REVIEW OF CONTINUOUS-TIME SAMPLING

8/28/24 (OSYP 4.0-4.6; NOTES 1.1-1.3)

$\boxed{\text{CTFT}}$

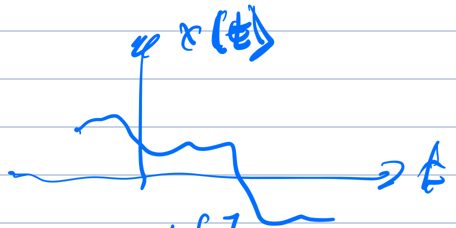
IF $\int_{-\infty}^{\infty} |x(t)|^2 \, dt < \infty$

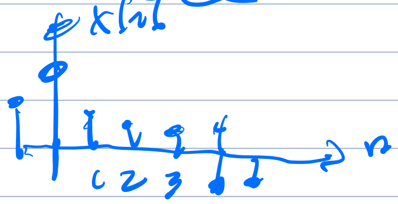
$$\begin{cases} x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} d\Omega \\ X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt \end{cases}$$

IF $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$ DTFT

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{+j\omega n} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$



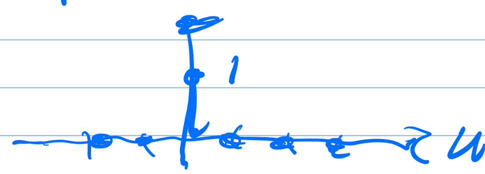


$e^{j\omega_0 n}$, $e^{j(\omega_0 + k 2\pi)n}$

$= e^{j\omega_0 n} \cdot \cancel{e^{j k 2\pi n}}$

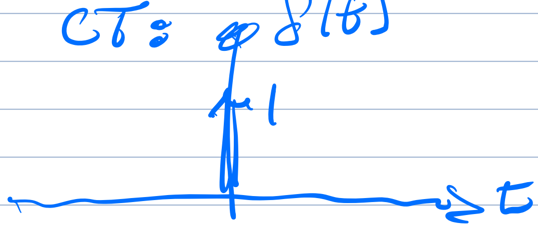
DELTA FUNCTIONS IN DT, CT

DT: $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$



$\delta[n-k] = \begin{cases} 1, & n = k \\ 0, & n \neq k \end{cases}$

CT: $\delta(t)$



$\delta(t) = 0, \; t \neq 0$

$\delta(0) = ?$

$\int_{-\infty}^{\infty} \delta(t) \, dt = 1$

Let $\phi(t)$ = CONTINUOUS FUNCTION

DEFINE $\displaystyle\int_{-\infty}^{\infty} \delta(t-a)\,\phi(t)\,dt \triangleq \phi(a)$

TO EVALUATE EXPRESSION...

1. WHAT VARIABLE IS INTEGRATED? (t)
2. WHAT IS VALUE OF $[t]$ THAT CAUSES $(\cdot)$ FOR ARG. TO BECOME ZERO? $(t=a)$
3. WHAT IS THE REST OF THE INTEGRAND AT $t=a$? $(\phi(a))$

$$\int_{-\infty}^{\infty} \delta(t)\cdot 1\,dt \overset{?}{=} 1$$

LTI $\quad x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

Let $h(t) = \delta(t-T)$

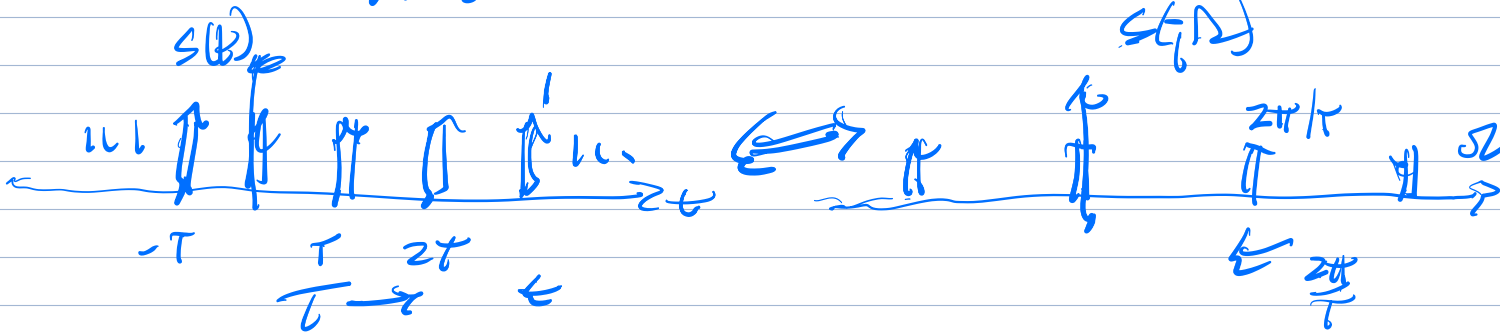
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} x(t-\tau)\,h(\tau)\,d\tau$$

