11/4/24 INTRO TO LATTICE FILTERS

COSYP 6.6, RHS NOTES 7.3–7.5

REMINDER: QUIZ 2 11/20/24, IN CLASS

COVERAGE THROUGH PS 9

REVIEW: LINEAR PREDICTION, LD RECURSION

Given $\{\phi[m]\}$, $|m| \le P$ Find $\{\alpha_k^{(P)}\}$

$$H(z) = \frac{G}{1 - \sum_{k=1}^{P} \alpha_k^{(P)} z^{-k}} \qquad w[n] \rightarrow \boxed{H(z)} \rightarrow x[n]$$

SOLVE WITH LEVINSON-DURBIN RECURSION

0. Let $E^{(0)} = \phi[0]$

1. $k_i = \dfrac{\phi[i] - \sum_{l=1}^{i-1} \alpha_l^{(i-1)} \phi[i-l]}{E^{(i-1)}}$ $\quad 1 \le i \le P$

2. $\alpha_i^{(i)} = k_i$

3. $\alpha_l^{(i)} = \alpha_l^{(i-1)} - k_i \alpha_{i-l}^{(i-1)}$ $\quad 1 \le l \le i-1$

4. $E^{(i)} = (1-k_i^2) E^{(i-1)}$

$i = i+1$

TO CALCULATE $\{\alpha_i\}$ FROM $\{k_i\}$

Let $\alpha_i^{(i)} = k_i$, BEGINNING WITH $i=1$

$\alpha_l^{(i)} = \alpha_l^{(i-1)} - k_i \alpha_{i-l}^{(i-1)}$

$i = i+1$

FROM $\{\alpha_i\}$ TO $\{k_i\}$ $\quad 1 \le i \le P$

Let $k_P = \alpha_P^{(P)}$

STARTING WITH $i = P$, let $k_i = \alpha_i^{(i)}$

$$\alpha_\ell^{(i-1)} = \frac{\alpha_\ell^{(i)} + k_i \alpha_{i-\ell}^{(i)}}{1 - k_i^2} \qquad \text{FOR } i = P \to 1, \quad \ell = i \to 1$$

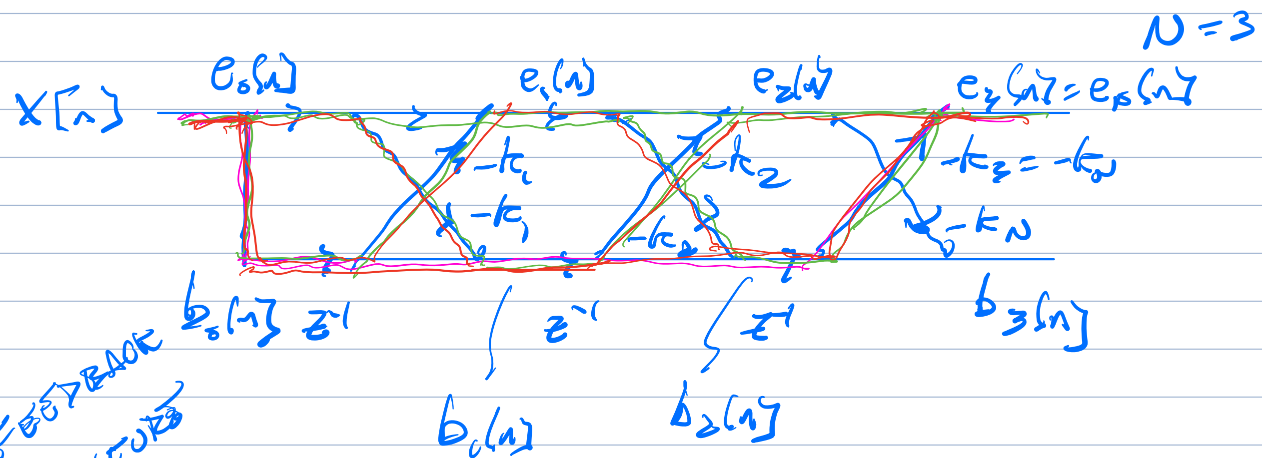
COMPUTING LPC GAIN $\qquad G^2$

$$\phi[0,0] = \sum_{k=1}^{P} \alpha_k \phi[0,k] + \cancel{\sigma_x^2}$$

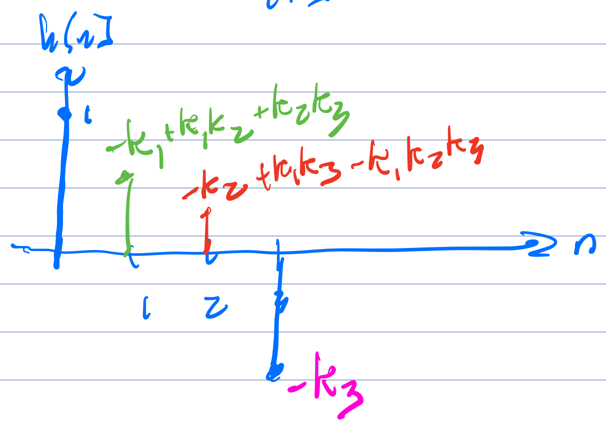
$$G^2 = \phi[0,0] - \sum_{k=1}^{P} \alpha_k \phi[0,k]$$

