# 18-600 Foundations of Computer Systems

## Lecture 13:
## "Cache Memories"

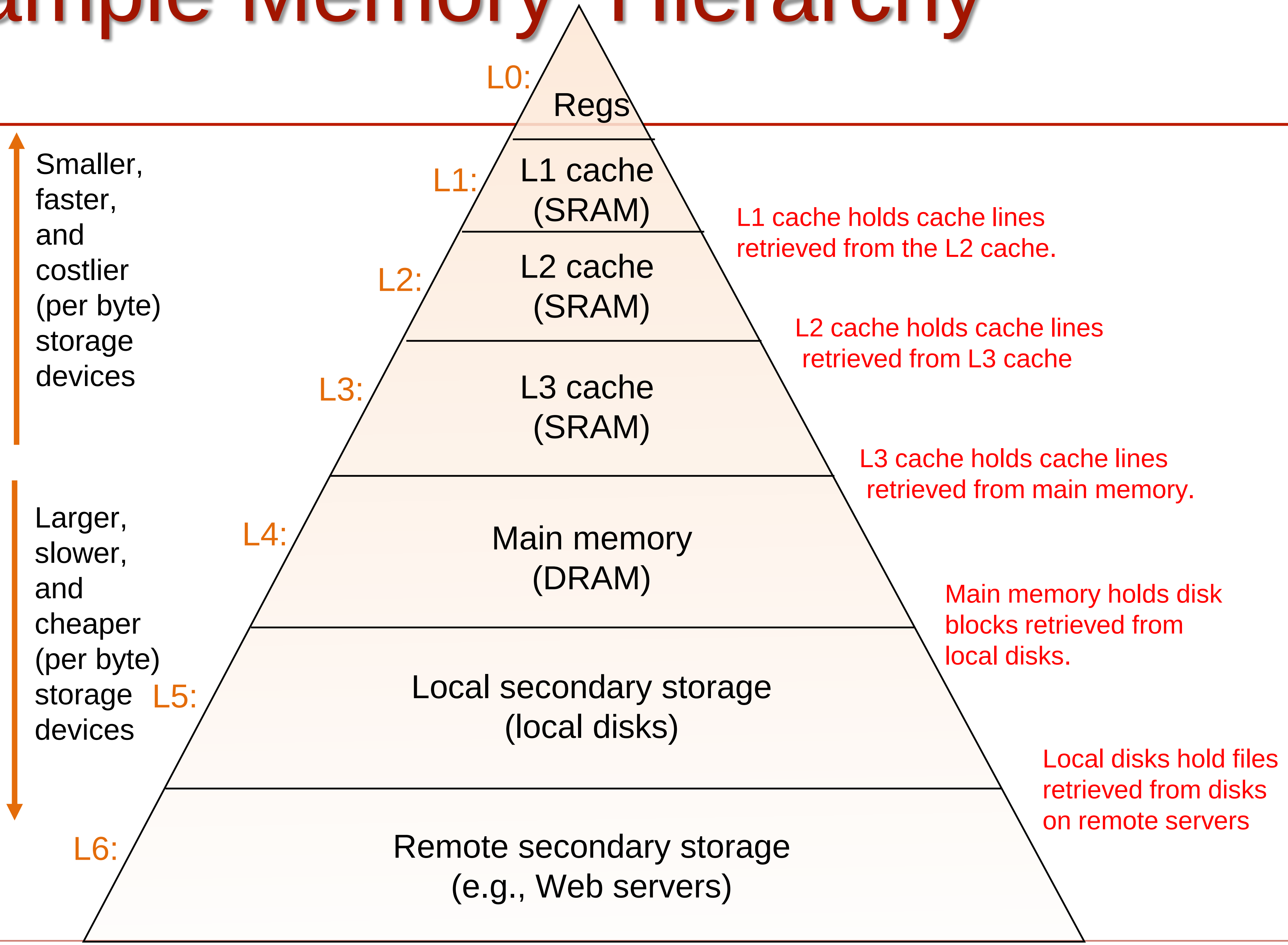
John Shen & Zhiyi Yu
October 12, 2016

- Required Reading Assignment:
  - **Chapter 6 of CS:APP (3rd edition) by Randy Bryant & Dave O'Hallaron**
- Assignments for This Week:
  - Lab 3 due, Lab 4 out

