9/6/23 CHANGE IN SAMPLING RATE

LO 3.0 - 3.3, BLS 1.4

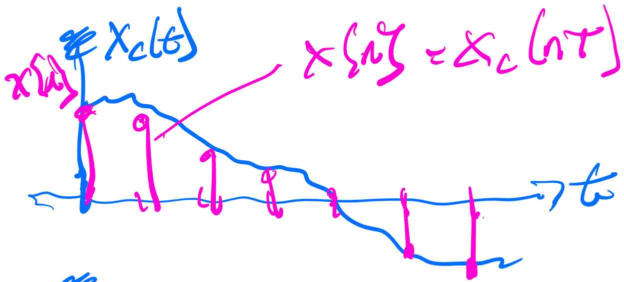
IF $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$

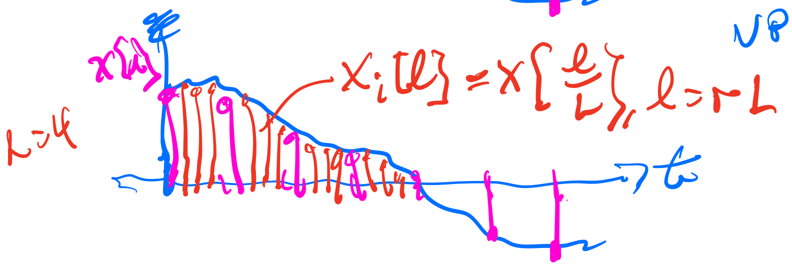
DTFT

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

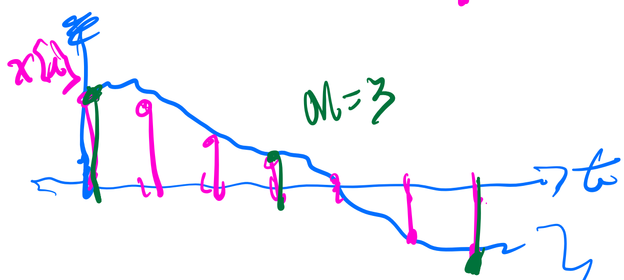
THINGS WE CAN DO





UPSAMPLING (INTERPOLATION) BY $L=4$

$$x[n] \to \boxed{\uparrow L} \to x_i[l]$$



DOWNSAMPLING (DECIMATION) BY $M=3$

$$x[n] \to \boxed{\downarrow M} \to x_d[m]$$

$$x_d[m] = x[mM]$$

INCORRECT FREQ. ANALYSIS

$$\cancel{x_d[m] = x[mM]}$$

$$\cancel{X_d(e^{j\omega'}) = \sum_{m=-\infty}^{\infty} x_d[m] e^{-j\omega' m}}$$

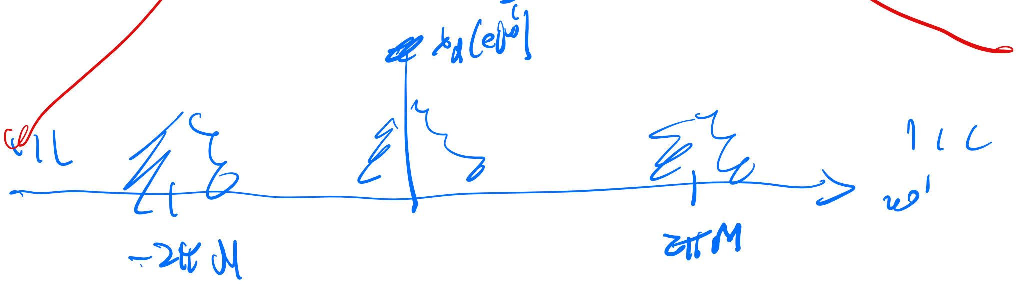
$$\cancel{X_d(e^{j\omega'}) = \sum_{m=-\infty}^{\infty} x[mM] e^{-j\omega' m}}$$

WRONG!