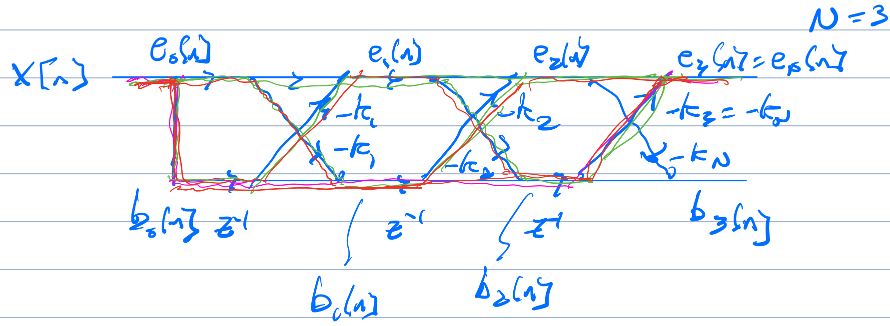
BASIC LATTICE FILTER STRUCTURE

$N = 3$



NO FEEDBACK ⇒ FIR STRUCTURE





RECURSIVE CHARACTERIZATION OF $e_i[n]$, $b_i[n]$

$$e_i[n] = e_{i-1}[n] - k_i\, b_{i-1}[n-1]$$

$$b_i[n] = -k_i\, e_{i-1}[n] + b_{i-1}[n-1]$$

$$e_0[n] = b_0[n] = x[n]\ ; \ y[n] = e_N[n]$$

$$E_i(z) = E_{i-1}(z) - k_i z^{-1} B_{i-1}(z)$$

$$B_i(z) = -k_i E_{i-1}(z) + z^{-1} B_{i-1}(z)$$

$$E_0(z) = B_0(z) = X(z)\ ; \ Y(z) = E_N(z)$$

$$x[n] \rightarrow \boxed{A(z)} \rightarrow y[n]$$

$$A(z) = 1 - \sum_{\ell=1}^{N} \alpha_\ell^{(N)} z^{-\ell} = \frac{Y(z)}{X(z)} = \frac{E_N(z)}{E_0(z)}$$

DEFINE

$$\boxed{A_i(z) \triangleq \frac{E_i(z)}{E_0(z)} \qquad \tilde{A}_i(z) \triangleq \frac{B_i(z)}{B_0(z)}}$$

CAN BE SHOWN THAT IF αs AND k_i OBEY LEVINSON-DURBIN

THEN

$$\boxed{\begin{aligned} A_i(z) &= A_{i-1}(z) - k_i z^{-i} A_{i-1}(z^{-1}) \\ \tilde{A}_i(z) &= z^{-i} A_i(z^{-1}) \end{aligned}}$$

$$A_i(z) = A_{i-1}(z) - k_i z^{-i} A_{i-1}(z^{-1}) \qquad -k_i\left(z^{-i} - \sum_{\ell=1}^{i-1} \alpha_\ell^{(i-1)} z^{\ell-i}\right)$$

$$\underbrace{1 - \sum_{\ell=1}^{i} \alpha_\ell^{(i)} z^{-\ell}}_{A_i(z)} = \underbrace{1 - \sum_{\ell=1}^{i-1} \alpha_\ell^{(i-1)} z^{-\ell}}_{A_{i-1}(z)} - k_i z^{-i} \underbrace{\left(1 - \sum_{\ell=1}^{i-1} \alpha_\ell^{(i-1)} z^{+\ell}\right)}_{A_{i-1}(z^{-1})}$$

$-r = \ell - i$
$\ell = i - r$

CONSIDER COEFFS of z^{-r}

LD RECURSION $\alpha_r^{(i)} \cancel{z^{-r}} = \alpha_r^{(i-1)} \cancel{z^{-r}} - k_i \alpha_{i-r}^{(i-1)} \cancel{z^{-r}}$

VISIBLE COEFFS OF LATTICE FILTER ARE $\{k_i\}$!

OTHER FILTER STRUCTURES WITH VISIBLE COEFFS.