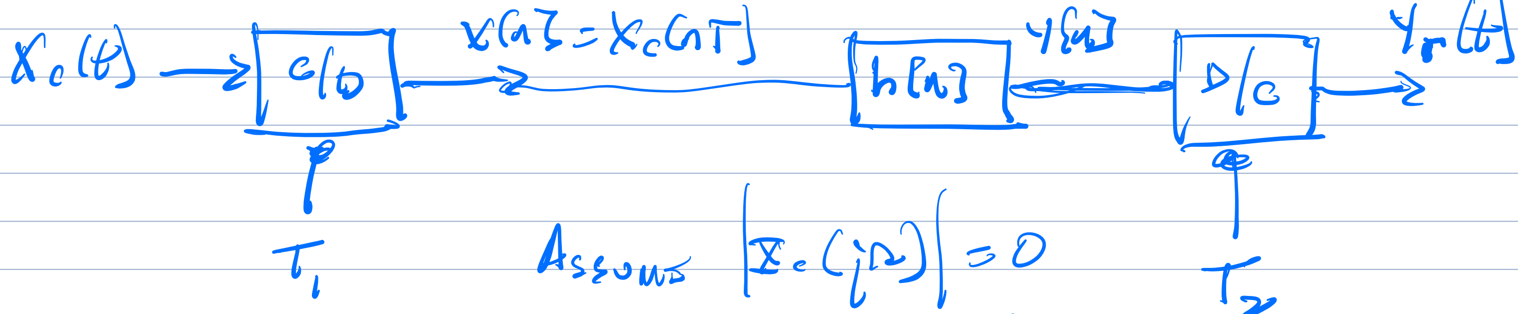
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau-T)\,d\tau = x(t-T)$$

SAMPLING IN CONTINUOUS TIME

LET $s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$ $\quad S(j\Omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\delta\left(\Omega - \frac{2\pi k}{T}\right)$