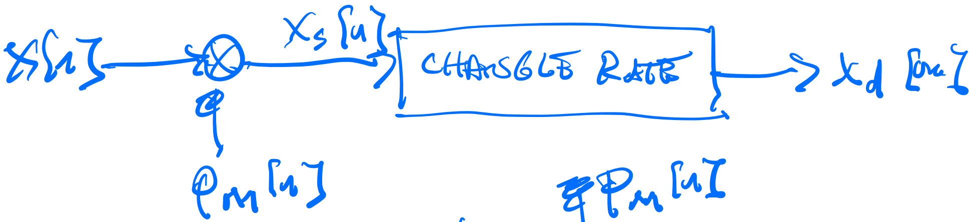
let $s = mM$
$m = s/M$

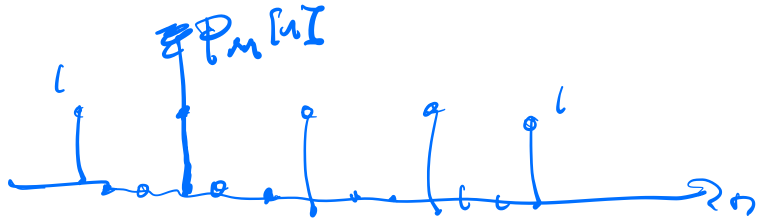
$$X_d(e^{j\omega}) = \sum_{s=-\infty}^{\infty} x[s]\, e^{-j\omega \frac{s}{M}} \qquad s = rM$$

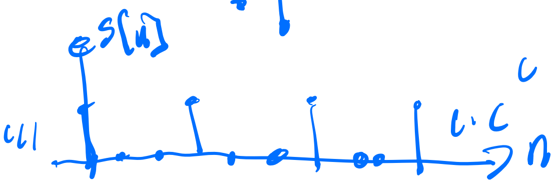
$$= X(e^{j\frac{\omega}{M}})$$

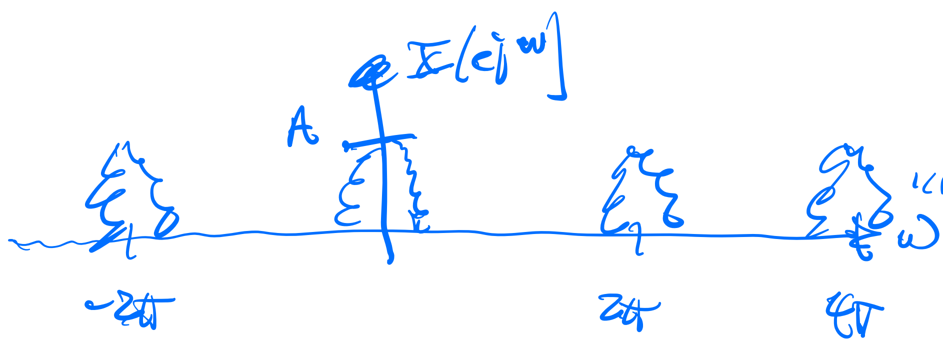


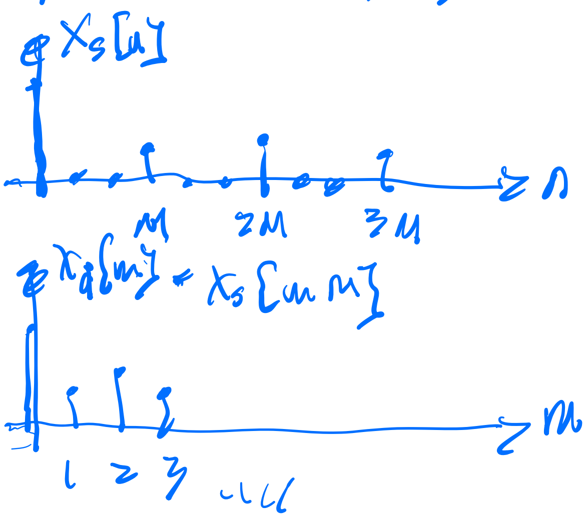
CORRECT WAY TO DECIMATE BY M:











$$x_s[n] = x[n]\, p_M[n]$$

$$\Updownarrow$$

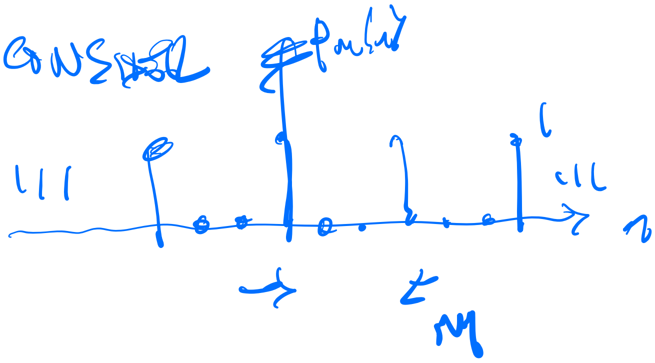
$$X_s(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \circledast P_M(e^{j\omega})$$

$$X_s(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})\, P_M(e^{j(\omega-\theta)})\, d\theta$$

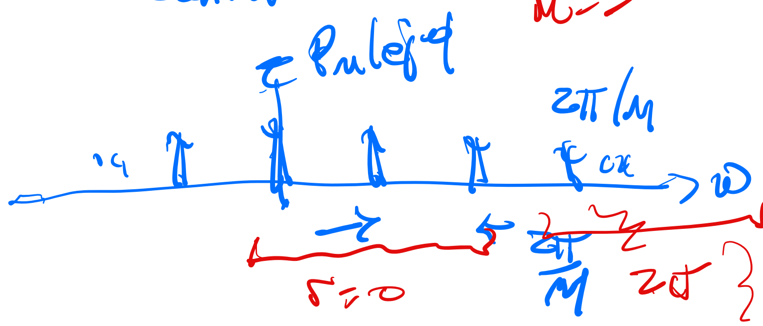
$= X(e^{j\omega}) *$ ONE PERIOD of $P_M(e^{j\omega})$

DTFT of $p_M[n]$

CONSIDER $p_M[n]$



CLAIM:



$$p_M[n] = \sum_{l=-\infty}^{\infty} \delta[n - lM]$$

$$P_M(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{M}\,\delta\left(\omega - \frac{2\pi k}{M}\right)$$

$$= \sum_{r=-\infty}^{\infty}\sum_{l=0}^{M-1} \frac{2\pi}{M}\,\delta\left(\omega - \frac{2\pi l}{M} - 2\pi r\right)$$

$$p_M[n] = \frac{1}{2\pi}\int_{0-\epsilon}^{2\pi-\epsilon} P_M(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{0-\epsilon}^{2\pi-\epsilon} \frac{2\pi}{M}\sum_{l=0}^{M-1}\delta\left[\omega - \frac{2\pi l}{M}\right] e^{j\omega n}\, d\omega$$

$$= \sum_{l=0}^{M-1}\frac{1}{M}\underbrace{\int_{0-\epsilon}^{2\pi-\epsilon}\delta\left(\omega-\frac{2\pi l}{M}\right) e^{j\omega n}\, d\omega}$$

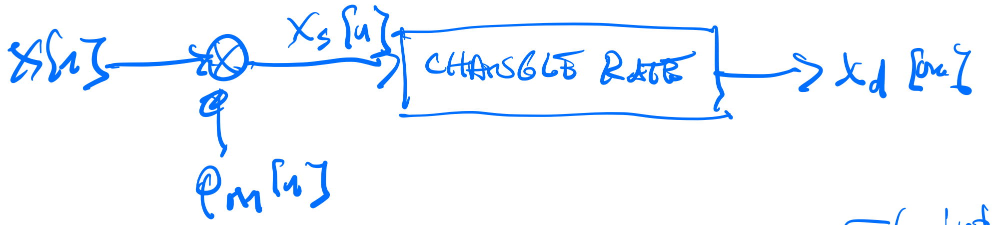
$$\sum_{l=0}^{M-1}\frac{1}{M}e^{j\frac{2\pi l n}{M}} = 1, \quad l = rM$$

