9/18/24

SHORT-TIME FOURIER SYNTHESIS

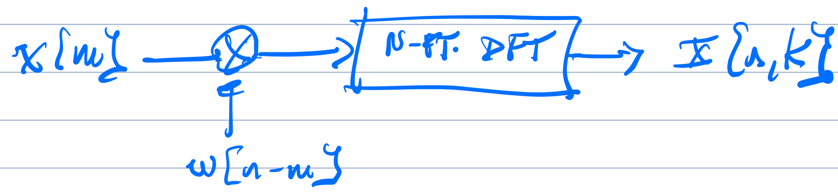
(L+D 6.3, NOTES 3.4 – 3.5)

STFT
$$X[n,k] = \sum_{m=n-(N-1)}^{n} x[m]\, w[n-m]\, e^{-j\omega_k n}$$

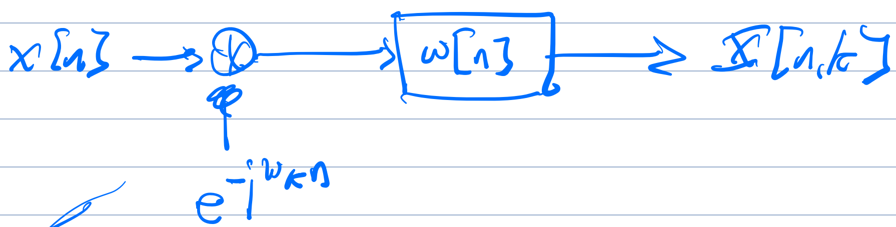
$$\omega_k = \frac{2\pi k}{N}$$

$$x[m]\, w[n-m] = \frac{1}{N}\sum_{k=0}^{N-1} X[n,k]\, e^{j\omega_k n}$$

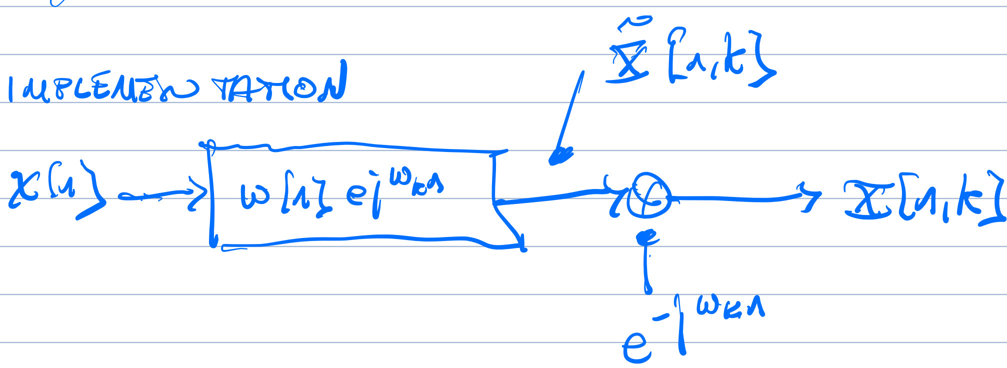
DFT IMPLEMENTATION



LP FILTER IMPLEMENTATION



BP FILTER IMPLEMENTATION



INVERSION OF THE STFT

$$X[n,\omega) = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega m}$$

$$x[m]\, w[n-m] \Longleftrightarrow X[n,\omega)$$

$$x[m]\, w[n-m] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X[n,\omega)\, e^{j\omega m}\, d\omega$$

let $n = m$

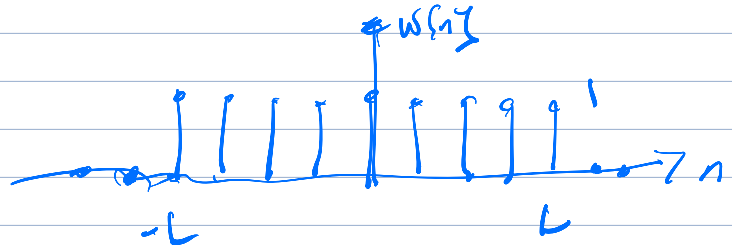
$$x[n]\, w[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X[n,\omega)\, e^{j\omega n}\, d\omega$$

$$x[n] = \frac{1}{2\pi\, w[0]}\int_{-\pi}^{\pi} X[n,\omega)\, e^{j\omega n}\, d\omega$$