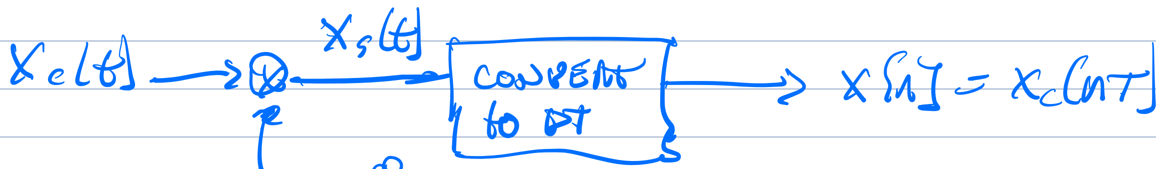


SAMPLING IN CONTINUOUS TIME

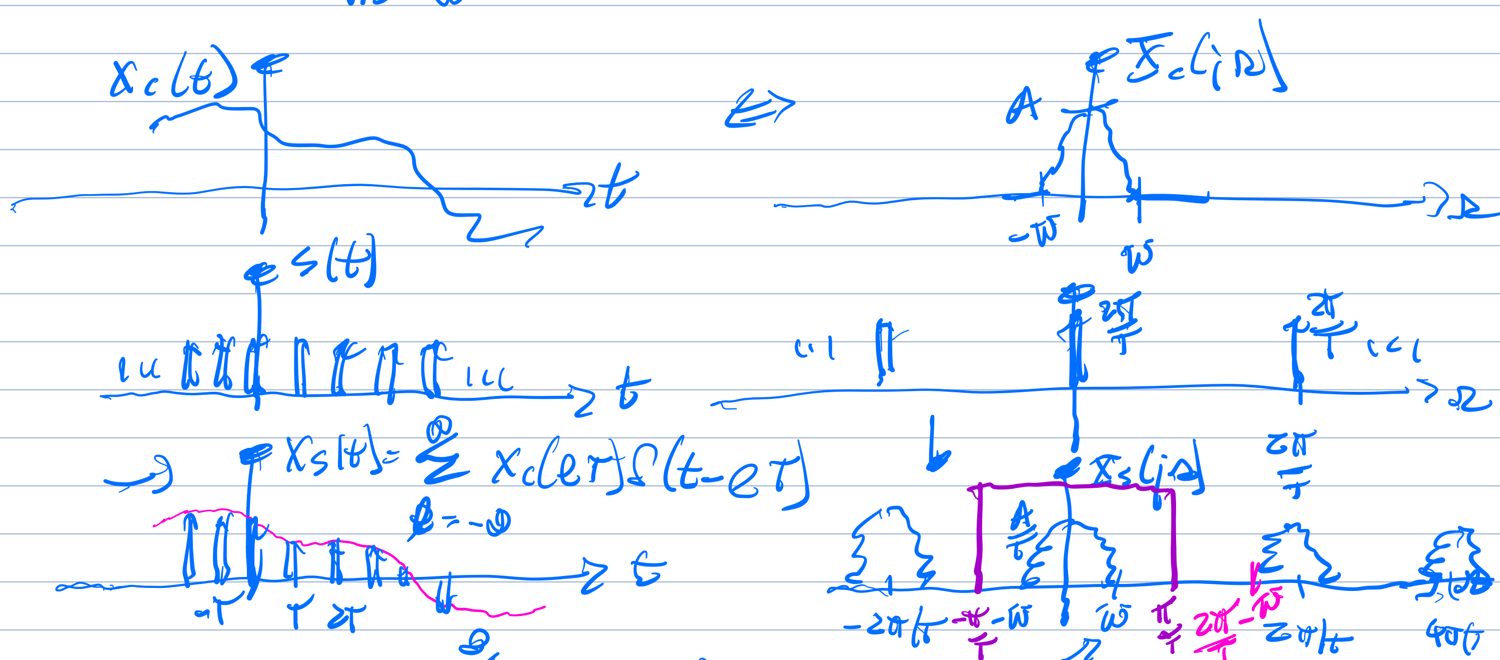


ASSUME $|X_c(j\Omega)| = 0$ $\quad |\Omega| > W$

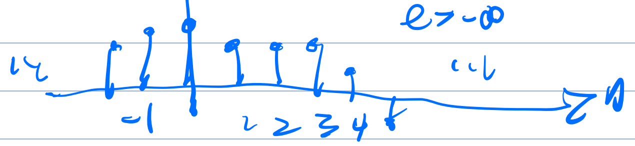
C/D CONVERSION IN DETAIL

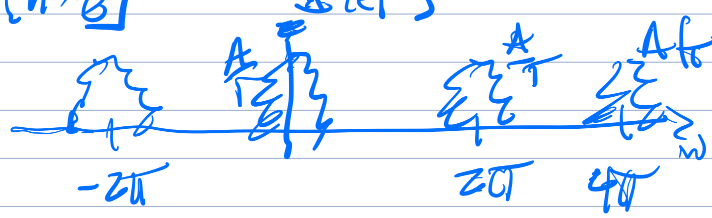


$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$



$$x[n] = \sum_{l=-\infty}^{\infty} x_c(lT)\,\delta[n-l]$$





CT MULTIPLICATION

$$x_1(t)\cdot x_2(t) \Longleftrightarrow \frac{1}{2\pi} X_1(j\Omega) * X_2(j\Omega)$$

