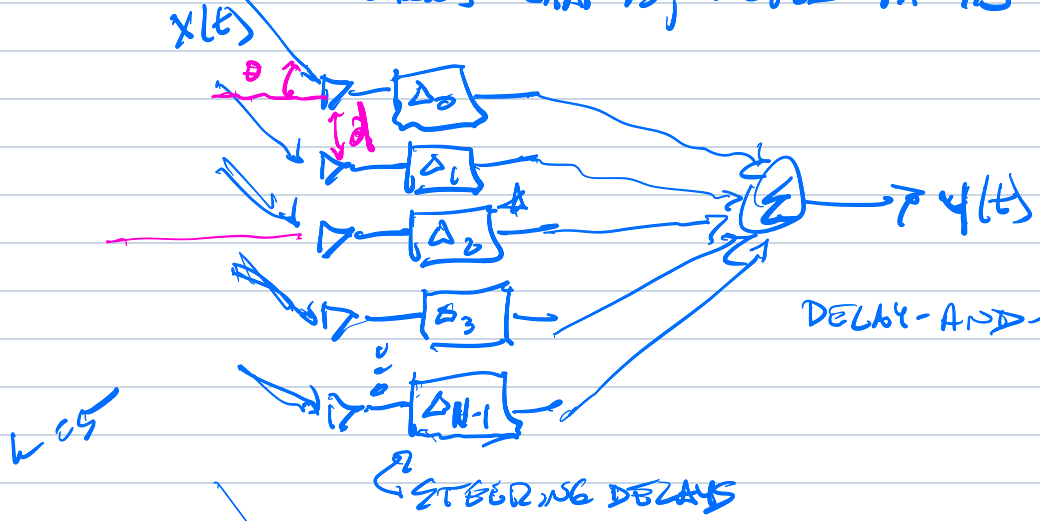
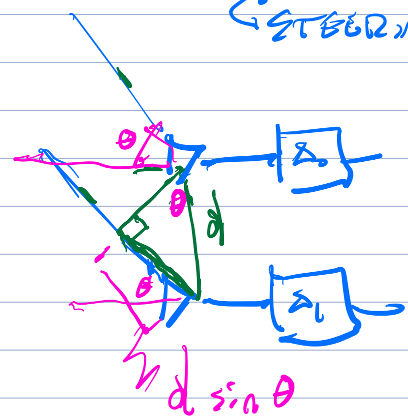
11/18/24 INTRO TO ADAPTIVE ARRAY PROCESSING

(NARROWBAND BEAMFORMING)

(WIDROW + STEARNS CHAP 13, NOTES 9.1-9.3)



DELAY-AND-SUM



DIFFERENCE IN DISTANCE

$= d \sin\theta$ meters

DIFFERENCE IN TIME

$\Delta t = \frac{d}{c} \sin\theta$ sec

$c \approx 340$ m/s

BEAM PATTERNS / POLAR RESPONSE

ASSUME $\{\Delta_k\} = 0$

LET $x(t) = \cos(\omega t) = \mathrm{Re}[e^{j\omega t}]$

$$y(t) = \sum_{k=0}^{N-1} \cos(\omega(t - k\Delta t))$$

COMPLEX AMPLITUDE FOR $x(t) = A\cos(\omega t + \phi)$

$$= \mathrm{Re}[A e^{j(\omega t + \phi)}]$$

$$= \mathrm{Re}[\underline{A e^{j\phi}} \cdot e^{j\omega t}]$$

Here $\quad y(t) = \sum_{k=0}^{N-1} \cos(\omega(t - k\Delta t))$

$$= \sum_{k=0}^{N-1} \mathrm{Re}\left[e^{i\omega t} \cdot e^{-i k\Delta t} \right]$$

$$= \mathrm{Re}\left[e^{i\omega t} \underbrace{\sum_{k=0}^{N-1} e^{-i\omega k\Delta t}}_{A(\omega)} \right]$$

"PHASOR"

$$A(\omega) = \sum_{k=0}^{N-1} \left(e^{-i\omega\Delta t}\right)^k = \frac{1 - e^{-i\omega\Delta t N}}{1 - e^{-i\omega\Delta t}}$$

$$= \frac{e^{-i\omega\Delta t \frac{N}{2}}\left(e^{i\omega\Delta t\frac{N}{2}} - e^{-i\omega\Delta t\frac{N}{2}}\right)}{e^{-i\frac{\omega\Delta t}{2}}\left(e^{i\frac{\omega\Delta t}{2}} - e^{-i\frac{\omega\Delta t}{2}}\right)}$$

$$A(\omega) = e^{-i\omega\Delta t\frac{(N-1)}{2}} \frac{\cancel{2i}\sin\left(\omega\Delta t\frac{N}{2}\right)}{\cancel{2i}\sin\left(\frac{\omega\Delta t}{2}\right)}$$

$$A(\omega) = e^{-i\omega\Delta t\frac{(N-1)}{2}} \frac{\sin\left(\omega\Delta t\frac{N}{2}\right)}{\sin\left(\frac{\omega\Delta t}{2}\right)}$$

