11/13/24 THE ADAPTIVE LATTICE FILTER; FREQUENCY-DOMAIN ADAPTIVE FILTERING (FDAF)

(LO 5.8, NOTES 8.8, SHYNK ARTICLE)

Q2 NEXT WEEK

RECITATION FRIDAY

QUIZ REVIEW SESSION MONDAY EVE 5

RE ADAPTIVE FILTERS,

LMS IS SLOW WHEN THE COMPONENTS of $\underline{x}_k$ ARE CORRELATED

WAYS OF DEALING W CORRELATION PROBLEM:

1. RLS ALGORITHM

NEWTON GRADIENT DESCENT

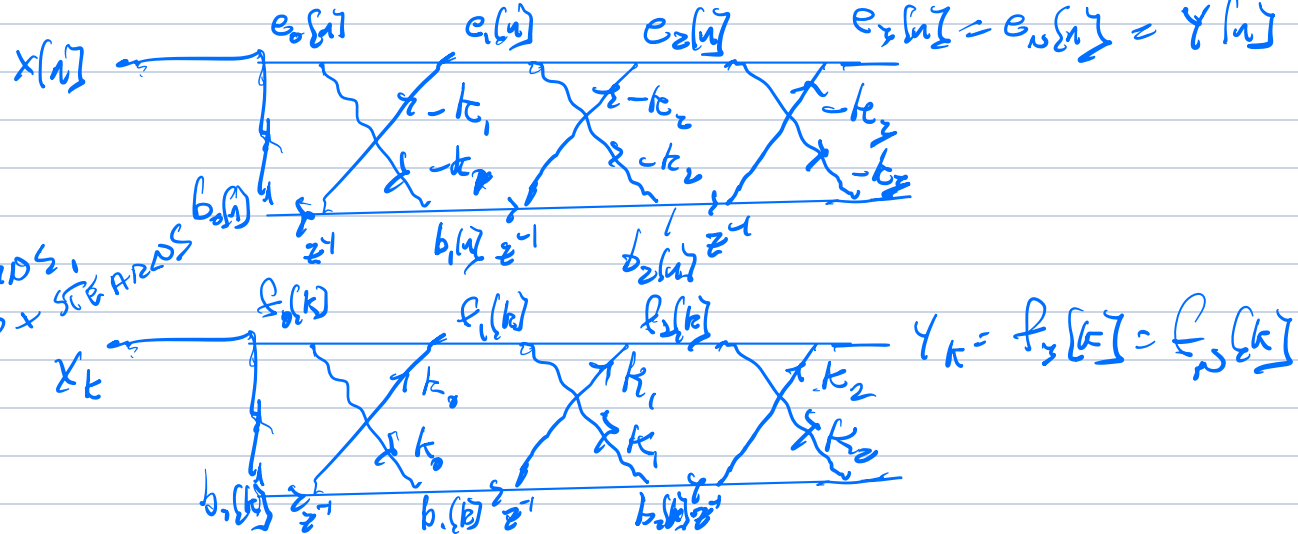
$$\underline{w}_{k+1} = \underline{w}_k - \mu \underline{\underline{R}}^{-1} \underline{\nabla}$$

2. ADAPTIVE LATTICE FILTER (DECORRELATES OBSERVATIONS IMPLICITLY)

3. FREQUENCY-DOMAIN ADAPTIVE FILTERS (FDAF)

ADAPTIVE LATTICE FILTERS

FOR LATTICE RE, OSAP



NO3

STEARNS, WIDROW + STEARNS

UPDATE EQS

$$f_{i+1}[k] = f_i[k] + k_i\, b_i[k-1]$$

$$b_{i+1}[k] = k_i f_i[k] + b_i[k-1]$$

LET $\xi_i^2 = E\left[f_{i+1}[k]\right] = E\left[\left(f_i[k] + k_i b_i[k-1]\right)^2\right]$

TO FIND k_i^*

$$\frac{\partial \xi_i^2}{\partial k_i} = E\left[2\left(f_i[k] + k_i b_i[k-1]\right) b_i[k-1]\right] = 0$$

$$E\left[f_i[k]\, b_i[k-1]\right] = -k_i\, b_i^2[k-1]$$

$$k_i^* = -\frac{E\left[f_i[k]\, b_i[k-1]\right]}{E\left[b_i^2[k-1]\right]}$$

CORRELATION of $\{f_i[k]\}$

$$E\left[f_{i+1}[k]\, f_i[k]\right] = E\left[\left(f_i[k] + k_i b_i[k-1]\right) f_i[k]\right]$$

FOR $k_i = k_i^*$

$$E\left[f_{i+1}[k] f_i[k]\right] = E\left[\left(f_i[k] - \frac{E\left[f_i[k]\, b_i[k-1]\right]}{E\left[b_i^2[k-1]\right]}\, b_i[k-1]\right) f_i[k]\right]$$

$$= E\left[f_i[k] - \frac{f_i[k]\, b_i[k-1]\, b_i[k-1]}{b_i^2[k-1]}\right) f_i[k]$$

$$= 0$$

LMS ADAPTIVE LATTICE

CONVENTIONAL LMS $\underline{W}_{k+1} = \underline{W}_k - \mu \underline{\nabla}_k$

$$= \underline{W}_k - \mu\left(2R\underline{W}_k - 2\underline{P}\right)$$

$\underline{\nabla} = \frac{\partial \xi^2}{\partial \underline{W}_k}$

LMS LATTICE

$$k_i[k+1] = k_i[k] - \mu_i \nabla \qquad \nabla = \frac{\partial \xi^2}{\partial k_i}$$

$$\frac{\partial \xi^2}{\partial k_i} = E\left[2\left(f_i[k] + k_i b_i[k-1]\right) b_i[k-1] \right]$$

