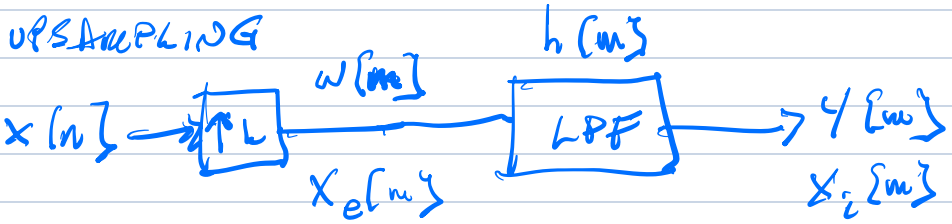
9/11/24 EFFICIENT CHANGES IN SAMPLING RATE (CONT.)

WO 3.3, NOTES 3.2 - 3.3

UPSAMPLING



$$w[m] = x_e[m] = \begin{cases} x[m/L], & m/L \text{ INTEGER} \\ 0, & \text{OTHERWISE} \end{cases}$$

$$y[m] = x_i[m] = \begin{cases} x[m/L], & m/L \text{ INTEGER} \\ \text{SOMETHING ELSE}, & \text{OTHERWISE} \end{cases}$$

$$y[m] = w[m] * h[m] = \sum_{k=-\infty}^{\infty} h[m-k]\, w[k]$$

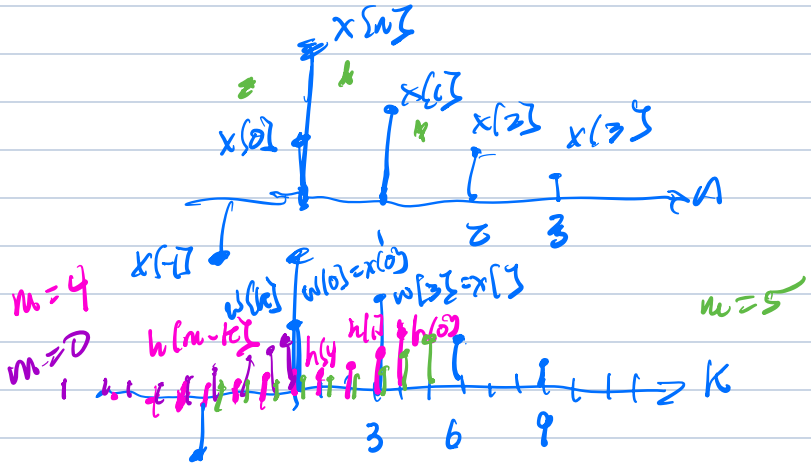
$x[m/L]$, k/L INTEGER

LET $r = \frac{k}{L}$ $\quad y[m] = \sum_{r=-\infty}^{\infty} h[m - rL]\, x[r]$

EX. Let $L = 3$

$h[n] = \begin{cases} \alpha^n, & 0 \le n \le N-1 \\ 0, & \text{ELSE} \end{cases}$