$w[n]$ ARE LPF

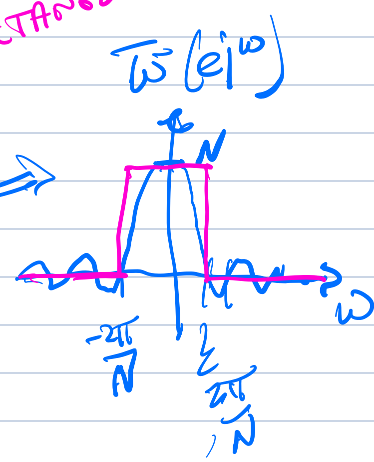
EX RECT-WINDOW

RECTANGULAR WINDOW



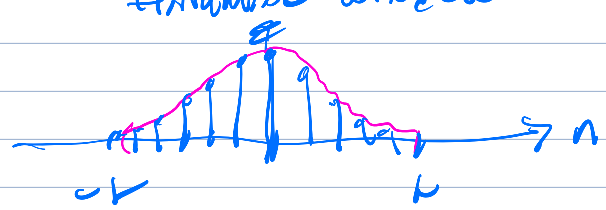
$N = 2L+1$

$\Longleftrightarrow$

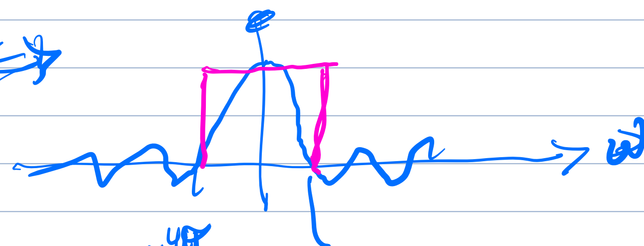


$$\frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

HAMMING WINDOW



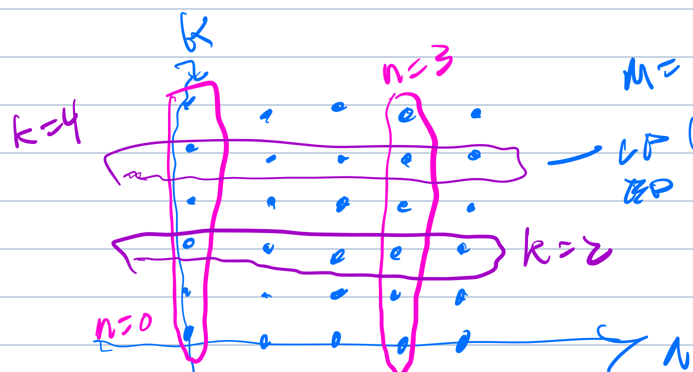
$\Longleftrightarrow$



$-\frac{4\pi}{M}$ $\approx$ $\qquad \frac{4\pi}{M} \approx \frac{4\pi}{N}$ $\approx$