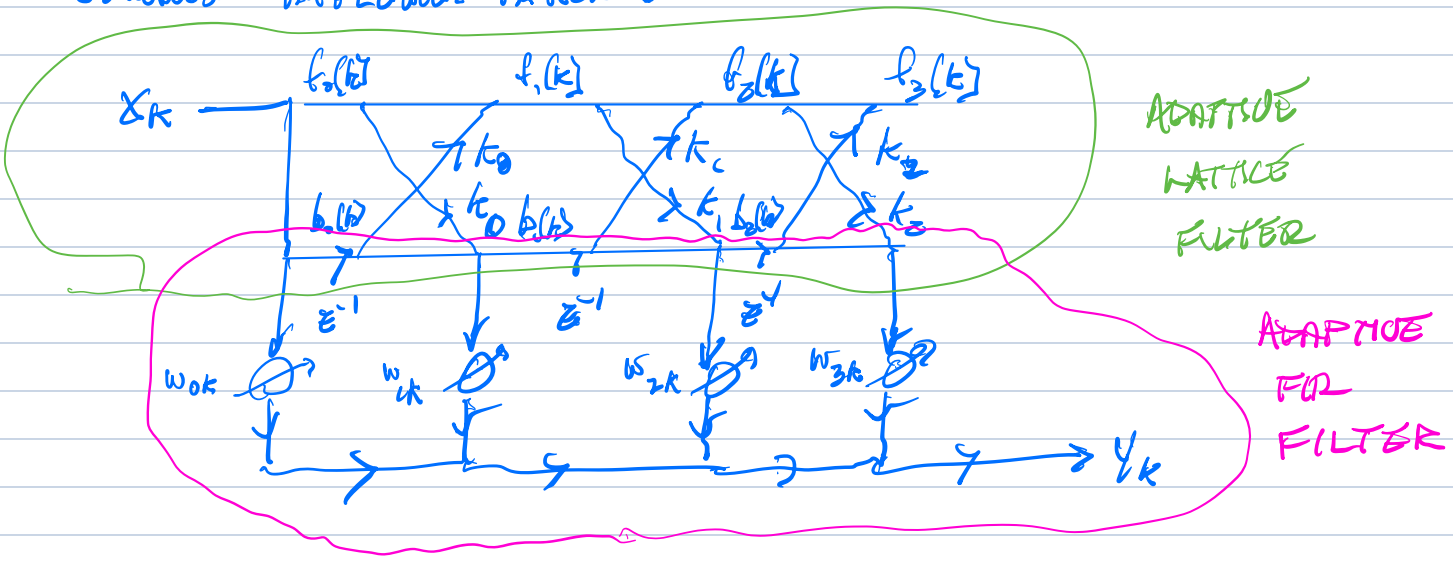
$$k_i[k+1] = k_i[k] - \mu_i 2\underbrace{\left(f_i[k] + k_i b_i[k-1]\right)}_{f_{i+1}[k]} b_i[k-1]$$

$$\boxed{k_i[k+1] = k_i[k] - 2\mu_i f_{i+1}[k]\, b_i[k-1]}$$

IMPLEMENTATION of ADAPTIVE LATTICE:

1. $f_0[k] = b_0[k] = x_k$
2. $k_i[k+1] = k_i[k] - 2\mu_i f_{i+1}[k]\, b_i[k-1]$
3. CALCULATE $f_i[k]$, $b_i[k]$
4. REPEAT FOR ALL STAGES

COMMON IMPLEMENTATION:



PRINCIPLES OF FREQUENCY-DOMAIN ADAPTIVE FILTERING (FDAF)

LMS $\underline{W}_{k+1} = \underline{W}_k + 2\mu \underline{X}_k \underline{E}_k$

SHENOK

$$W[n+1] = w[n] + 2\mu\, x[n] e[n]$$

Let N be size of ADAPTIVE FIR FILTER

$$e[n] = d[n] - y[n]$$

$$y[n] = \underline{x}^T[n]\, \underline{w}[n] = \underline{w}^T[n]\, \underline{x}[n]$$

$$\hat{\nabla}[n] = \underline{x}[n]\, e[n]$$

BLOCK UPDATES ... CHANGE ONLY AFTER L SAMPLES

$$\underline{w}[n+L] = w[n+(L-1)] + 2 \sum_{L=1} \underline{x}[n+L-1] e[n+L-1]$$

$$w[n+L] = w[n] + 2\mu \sum_{m=0} x[n+m]\, e[n+m]$$

$$y[n+m] = \underline{x}^T[n+m]\, \underline{w}[n]$$

Let $\underline{\underline{W}}[k] = \begin{bmatrix} \underline{W}_0[k] \\ \underline{W}_1[k] \\ \underline{W}_2[k] \\ \vdots \\ \underline{W}_{N-1}[k] \end{bmatrix}$ ← WEIGHT COEFFS

Let $\underline{\underline{X}}[k]$ = DIAGONAL OF $[X_0[k], X_1[k], X_2[k]]$ ← DFT of $\underline{x}(n)$

$$\underline{\underline{Y}}[k] = \underline{\underline{X}}[k] \cdot \underline{\underline{W}}[k]$$

$$\underline{\underline{W}}[k+1] = \underline{\underline{W}}[k] + 2\underline{\underline{G}}\,\underline{\underline{\mu}}[k]\, \underline{\underline{X}}[k]\, \underline{\underline{E}}[k]$$

DIAG. MATRIX