9/23/24 APPLICATIONS OF SHORT-TIME FOURIER ANALYSIS

SPECTRAL SUBTRACTION + PHASE VOCODING

NOTES 3.7

SS: BOLL (1979)

PV: FLANAGAN + GOLDEN (1962)

SPECTRAL SUBTRACTION

$$x[n] = s[n] + w[n]$$

$$X[n,k] = S[n,k] + W[n,k]$$

PROCEDURE:

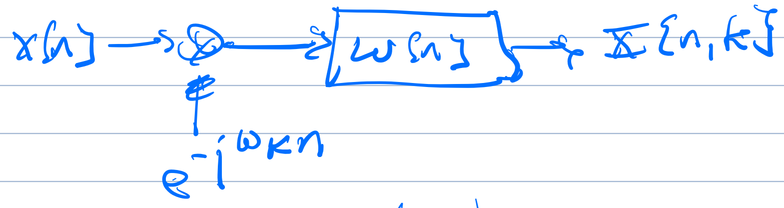
1. DETERMINE FRAMES FOR WHICH $S[n,k] = 0$
   - SPEECH ACTIVITY DETECTION (SAD)
   - VOICE ACTIVITY DETECTION (VAD)
2. ESTIMATE $|\hat{W}[n,k]|$ FOR $|S[n,k]| = 0$
3. $|\tilde{S}[n,k]| \approx |X[n,k]| - |\hat{W}[n,k]|$
4. ESTIMATE $\hat{S}[n,k] = |\tilde{S}[n,k]| \cdot e^{j\measuredangle X[n,k]}$
5. RECOVER $\tilde{s}[n]$ USING FBS OR OLA

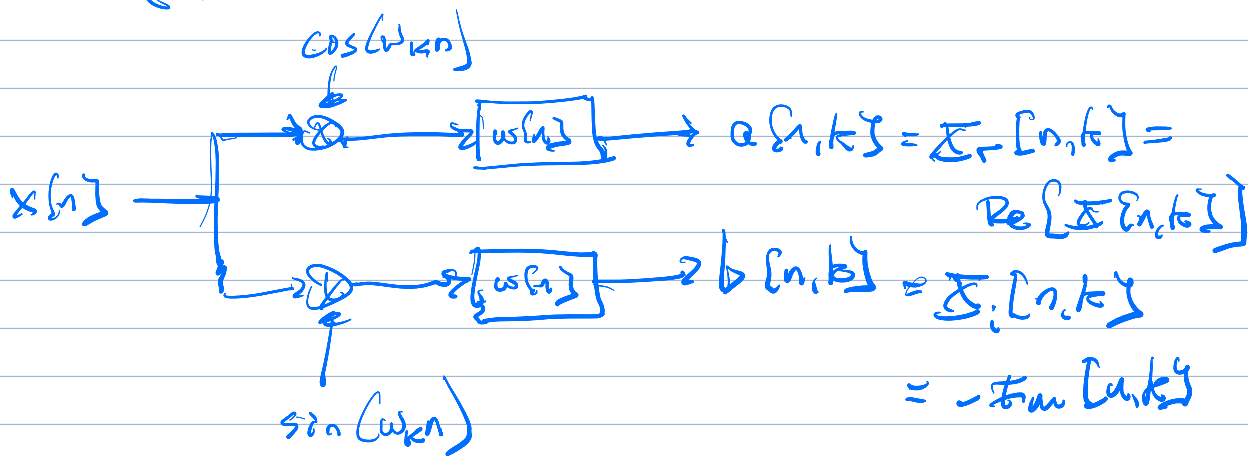
PROBLEMS / "ISSUES"

1. $|X[n,k]| - |\hat{W}[n,k]|$ COULD BE NEGATIVE
2. VAD ERRORS
3. PHASE DIFFERENCES BET. $\measuredangle X[n,k]$, $\measuredangle S[n,k]$