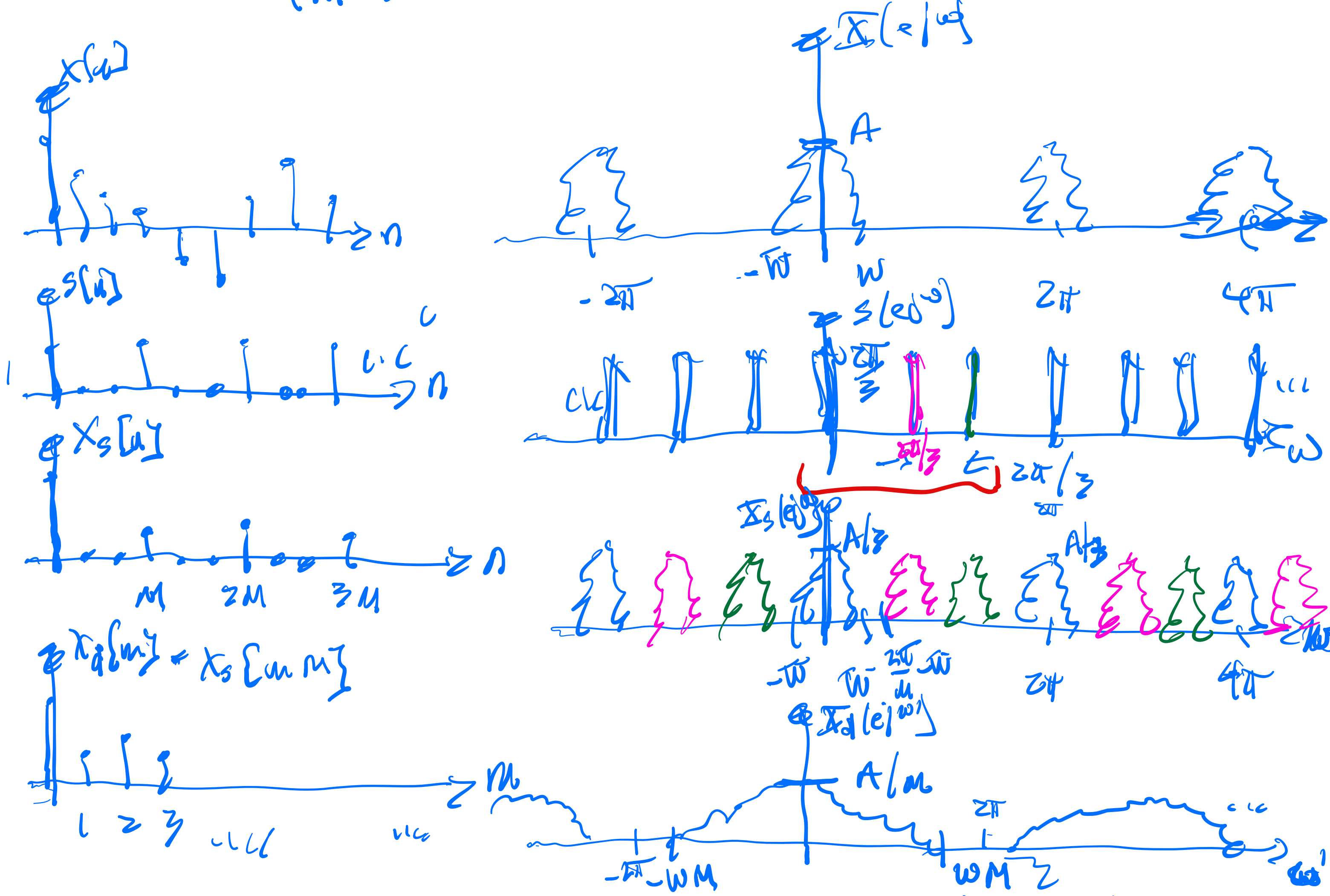
$$= \frac{1}{M}\,\frac{1-e^{j\frac{2\pi l M}{M}}}{1-e^{j\frac{2\pi l}{M}}} \qquad l \neq rM \quad = 0$$

$$\sum_{n=0}^{\infty}\alpha^n = \frac{1}{1-\alpha}, \quad |\alpha|<1$$

$$\sum_{n=0}^{N-1}\alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

$$p_M[n] = \begin{cases} 1, & n = rM \\ 0, & \text{ELSE} \end{cases}$$





$$x_s[n] = x[n]\,s[n] \Leftrightarrow \frac{1}{2\pi} X(e^{j\omega}) \circledast S(e^{j\omega})$$