1. FIR FILTER $H(z) = \sum_{\ell=0}^{M} b_\ell z^{-\ell}$

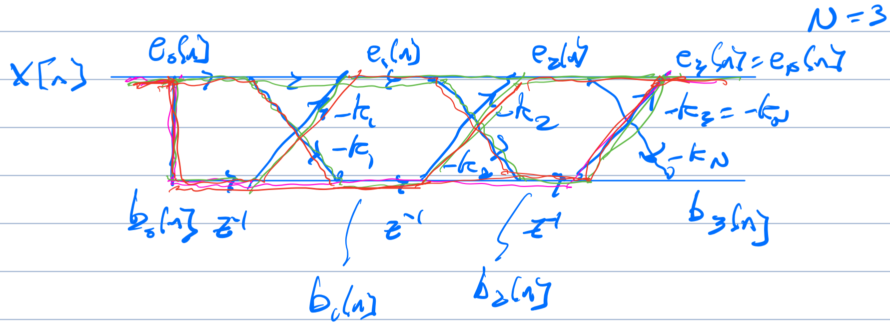
2. IIR FILTER $H(z) = \dfrac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$ DF-I, DF-II

2. FREQ. SAMPLING STRUCTURE

$$H(z) = (1 - z^{-N}) \sum_{k=1}^{N} \frac{H[k]}{1 - W_N^k z^{-1}}$$

3. FIR LATTICE FILTER

COEFFS ARE REFLECTION COEFFS.



MEANING of $e_i[n]$, $b_i[n]$

$$A_i(z) = \frac{E_i(z)}{E_0(z)} = \frac{E_i(z)}{X(z)} = 1 - \sum_{\ell=1}^{i} \alpha_\ell^{(i)} z^{-\ell}$$

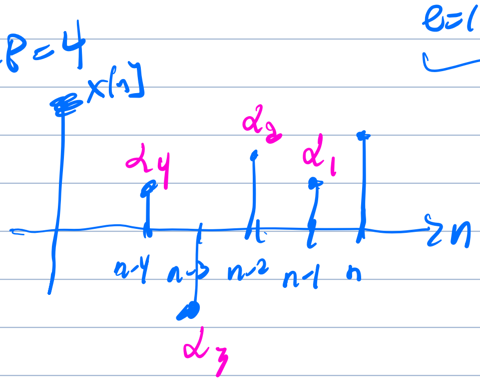
$$E_i(z) = X(z)\left(1 - \sum_{\ell=1}^{i} \alpha_\ell^{(i)} z^{-\ell}\right)$$

$$e_i[n] = x[n] - \underbrace{\sum_{\ell=1}^{i} \alpha_\ell^{(i)} x[n-\ell]}_{\tilde{x}[n]}$$ = 4^{th} ORDER LPC ERROR

" " FORWARD "

$$e_4[n] = x[n] - \tilde{x}[n]$$

ASSUME $N=P=4$



WHAT ARE $\{b_i[n]\}$?

FROM BEFORE $\tilde{A}_i(z) = z^{-i} A_i(z^{-1})$

$$\tilde{A}_i(z) = \frac{B_i(z)}{B_0(z)} = z^{-i}\left(1 - \sum_{\ell=1}^{i} \alpha_\ell^{(i)} z^{+\ell}\right)$$

$$\frac{B_i(z)}{E_0(z)} = \tilde{A}_i(z) = z^{-i} - \sum_{\ell=1}^{i} \alpha_\ell^{(i)} z^{-(i-\ell)}$$

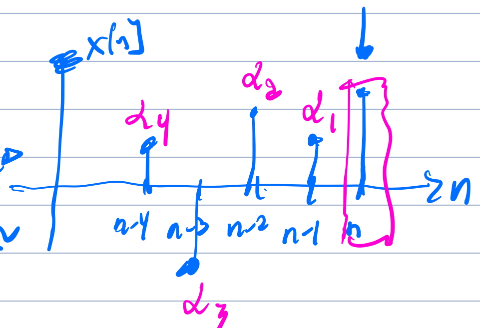
$$B_i(z) = X(z)\left(z^{-i} - \sum_{\ell=1}^{i} \alpha_\ell^{(i)} z^{-(i-\ell)}\right)$$

BACKWARD ERROR $$b_i[n] = x[n-i] - \sum_{\ell=1}^{i} \alpha_\ell^{(i)} x[n-(i-\ell)]$$

$N=P=i=4$

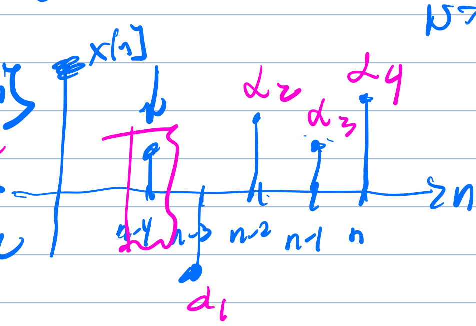
$e_i[n]$

4TH-ORDER FORWARD ERROR



$b_i[n]$

4TH-ORDER BACKWARD ERROR



THE PARCOR METHOD of ESTIMATING REFLECTION COEFS.