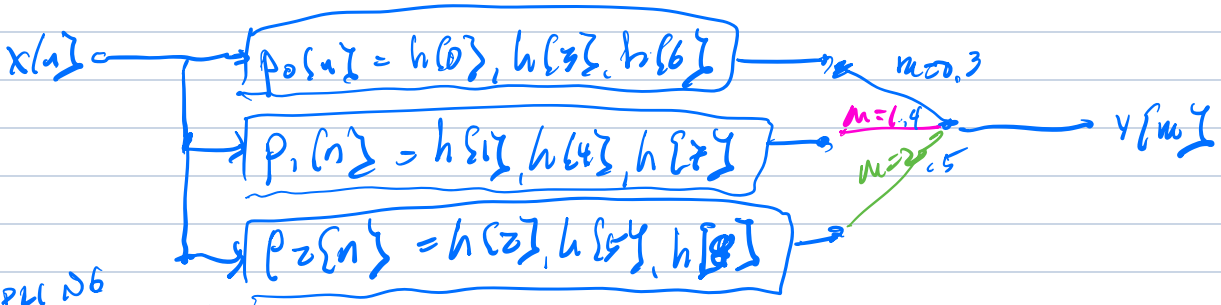
$N = 9$



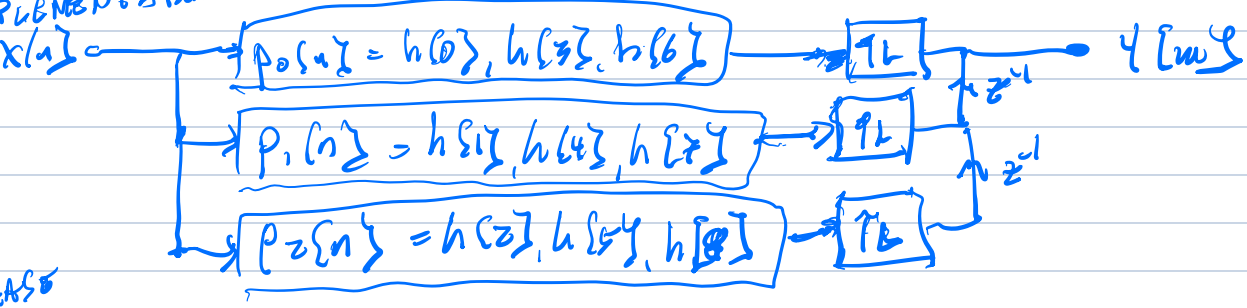
y[m]	m	INDEX of x	INDEX of h
	0	-2, -1, 0	0, 3, 6
	1	-2, -1, 0	1, 4, 7
	2	-2, -1, 0	2, 5, 8
	3	-1, 0, 1	0, 3, 6
	4	-1, 0, 1	1, 4, 7
	5	-1, 0, 1	2, 5, 8
	6	0, 1, 2	0, 3, 6

POLYPHASE FILTER IMPLEMENTATION OF INTERPOLATION

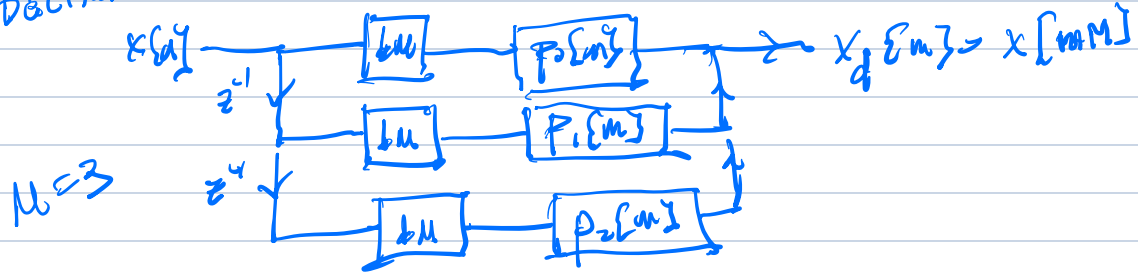
$L=3, N=9$ let $p_l[n] = h[nL+l], \quad 0 \le l \le L-1$



UPSAMPLING IMPLEMENTATION

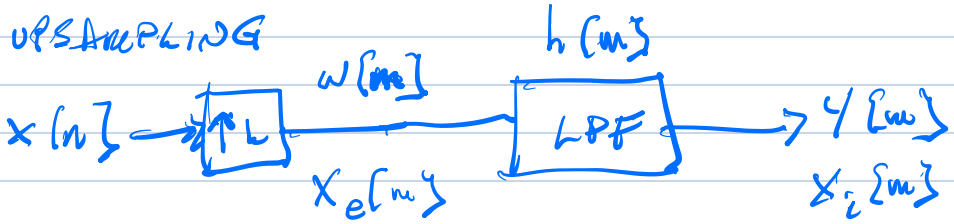


POLYPHASE DECIMATOR



EFFICIENT DECIMATION & INTERPOLATION USING TIME-VARYING COEFFS

UPSAMPLING



$$y[m] = \sum_{r=-\infty}^{\infty} h[m - rL]\, x[r]$$

LET $r = \left\lfloor \frac{m}{L} \right\rfloor - n$

CONSIDER $m - \left\lfloor \frac{m}{L} \right\rfloor L = ((m))_L \quad k = 3$; $m = \left\lfloor \frac{m}{L} \right\rfloor L + ((m))_L$

$$y[m] = \sum_{r=-\infty}^{\infty} h[m-r]\, x[r]$$

$$y[m] = \sum_{n=-\infty}^{\infty} h\left[m - \underbrace{\left(\left\lfloor \frac{m}{L} \right\rfloor - n\right)L}_{m - rL}\right] x\left[\left\lfloor \frac{m}{L} \right\rfloor - n\right]$$

$$y[m] = \sum_{n=-\infty}^{\infty} g_m[n]\, x\left[\left\lfloor \frac{m}{L} \right\rfloor - n\right]$$