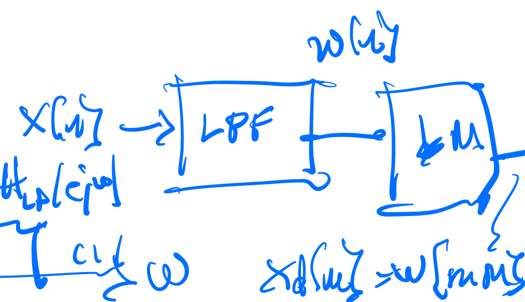
$\left(\frac{2\pi}{M} - \omega\right)M$

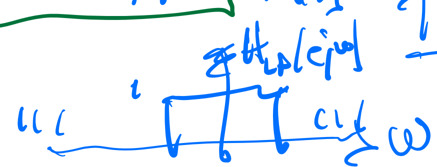
TO AVOID DT "ALIASING" NEED $W < \frac{2\pi}{M} - W$

TO AVOID ALIASING

$$2W < \frac{2\pi}{M}$$

$$\boxed{W < \frac{\pi}{M}}$$



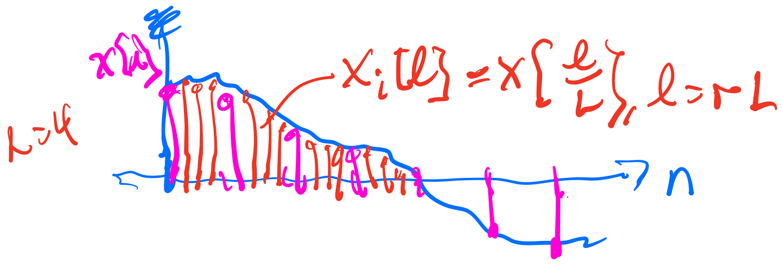


$$x_d[m] = x_s[mM]$$

$$X_d(e^{j\omega'}) = X_s(e^{j\omega'/M})$$

$\frac{\pi}{M}$ π/M

INTERPOLATION BY L



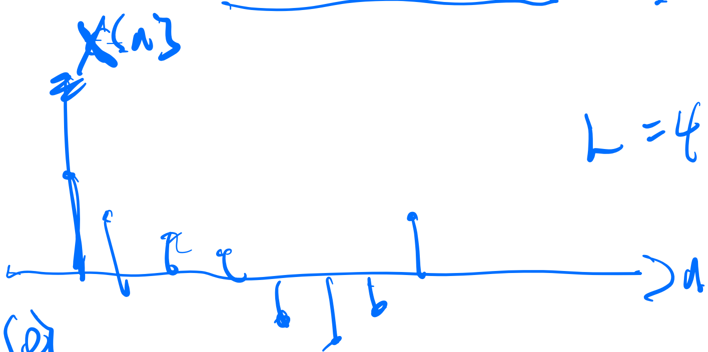
$x[n] \rightarrow \boxed{\uparrow L} \rightarrow x_i[\ell]$

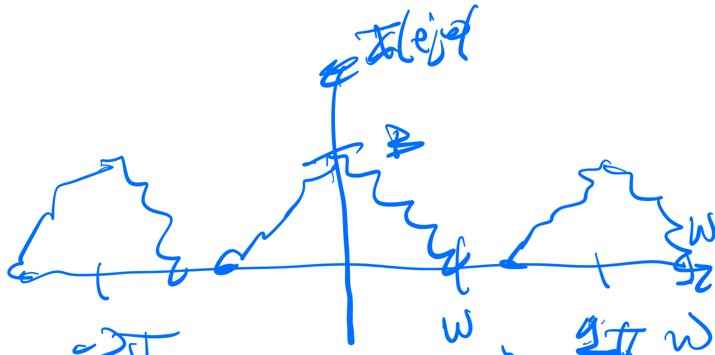
$x_i[\ell] = x\left[\frac{\ell}{L}\right],$