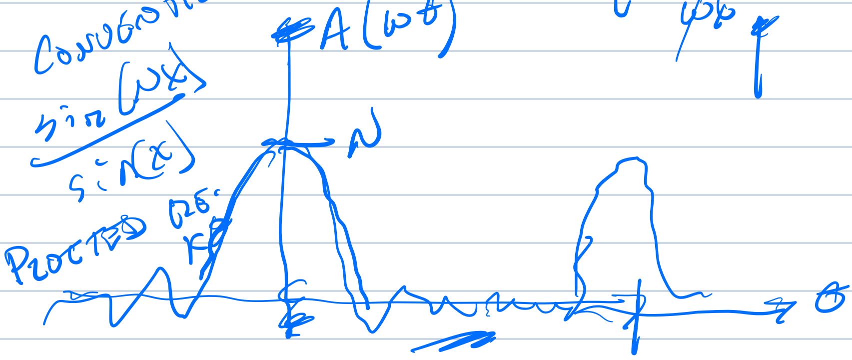
$$|A(\omega)| = \frac{\sin\left(\omega\Delta t\frac{N}{2}\right)}{\sin\left(\frac{\omega\Delta t}{2}\right)}$$

$$c = \nu\lambda = \frac{\omega}{2\pi}\lambda$$

$$\frac{d}{c} = \frac{2\pi}{\omega\lambda} d$$

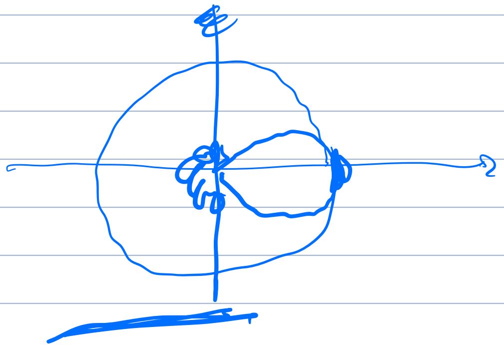
$$\Delta t = \frac{d}{c}\sin\theta = \frac{2\pi}{\omega\lambda} d \sin\theta$$

$$|A(\omega)| = \frac{\sin\left(\omega \frac{2\pi}{\omega/c} d \sin\theta \frac{N}{2}\right)}{\sin\left(\omega \frac{2\pi}{\omega/c} d \sin\theta \frac{1}{2}\right)} = \frac{\sin\left(\frac{\pi d}{\lambda} N \sin\theta\right)}{\sin\left(\frac{\pi d}{\lambda} \sin\theta\right)}$$



POLAR PLOT

$$\frac{\pi d}{\lambda} \sin\theta = 2\pi$$



PLOT $A(\theta)$ v. θ

SPATIAL ALIASING OCCURS WHEN $d \sin\theta > \frac{\lambda}{2}$

TO AVOID SPATIAL ALIASING,

REQUIRE $d \sin\theta < \frac{\lambda}{2} = \frac{2\pi c}{2\omega}$

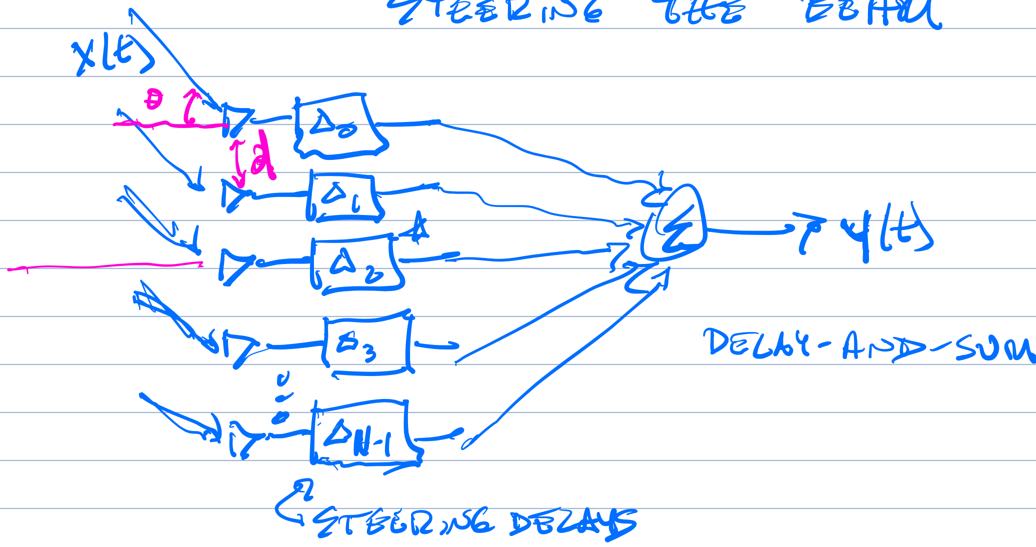
$$c = \nu\lambda = \frac{\omega}{2\pi}\lambda$$