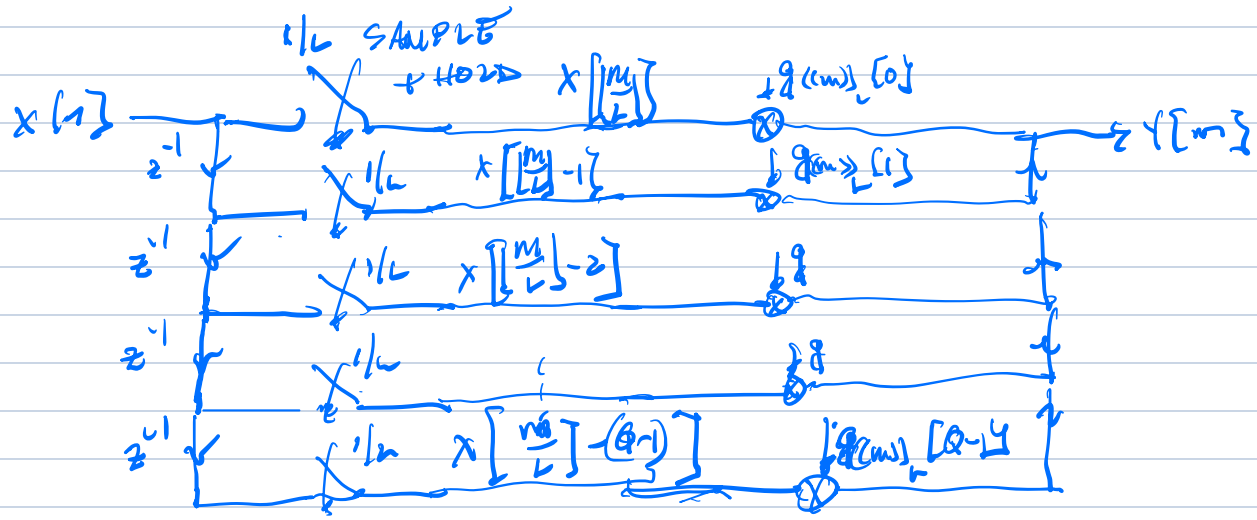
WHERE $g_m[n] = h[nL + ((m))_L]$

$$= h\left[nL + m - \left\lfloor \frac{m}{L} \right\rfloor L\right]$$

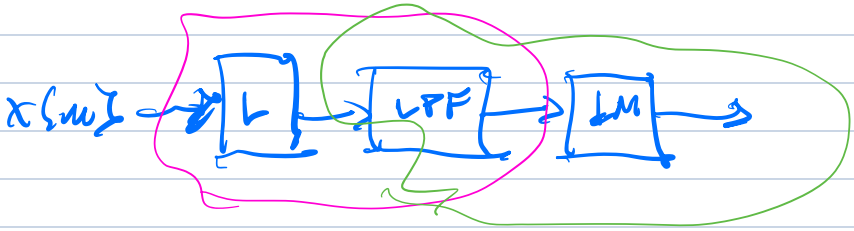
INTERPOLATION USING TIME-VARYING COEFFS

ASSUME FILTER LENGTH $N = QL$; $Q = N/L$

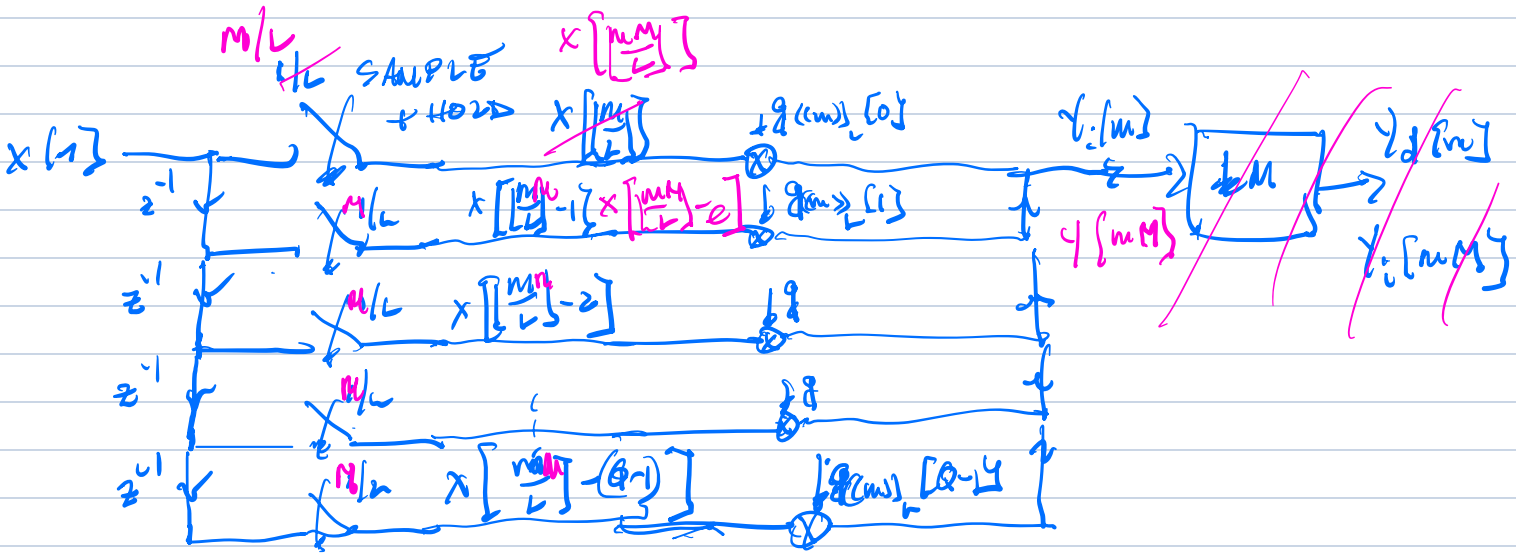
$L = 3$



CHANGE IN SAMPLING RATE BY L/M



EFFICIENT UP SAMPLING DOWN SAMPLING



