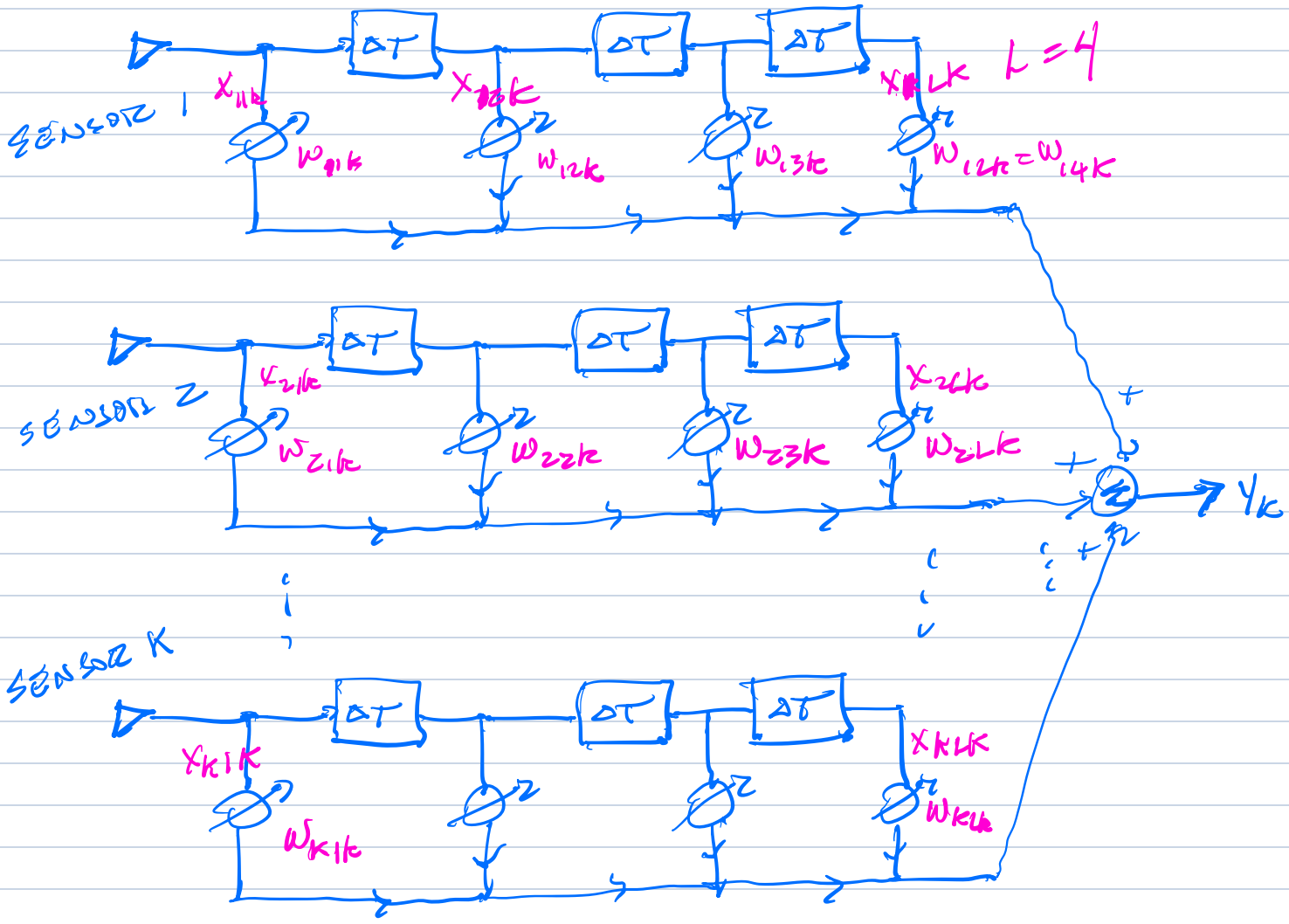
$$d \le \frac{c}{2f}$$

HOWEVER APPROACH NARROW BAND BEAMFORMING



PROBLEMS / LIMITATIONS	SOLUTIONS
1. SINGLE FREQ ONLY	REPLACE 2-CHANNEL CANCELLER WITH GENERAL FIR ADAPTIVE FILTER
2. CAN ONLY CANCEL SIGNALS FROM ONE DIRECTION	ADD MORE REFERENCE CHANNELS. WITH L CHANNELS, CAN CANCEL L-1 INTERFERING SOURCES
3. CAN ONLY DELAY SIGNALS BY INTEGER # of SAMPLES	INCREASE SAMPLING FREQ. OR UPSAMPLE INTERNALLY