$\frac{4\pi}{N-1}$

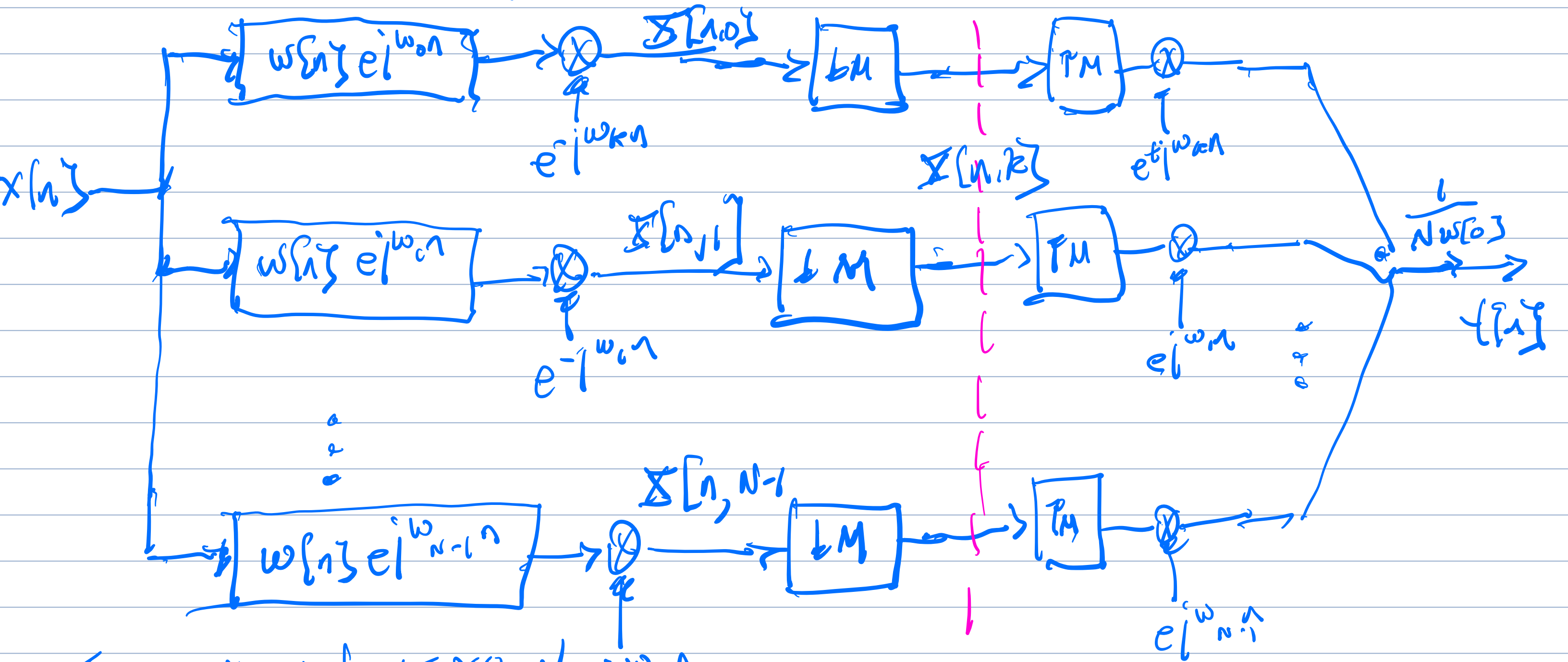
$M = N-1$



→ LP (IMPLEMENTATION)
BP "

STFT VIA BP IMPLEMENTATION

$\omega_k = \frac{2\pi}{N} k$



FOR WINDOWS of LENGTH N

DOWN SAMPLE BY $\frac{N}{2}$ FOR RECT WINDOW

" " $\frac{N-1}{4}$ FOR HAMMING WINDOW

$\boxed{\text{FILTERBANK SUMMATION (FBS) METHOD}}$

$$y[n] = \frac{1}{N w[0]} \sum_{k=0}^{N-1} X[n,k] e^{j\omega_k n}$$

$$= \frac{1}{N w[0]} \sum_{k=0}^{N-1} \underbrace{\sum_{m=-\infty}^{\infty} x[m] w[n-m] e^{-j\omega_k m}}_{X[n,k]} e^{j\omega_k n}$$

$$y[n] = \frac{1}{N w[0]} \sum_{m=-\infty}^{\infty} x[m] w[n-m] \underbrace{\sum_{k=0}^{N-1} e^{j\omega_k (n-m)}}$$

$$= \frac{1}{N w[0]} \left(x[n] * w[n] \sum_{k=0}^{N-1} e^{j\omega_k n} \right)$$

$y[n] = x[n]$ IF $w[n]\sum_{k=0}^{N-1} e^{j\omega_k n} = N\, w[0]\, \delta[n]$

$$r[n] = w[n]\sum_{k=0}^{N-1} e^{j\frac{2\pi k}{N} n} = w[n]\underbrace{\frac{1-e^{j2\pi n}}{1-e^{j\frac{2\pi}{N} n}}}_{\begin{cases} N, & n=0,\ N \\ 0, & \text{ELSE} \end{cases}}$$

DESIRED RESULT

$$r[n] = \begin{cases} w[0]N, & n=0 \\ 0, & n \neq \ell N, 0 \\ \neq 0, & n = \ell N \end{cases}$$

