18734 Recitation

Basic Probability Theory

Definition of Probability

- ***Experiment***: toss a coin twice
- ***Sample space***: possible outcomes of an experiment
  - S = {HH, HT, TH, TT}
- ***Event***: a subset of possible outcomes
  - A={HH}, B={HT, TH}, C={TT}
- ***Probability of an event*** : an number assigned to an event Pr(A)
  - Axiom 1: Pr(A) ≥ 0
  - Axiom 2: Pr(S) = 1
  - Axiom 3: For every sequence of disjoint events

$$\Pr\left(\bigcup_i A_i\right) = \sum_i \Pr(A_i)$$

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Assuming coin is fair,
P(A) = ¼ P(B) = ½ P(C) = ¼

What is the probability that we get at least one head?
P({HH, HT, TH}) = ¾

Joint Probability

- For events A and B, **joint probability** $Pr(A \cap B)$ stands for the probability that both events happen.
- Example: A={HH}, B={HT, TH}, what is the joint probability $Pr(A \cap B)$?
- $P(A \cap B) = 0$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Joint Probability

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- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- P({HH, HT, TH})

  $= P(\{HH\} \cup \{HT, TH\}) = P(A \cup B) = P(A)+P(B) - P(A \cap B)$

  $= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

Conditional Probability

- If A and B are events with $\Pr(A) > 0$, the ***conditional probability of B given A*** is

$$\Pr(B|A) = \Pr(A \cap B)/\Pr(A)$$

Random Variable (RV)

- A ***random variable X*** is a function from the sample space to a real number
- X : {HH} -> 2;   {HT, TH} -> 1;   {TT} -> 0

- Pr(X=0) = Pr(C), where C = {TT}

- A discrete RV takes on finite number of values
- A continuous RV can take an uncountable number of values

Discrete RV

- Probability Mass Function (PMF) p_X gives the probability that X will take on a particular value

- $p_X(a) = Pr(X=a)$

- $\sum_i p_X(a_i) = 1$

Continuous RV

- Probability Density Function (PDF) f_X is a non-negative function such that

$$Pr(a<=X<=b) = \int_a^b f_X(x)dx$$

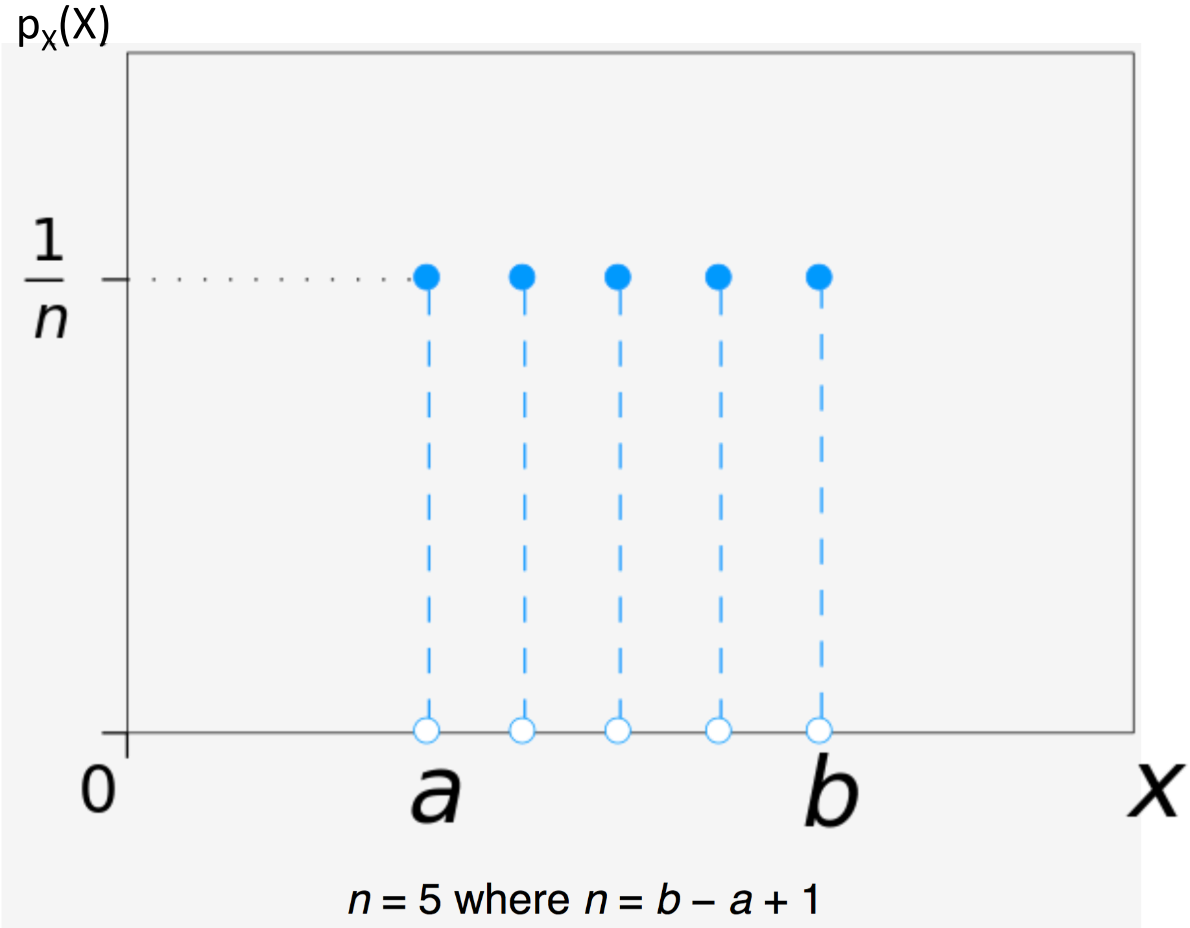
- The integral from $-\infty$ to $+\infty$ is 1