INTERP BY L: $y[n] = \sum_n g_{((n))_L}[n] \, x\left[\left\lfloor \frac{n}{L} \right\rfloor - n\right]$

INTERP BY L, DECIMATE BY M

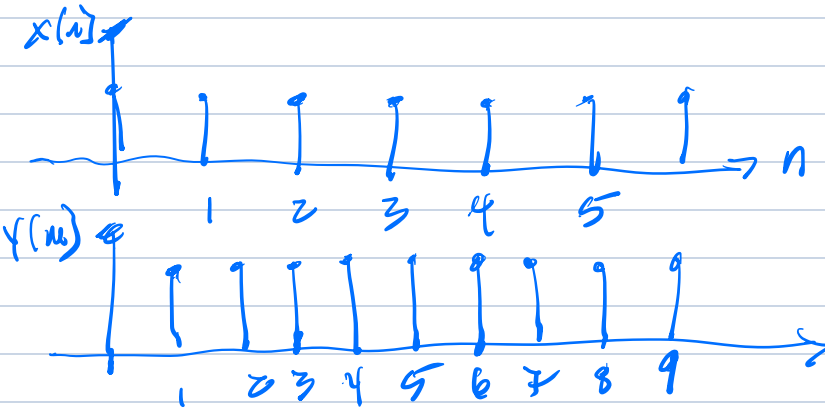
$$y[nM] = \sum_n g_{((n))_L}[n] \, x\left[\left\lfloor \frac{nM}{L} \right\rfloor - n\right]$$

WHERE $g_n[n]$ IS NOW $h[nL + ((nM))_L]$

WHAT ARE $((n))_L$ AND $\lfloor nM/L \rfloor$ (SEE EQ. 2.16 IN NOTES)

$((nM))_L = ((2n))_3$ $\quad \lfloor nM/L \rfloor$

LET $M = 2$, $L = 3$



0	0	0
1	2	0
2	1	1
3	0	2
4	2	2
5	1	3
6	0	4