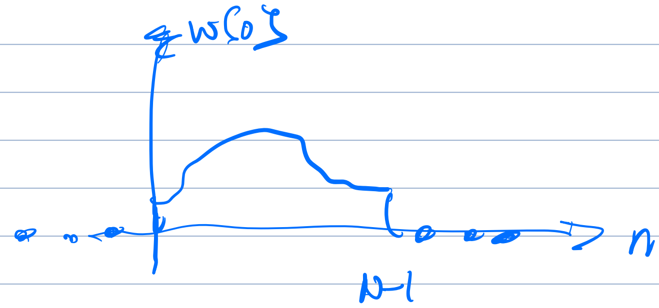
FBS CONSTRAINT: FOR $y[n] = x[n]$

REQUIRE $w[n] = 0,\ n = \ell N,\ \ell \neq 0$

$w[0] \neq 0$

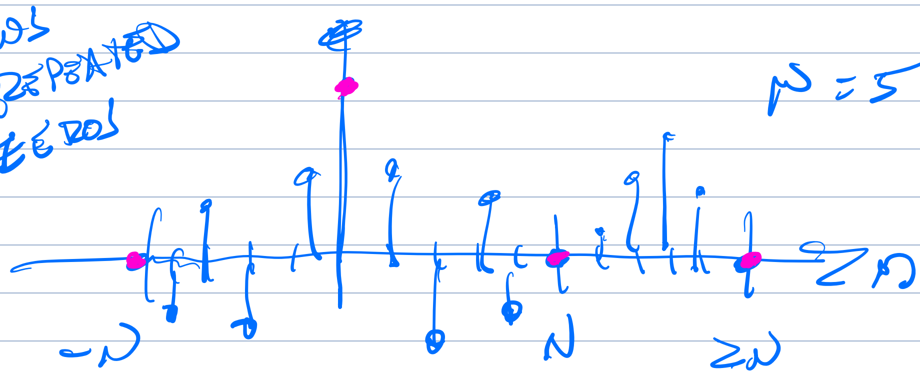
WINDOWS THAT WORKS

FIR WINDOWS



WINDOWS WITH REPEATED ZEROS

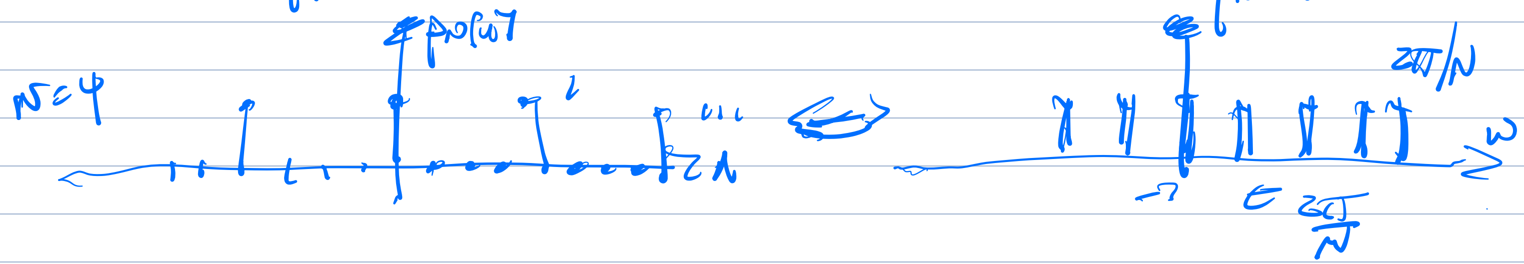
$N = 5$



ANOTHER WAY OF LOOKING AT FBS CONSTRAINT...

FBS CONSTRAINT REQUIRES

$$p_N[n] \cdot w[n] = \delta[n]$$



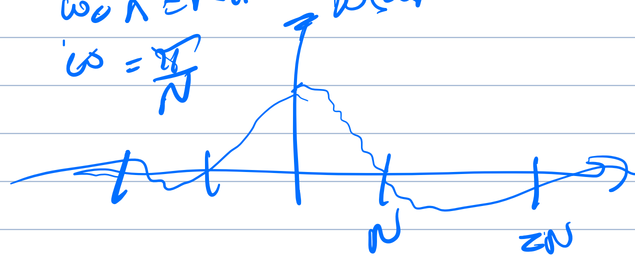
$$p_N[n] \cdot w[n] = \delta[n] \Longleftrightarrow \frac{1}{2\pi} P_N(e^{j\omega}) \circledast W(e^{j\omega}) = 1$$

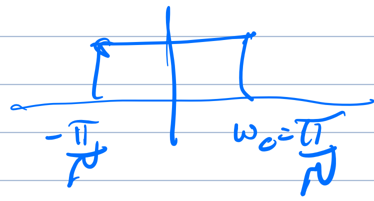
$$1 = \sum_{k=0}^{N-1} \frac{1}{\cancel{2\pi}} \cdot \frac{\cancel{2\pi}}{N} W\left(e^{j\left(\omega - k\frac{2\pi}{N}\right)}\right)$$