FILTER-AND-SUM BEAMFORMING

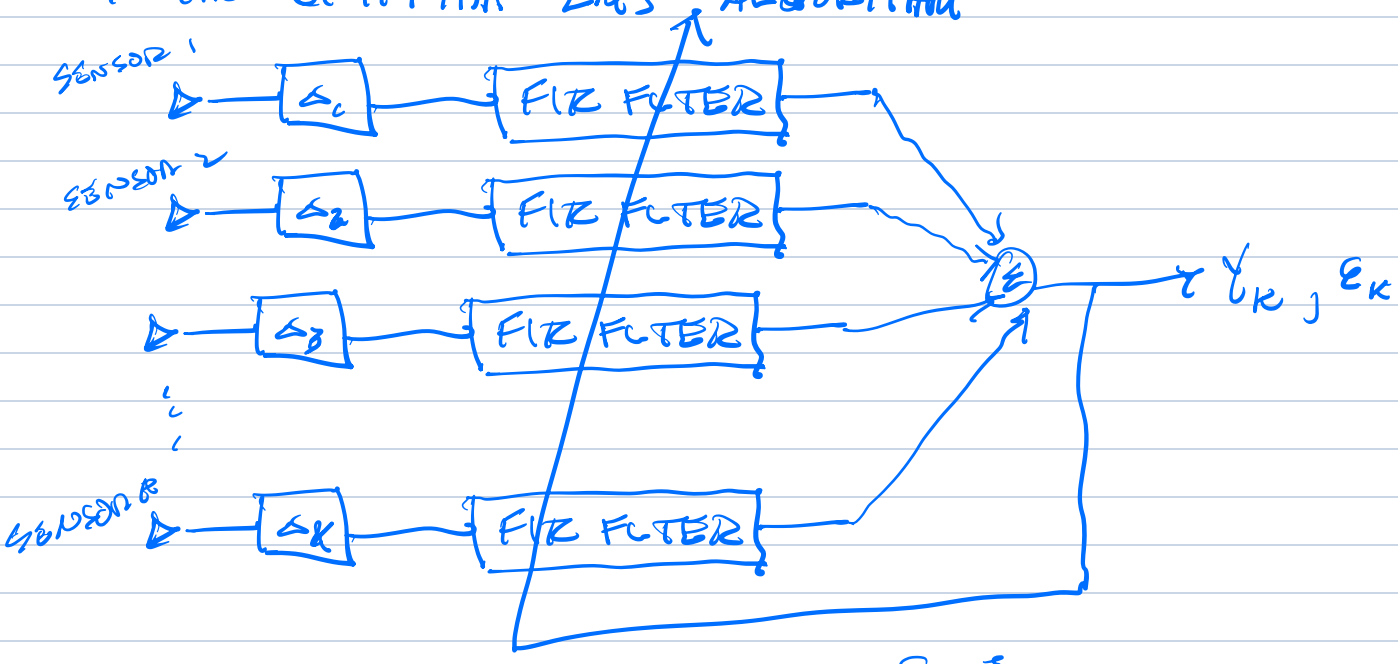


GOALS of ADAPTATION ALGORITHMS

1. MAINTAIN UNDISTORTED RESPONSE IN THE "LOOK" DIRECTION

2. CANCEL NOISE/INTERFERENCE FROM ELSEWHERE

1. THE GRIFFITHS LMS ALGORITHM



$$\min \xi^2 = E[\varepsilon_k^2] = E[y_k^2]$$

$$\underline{w}_k = \begin{bmatrix} w_{1,1,k} \\ w_{1,2,k} \\ \vdots \\ w_{1,L,k} \\ w_{2,1,k} \\ w_{2,2,k} \\ \vdots \\ w_{2,L,k} \\ \vdots \\ w_{K-1,L,k} \\ w_{K,1,k} \\ \vdots \\ w_{K,L,k} \end{bmatrix} \qquad \underline{x}_k = \begin{bmatrix} x_{1,1,k} \\ x_{1,2,k} \\ \vdots \\ x_{1,L,k} \\ x_{2,1,k} \\ x_{2,2,k} \\ \vdots \\ x_{2,L,k} \\ \vdots \\ x_{K-1,L,k} \\ x_{K,1,k} \\ \vdots \\ x_{K,L,k} \end{bmatrix}$$