- $Pr(X=a) = 0$

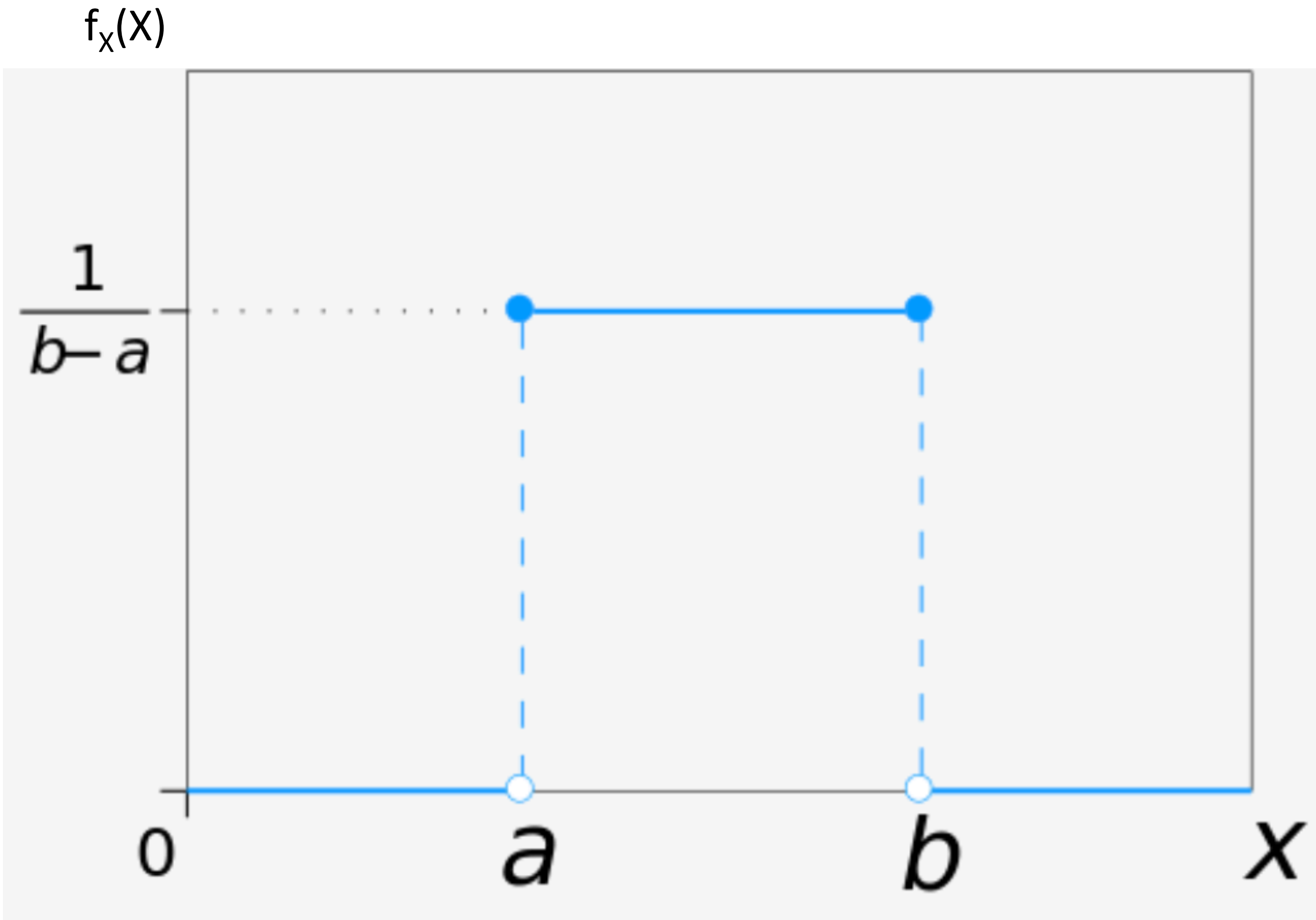
Probability Distribution

- A distribution assigns a probability to each event in the sample space

Discrete Uniform Distribution



Continuous Uniform Distribution



Laplace Distribution

PDF = $$\frac{1}{2b} exp(-\frac{|y-\mu|}{b})$$