GIVEN $\{x[n]\}$ FIND $\{k_i\}$ THAT MIN $\xi^2 = E[e_i^2[n]]$

$$e_i[n] = e_{i-1}[n] - k_i b_{i-1}[n-1]$$

Let $\xi^2 = E[e_i^2[n]] = E[(e_{i-1}[n] - k_i b_{i-1}[n-1])^2]$

$$\frac{d\xi^2}{dk_i} = E[2(e_{i-1}[n] - k_i b_{i-1}[n-1])\, b_{i-1}[n-1]] = 0$$

$$E[e_{i-1}[n]\, b_{i-1}[n-1]] = k_i E[b_{i-1}^2[n-1]]$$

$$\boxed{k_i^f = \frac{E[e_{i-1}[n]\, b_{i-1}[n-1]]}{E[b_{i-1}^2[n-1]]}}$$

RECURSIVE PROCEDURE:

0. Let $b_0[n] = e_0[n] = x[n]$

1. Let $k_i = \dfrac{E[e_{i-1}[n]\, b_{i-1}[n-1]]}{E[b_{i-1}^2[n-1]]}$

$i = i+1$

2. Let $e_i[n] = e_{i-1}[n] - k_i b_i[n-1]$

$b_i[n] = -k_i e_{i-1}[n] + b_{i-1}[n-1]$

COMMENTS

1. USE TIME AVERAGES

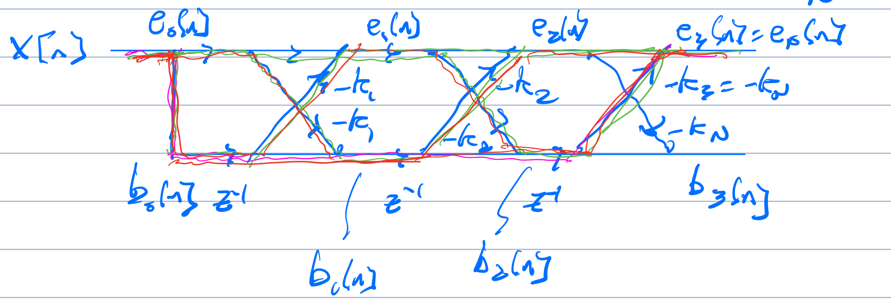
$$E[b_{i-1}^2[n-1]] = \frac{1}{N}\sum_{n=0}^{N-1} b_{i-1}^2[n-1]$$

2. CAN OBTAIN α_i FROM LD RECURSION

3. $|k_i| < 1$

4. $k_i^b = \dfrac{E[e_{i-1}[n]\, b_{i-1}[n-1]]}{E[e_i^2[n]]}$

ALL-ZERO LATTICE



$$e_i[n] = e_{i-1}[n] - k_i\, b_{i-1}[n-1]$$

$$b_i[n] = -k_i\, e_{i-1}[n] + b_{i-1}[n-1]$$

$$e_0[n] = b_0[n] = x[n]\ ;\ y[n] = e_N[n]$$

FOR ALL-POLE LATTICE

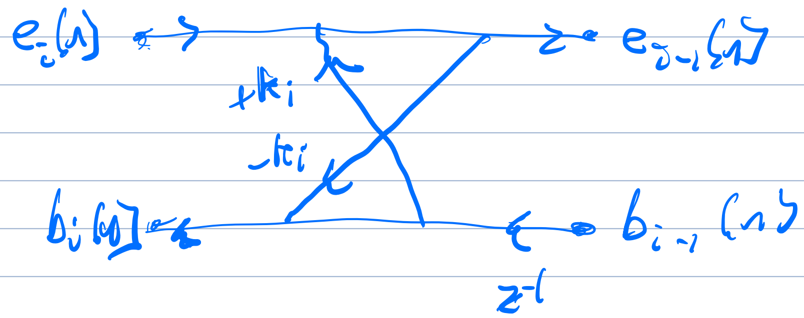
$$b_i[n] = -k_i\, e_{i-1}[n] + b_i[n-1]$$

$$e_{i-1}[n] = e_i[n] + k_i\, b_{i-1}[n-1]$$

ALL-ZERO:

INPUT $x[n] = e_0[n]$

OUTPUT $x_N[n] = y[n]$



FIR LATTICE $\dfrac{Y(z)}{X(z)} = \dfrac{E_N(z)}{E_0(z)} = A(z)$

IIR LATTICE $\dfrac{Y(z)}{X(z)} = \dfrac{E_0(z)}{E_N(z)} = \dfrac{1}{A(z)} \Rightarrow \dfrac{G}{A(z)}$