GRIFFITHS LMS

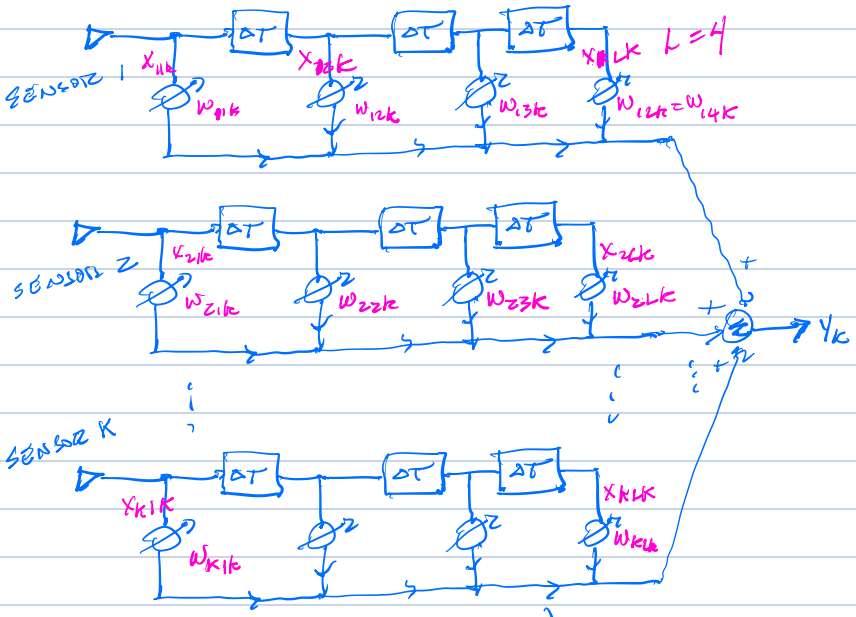
$d_k - y_k$

$$\underline{w}_{k+1} = \underline{w}_k + 2\mu \varepsilon_k \underline{x}_k$$

$$= \underline{w}_k + 2\mu d_k \underline{x}_k - 2\mu y_k \underline{x}_k$$

$$= \underline{w}_k + 2\mu \underline{p} - 2\mu y_k \underline{x}_k = \underline{w}_k + 2\mu (\underline{p} - \varepsilon_k \underline{x}_k)$$

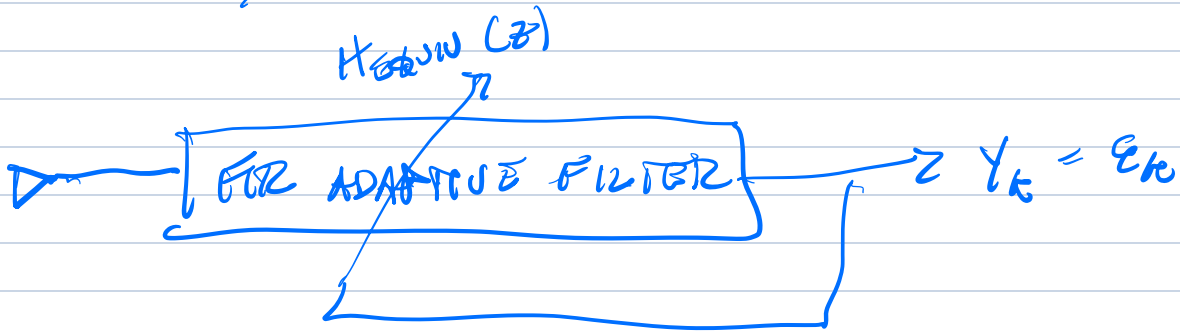
FROST ALGORITHM



ASSUME

$\sum_k \xi_k^2 = 0$

TARGET IS AT $\theta = 0$



FOR FROST

$$\underline{W}_k = \begin{bmatrix} W_{1k} & W_{21k} & \cdots & W_{L1k} \\ W_{12k} & W_{22k} & \cdots & W_{L2k} \\ \vdots & & & \\ W_{1Kk} & W_{2Kk} & & W_{LKk} \end{bmatrix}$$

let $\underline{C} = [\ \cdots\ ]$ $\underline{W}_k = \underline{W}_{EQUIV}$

WANT $\underline{C}^* = [1\ 0\ 0\ 0\ \cdots]$ FOR DISTORTIONLESS RESPONSE

FROST A26 :

1. $\underline{\underline{W}}_{k+\frac{1}{2}} = \underline{\underline{W}}_k + 2\mu\, y_k \underline{\underline{X}}_k$

LCMV CONSTRAINT VECTOR

2. $\underline{\underline{E}}_{k+\frac{1}{2}} = \frac{C}{K}\left[\underline{\underline{C}}^{(k)} - [1\,1\,1\,1]\,\underline{\underline{W}}_{k+\frac{1}{2}}\right]$

3. $\underline{\underline{W}}_{k+1} = \underline{\underline{W}}_{k+\frac{1}{2}} + \underline{\underline{E}}_{k+\frac{1}{2}}$ ROW BY ROW

$$\underline{W}_{k+1} = 2\mu \underline{\underline{X}}_k y_k + \underline{\underline{E}}_{k+\frac{1}{2}}$$

GRIFFITHS – JIM

GENERALIZED SIDELOBE CANCELLOR (GSC)