Phase Vocoding

LPF implementation of STFT





$$X[n,k] = a[n,k] - j\, b[n,k]$$

$$X[n,k] = |X[n,k]|\, e^{j\theta[n,k]} \qquad \theta[n,k] = \measuredangle X[n,k]$$

$$|X[n,k]| = \sqrt{a^2[n,k] + b^2[n,k]}$$

$$\theta[n,k] = \measuredangle X[n,k] = -\tan^{-1}\left[\frac{b[n,k]}{a[n,k]}\right]$$

IF $x[n]$ REAL

$$X[n,-k] = X[n,N-k] = X^*[n,k]$$

$$|X[n,-k]| = |X[n,N-k]| = X[n,k]$$

$$\theta[n,-k] = \theta[n,N-k] = -\theta[n,k]$$

IDFT $\quad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{j\omega_k n}, \quad \omega_k = 2\pi k/N$

$$e^{j\omega_{-k} n} = e^{j\omega_{N-k} n} = e^{j\frac{2\pi(N-k)}{N}} = e^{-j\frac{2\pi k}{N}} = e^{-j\omega_k n}$$

Let $x_k[n] = X[n,k]\, e^{j\omega_k n} + X[n, N-k]\, e^{-j\omega_k n}$

$$= |X[n,k]|\, e^{j\theta[n,k]} e^{j\omega_k n} + |X[n,k]|\, e^{-j\theta[n,k]} e^{-j\omega_k n}$$

$$= |X[n,k]| \left(e^{j(\omega_k n + \theta[n,k])} + e^{-j(\omega_k n + \theta[n,k])} \right)$$

$$= 2\, |X[n,k]| \cos(\omega_k n + \theta[n,k])$$

ORIGINAL GOAL of PHASE VOCODING

1. EXTRACT MAGNITUDE + PHASE
2. SEND MAG. + PHASE SEPARATELY

PROBLEM w PHASE SIGNAL

UNWRAPPED ⇒ INFINITE AMPLITUDE

WRAPPED ⇒ INFINITE BANDWIDTH

SOLUTION: TRANSMIT MAGNITUDE + PHASE DERIVATIVE

CONSIDER

$$x_k(t) = 2A(t)\cos(\Omega_k t + \theta(t))$$

$$= 2A(t)\cos(\phi(t))$$

$$\frac{d\phi(t)}{dt} = \Omega_k + \frac{d\theta(t)}{dt}$$

INSTANTANEOUS FREQUENCY

OBTAINING PHASE DERIVATIVE

CONSIDER CT STFT

$$X_c(t,\Omega_k) = \int_{-\infty}^{\infty} x_c(\tau)\, w_c(t-\tau)\, e^{-j\Omega_k \tau}\, d\tau$$

$$X_c(t,\Omega_k) = |X_c(t,\Omega_k)|\, e^{-\theta(t,\Omega_k)}$$

$$= a_c(t,\Omega_k) - j\, b_c(t,\Omega_k)$$

Let $x_k(t) = 2|X_c(t,\Omega_k)| \cos(\Omega_k t + \theta[t,\Omega_k])$

COMPUTING MAGNITUDE + PHASE

$$X_c(t,\Omega_k) = |X_c(t,\Omega_k)|\, e^{j\theta(t,\Omega_k)}$$

$$\theta(t,\Omega_k) = -\tan^{-1}\left[\frac{b(t,\Omega_k)}{a(t,\Omega_k)}\right]$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \; ; \; \frac{b(t,\Omega_k)}{a(t,\Omega_k)} = x$$

$$\frac{\partial\theta_c(t,\Omega_k)}{\partial t} = \dot{\theta}(t) = \frac{b\dot{a} - a\dot{b}}{a^2+b^2}$$

IN DISCRETE TIME

$$\dot{\theta}_c[n,\omega_k] \approx \frac{b[n,\omega_k]\,\dot{a}[n,\omega_k] - a\,\dot{b}[n,\omega_k]}{a^2[n,\omega_k] + b^2[n,\omega_k]}$$

$$\dot{a}[n,k] = \frac{a[n,k] - a[n-1,k]}{1}$$

PHASE ANALYSIS