$$\lambda = \frac{2\pi c}{\omega} = \frac{\cancel{2\pi} c}{\cancel{2\pi}\nu}$$

$$d < \frac{\pi c}{\omega \sin\theta} = \frac{c}{2 \sin\theta\, \nu}$$

OR $\nu < \frac{c}{2 d \sin\theta}$

STEERING THE BEAM



FOR MAJOR LOBE @ $\theta = \theta_0$

Let $\Delta_k = (N-1-k)\Delta t$, $0 \le k \le N-1$, $\Delta t = \frac{d}{c}\sin(\theta_0)$

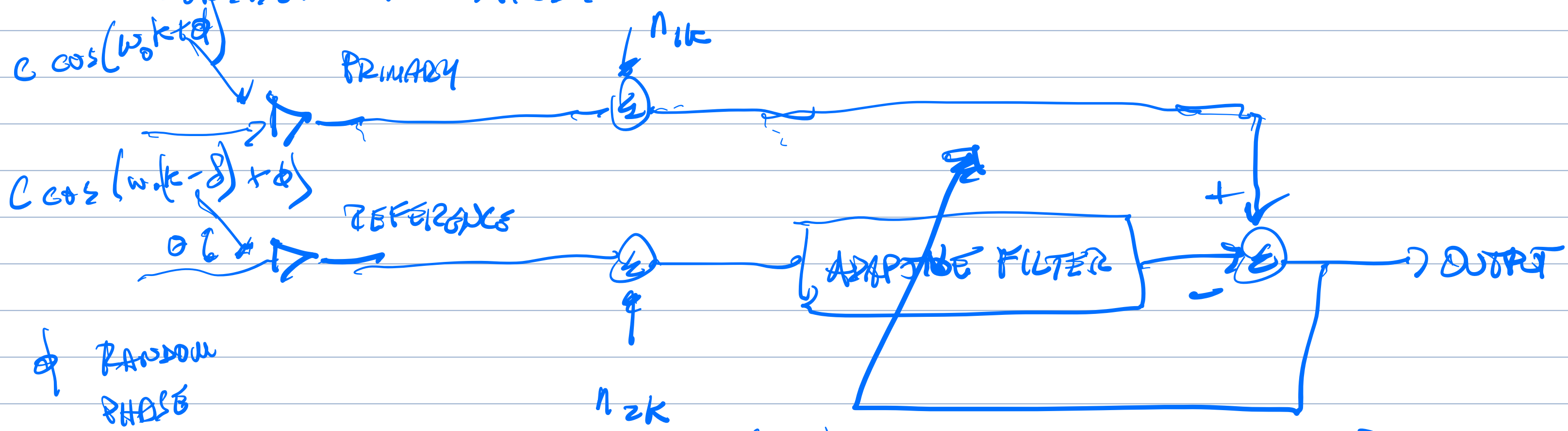
REMINDER: REVIEW AT 5, HH B206 412 916-7386