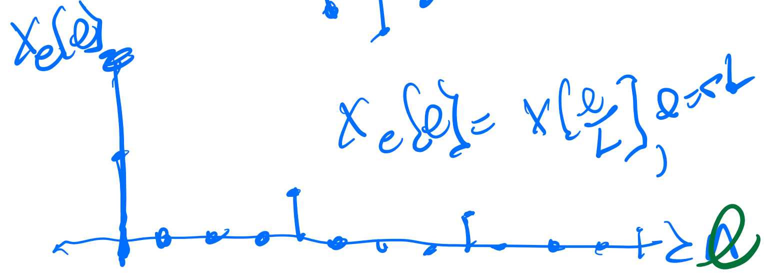
$\ell = rL$

METHOD:

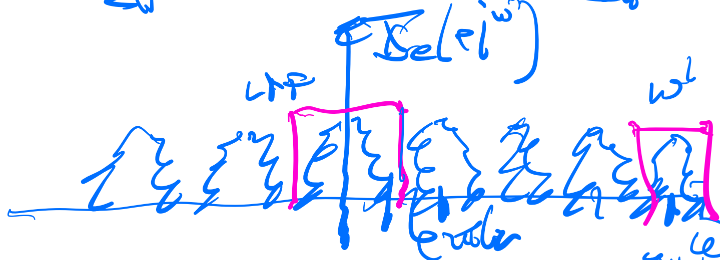
$x[n] \rightarrow$ INSERT L−1 SAMPLES OF 0 $\xrightarrow{x_e[\ell]}$ $\boxed{\text{LPF}} \rightarrow x_i[\ell]$

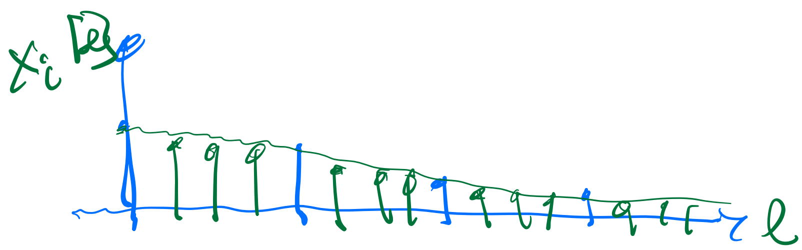


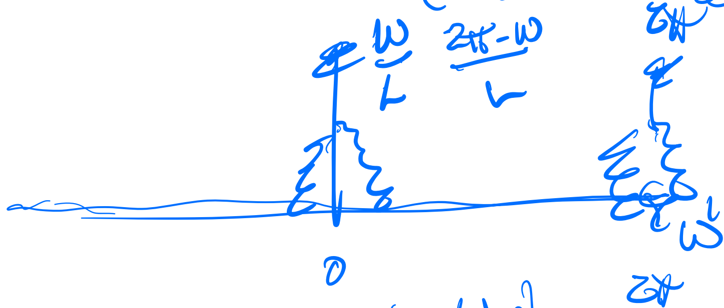




$x_e[\ell] = x\left[\frac{\ell}{L}\right],\ \ell = rL$



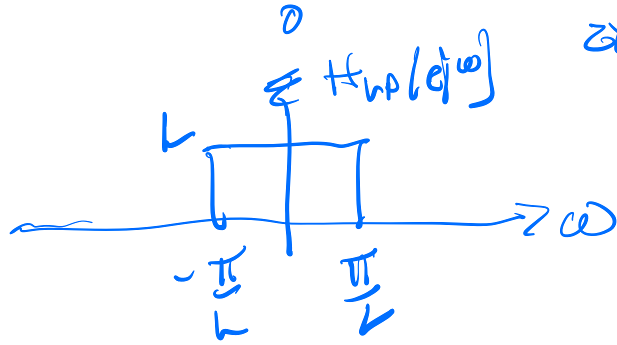




$$X_e(e^{j\omega'}) = \sum_{\ell=-\infty}^{\infty} x_e[\ell]\, e^{j\omega'\ell}$$

$$= \sum_{\substack{\ell=-\infty \\ \ell = rL}}^{\infty} x\left[\frac{\ell}{L}\right] e^{j\omega'\ell}$$

$r = \frac{\ell}{L};\ \ \ell = rL$



$$= \sum_{s=-\infty}^{\infty} x[s]\, e^{j\cancel{\omega' s L}\ \omega' L s}$$

$$X_e(e^{j\omega'}) = X(e^{jL\omega'})$$