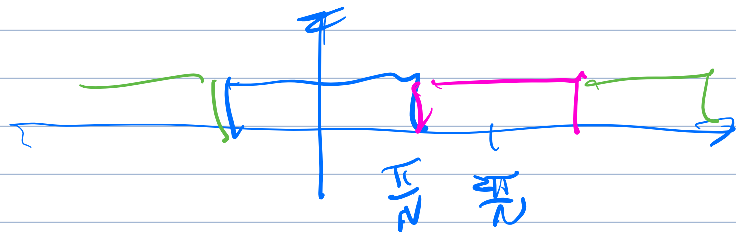
EX... $w[n] = \dfrac{\sin(\omega_0 n)}{\pi n}$

$\omega_0 n = k\pi$

$\omega = \frac{\pi}{N}$







(OLA)
OVERLAP-ADD METHOD of STFT SYNTHESIS

CONSIDER $X[n,\omega] = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega m}$

1. COMPUTE $X[n,\omega)$ FOR $n = rR$

2. COMPUTE IDTFT of $X[rR,\omega)$

SHOULD BE $x[m]\, w[rR-m]$

3. LET $y[n] = \sum_r$ IDTFTS

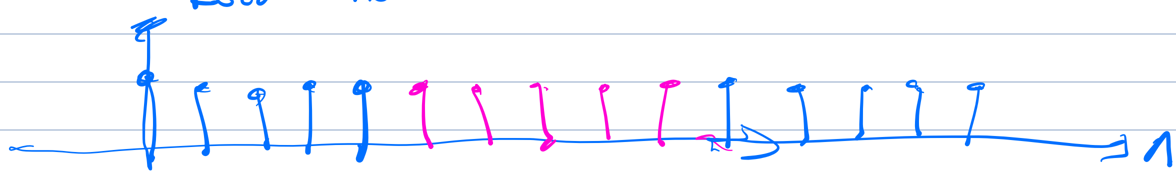
$= \sum_r x[m]\, w[rR-m]$

$= x[m] \sum_r w[rR-m]$

→ SUM OF SHIFTED WINDOWS = 1

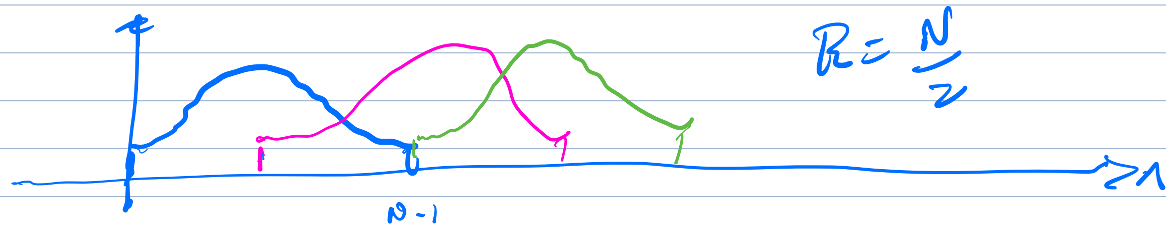
OLA CONSTRAINT $\sum_{r=-\infty}^{\infty} w[rR-m] = 1$

EX: $w[n]$ RECT. WINDOW



$R = N$

HAMMING WINDOW



$R = \frac{N}{2}$

EFFICIENCY of STFT REPRESENTATION

ASSUME $x[n]$ BANDLIMITED

$w[n]$ FIR, LENGTH N_w

F_s SAMPLING FREQ.

N NUMBER of CHANNELS $N \geq N_w$

$\Rightarrow$ TOTAL NUMBER of NUMBERS / SEC

$$= \frac{F_s N}{M}$$

BASELINE F_s

	DOWN SAMP RATIO	RECT. WINDOW	HAMMING WINDOW
		TOTAL # of NUMBERS $F_s N / M$	
FBS, N CHANNELS	M	$\frac{F_s N}{N/2} = 2F_s$	$\frac{F_s N}{(N-1)/4} \approx 4F_s$
OLA	R	$\frac{F_s N}{R} = \frac{F_s N}{N_w} = F_s$	$\frac{F_s N}{R} = \frac{F_s N}{N_w/2} = 2F_s$