EXAM: COVERAGE THROUGH PS 9, CLOSED BOOK, 2 SHEETS of NOTES

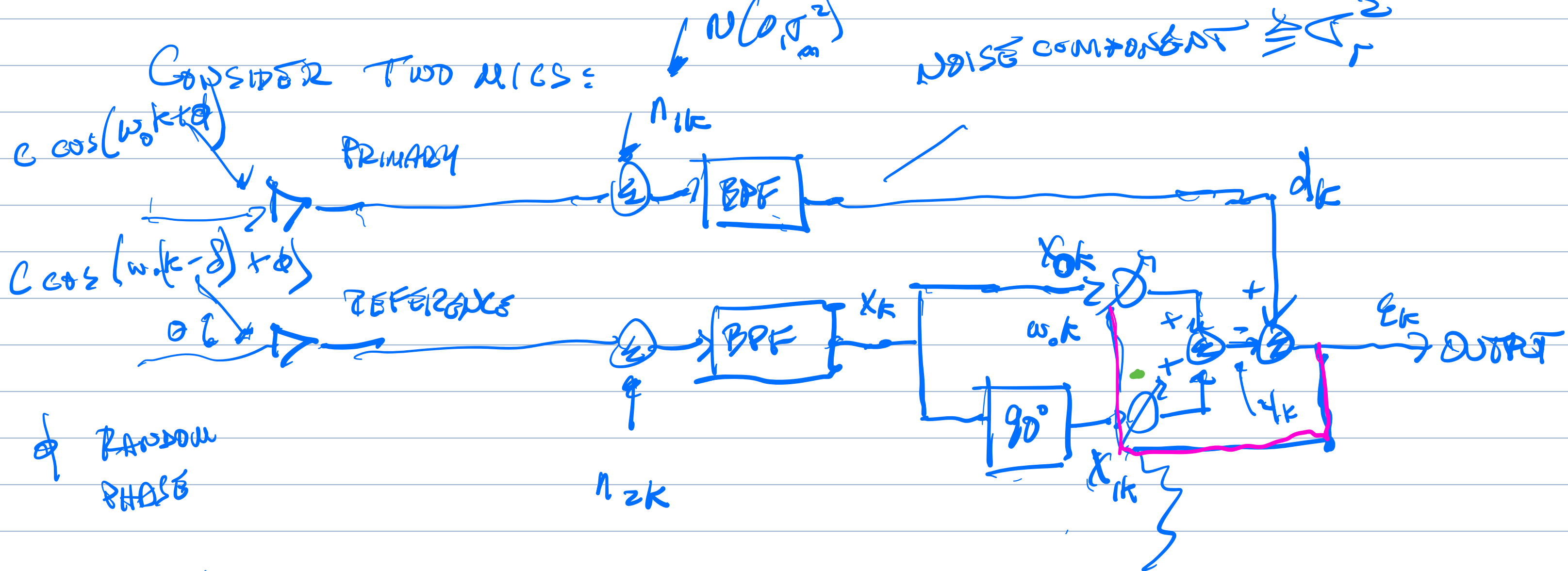
NARROW BAND ADAPTIVE ARRAY

[HOWELLS-APPLEBAUM SIDELOBE CANCELLER]

CONSIDER TWO MICS:



CONSIDER TWO MICS:



$$\delta = \frac{d\sin\theta}{cT} = \frac{2\pi d\sin\theta}{\lambda_0\omega_0 T}$$ DELAY IN SAMPLES $\omega_1 k$

$1/T$ SAMPLING FREQ

$$y_k = \underline{x}_k^T \underline{w}_k \qquad \text{let } \underline{w}_k = \begin{pmatrix} w_{0k} \\ w_{1k} \end{pmatrix}, \quad \underline{x}_k = \begin{pmatrix} x_{0k} \\ x_{1k} \end{pmatrix}$$

$$\underline{\underline{R}} = E\left[\underline{x}\,\underline{x}^T\right] = \begin{bmatrix} \overline{x_{0k}x_{0k}} & \overline{x_{0k}x_{1k}} \\ \overline{x_{1k}x_{0k}} & \overline{x_{1k}x_{1k}} \end{bmatrix} = \begin{pmatrix} \sigma_r^2 + \frac{C^2}{2} & 0 \\ 0 & \sigma_r^2 + \frac{C^2}{2} \end{pmatrix}$$

σ_r^2 = POWER IN n_{1k}, n_{2k} THAT GETS THROUGH BPF

So $x_{0k} = c\left(\cos((k-\delta)\omega_0+\phi)\right) + n_{2k}$

$$x_{1k} = c\left(\sin((k-\delta)\omega_0+\phi)\right) + n_{2k}$$

$$d_k = c\cos(\omega_0 k+\phi)+n_{1k}$$

$$\underline{\underline{R}} = \left(\sigma_r^2 + \frac{c^2}{2}\right)\underline{\underline{I}}$$

$$\underline{P} = E[d_k \cdot \underline{x}_k] = \frac{c^2}{2}\begin{bmatrix}\cos(\omega_0\delta)\\ \sin(\omega_0\delta)\end{bmatrix}$$

$$\underline{w}^* = \underline{\underline{R}}^{-1}\underline{P} = \frac{1}{\sigma_r^2+\frac{c^2}{2}}\cdot\frac{c^2}{2}\begin{bmatrix}\cos(\omega_0\delta)\\ \sin(\omega_0\delta)\end{bmatrix}$$

$Y(\theta)$

FOR $\delta=\phi=0$