$\omega = \Omega T$

$\Omega = \frac{2\pi}{T} \rightarrow \omega = \frac{2\pi}{T} T$

$$X_s(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(j\left(\Omega - k\frac{2\pi}{T}\right)\right)$$

$$\text{CTFT}\{x_s(t)\} = X_s(j\Omega) = \int_{-\infty}^{\infty} x_s(t)\, e^{-j\Omega t}\, dt$$

$$X_s(j\Omega) = \int_{-\infty}^{\infty} \left(\sum_{l=-\infty}^{\infty} x_c(lT)\,\delta(t-lT)\right) e^{-j\Omega t}\, dt$$

$$= \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} x_c(lT)\,\delta(t-lT)\, e^{-j\Omega t}\, dt$$

$$= \sum_{l=-\infty}^{\infty} x_c(lT) \underbrace{\int_{-\infty}^{\infty} \delta(t-lT)\, e^{-j\Omega t}\, dt}_{e^{-j\Omega l T}}$$

CTFT of $x_s(t)$

$$X_s(j\Omega) = \sum_{l=-\infty}^{\infty} x_c(lT)\, e^{-j\Omega l T}$$

$\omega = \Omega T$

DTFT of $x[n]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-j\omega n}$$

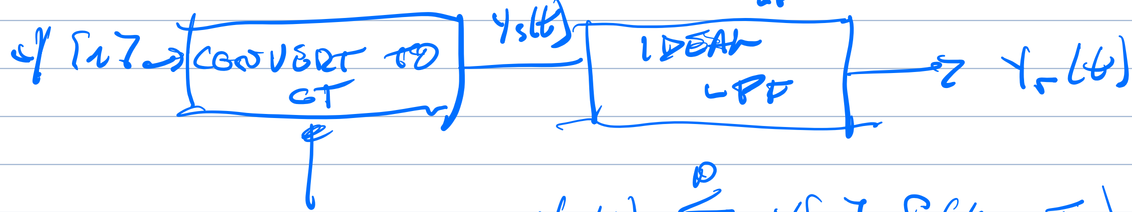
$\text{RAD} = \frac{\text{RAD}}{\text{SEC}}\ \text{SEC}$

To avoid overlap $\frac{2\pi}{T} - W \geq W$

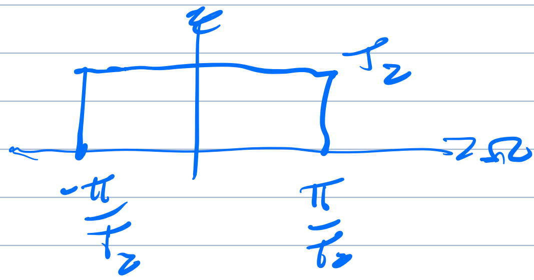
NYQUIST WIDTH $\boxed{W < \frac{\pi}{T}}$

Recovery from sampled signals

D/C conversion in detail



$$y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t - nT_2)$$

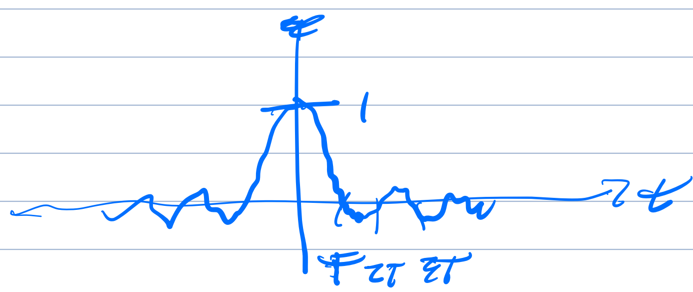


Let $T_1 = T_2 = T$

Time domain analysis

$$y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t - nT)$$

$$h_{LP}(t) = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} T\, e^{j\Omega t}\, d\Omega = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}}$$



$\frac{\pi t}{T} = k\pi$

$t = kT$

$$y(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} * \sum_{n=-\infty}^{\infty} y[n]\, \delta(t - nT)$$

$$= \sum_{n=-\infty}^{\infty} y[n] \cdot \frac{\sin\left(\frac{\pi (t-nT)}{T}\right)}{\frac{\pi (t-nT)}{T}}$$