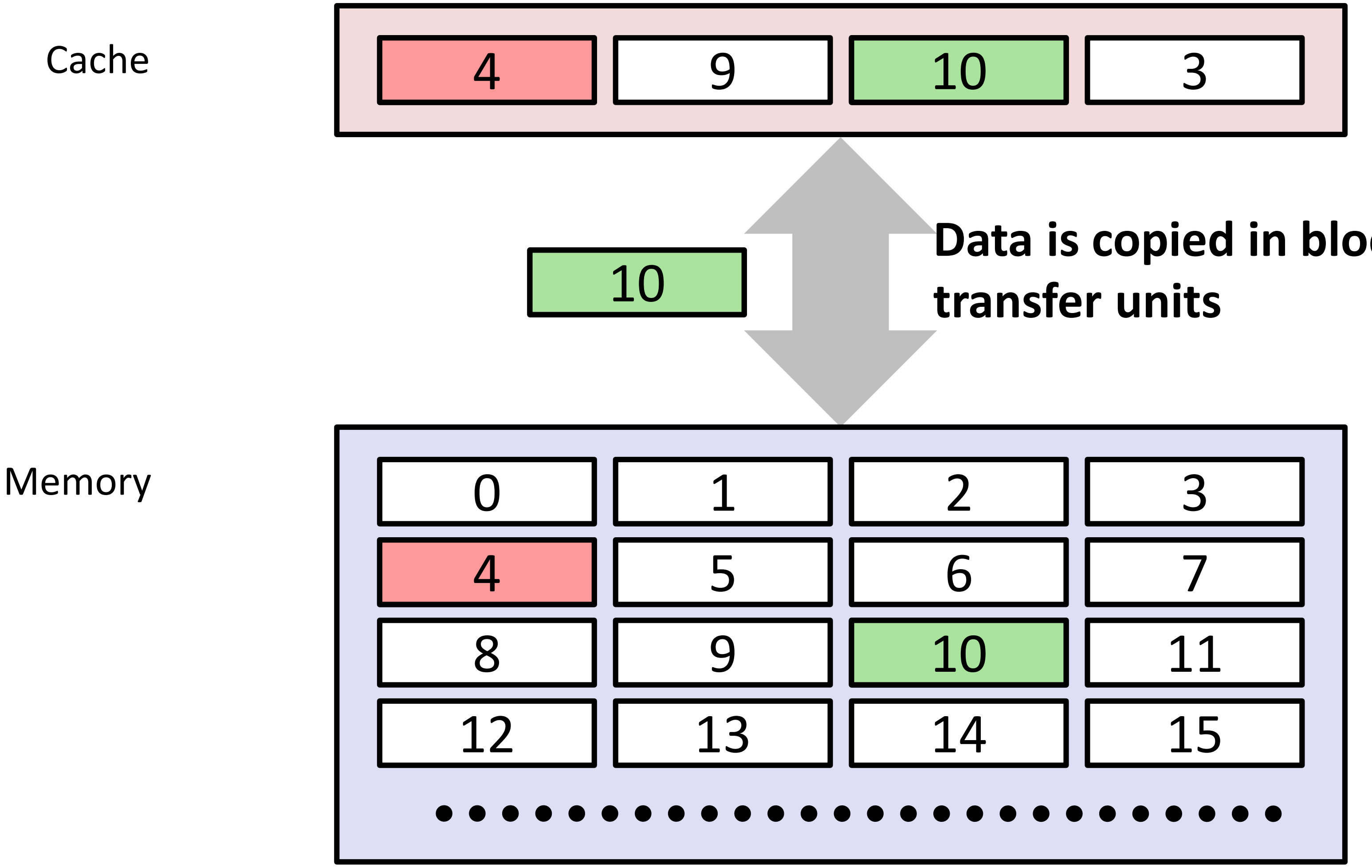
# Today

- Cache memory organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

# Example Memory Hierarchy



# General Cache Concept

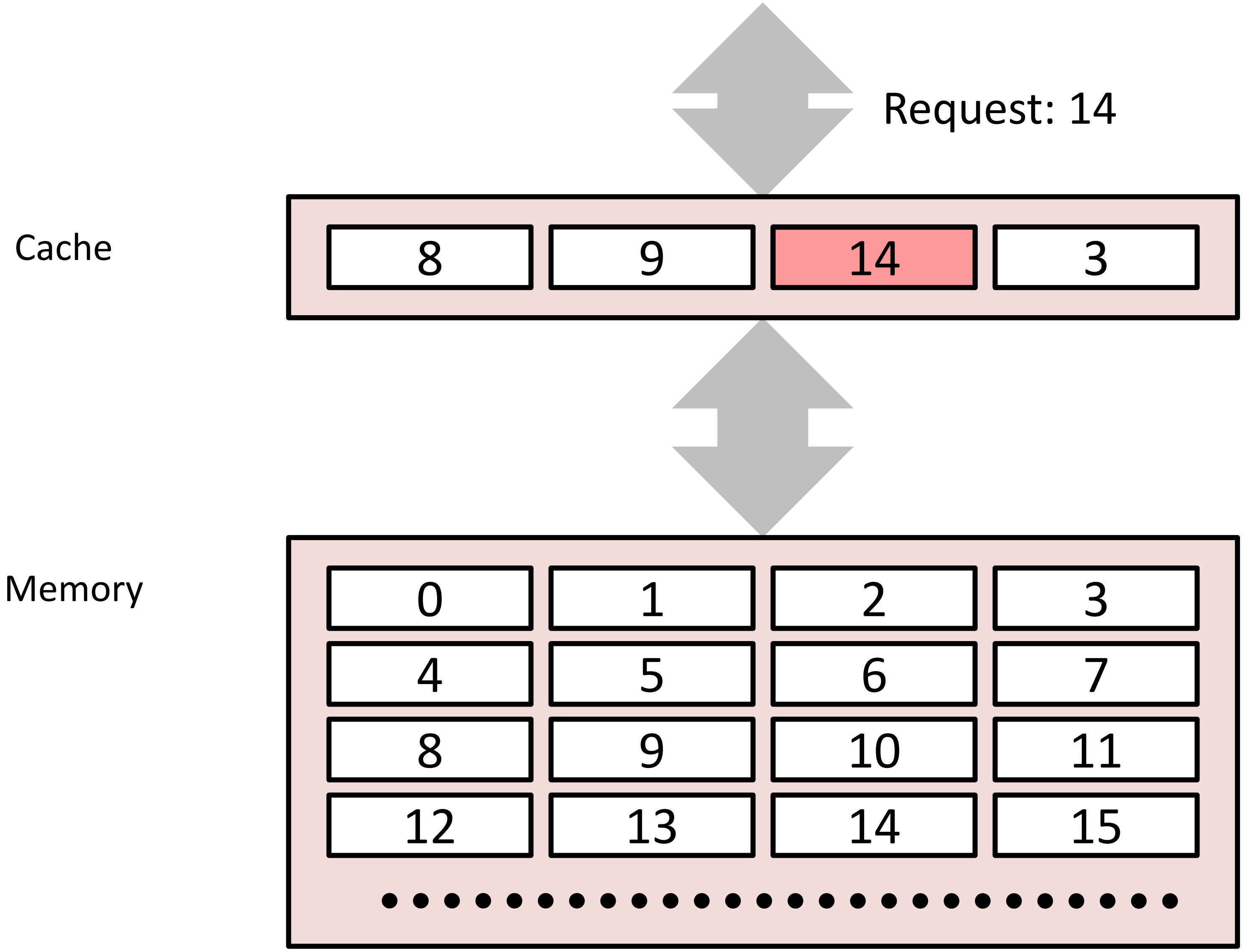



**Smaller, faster, more expensive memory caches a subset of the blocks**

**Data is copied in block-sized transfer units**

**Larger, slower, cheaper memory viewed as partitioned into "blocks"**

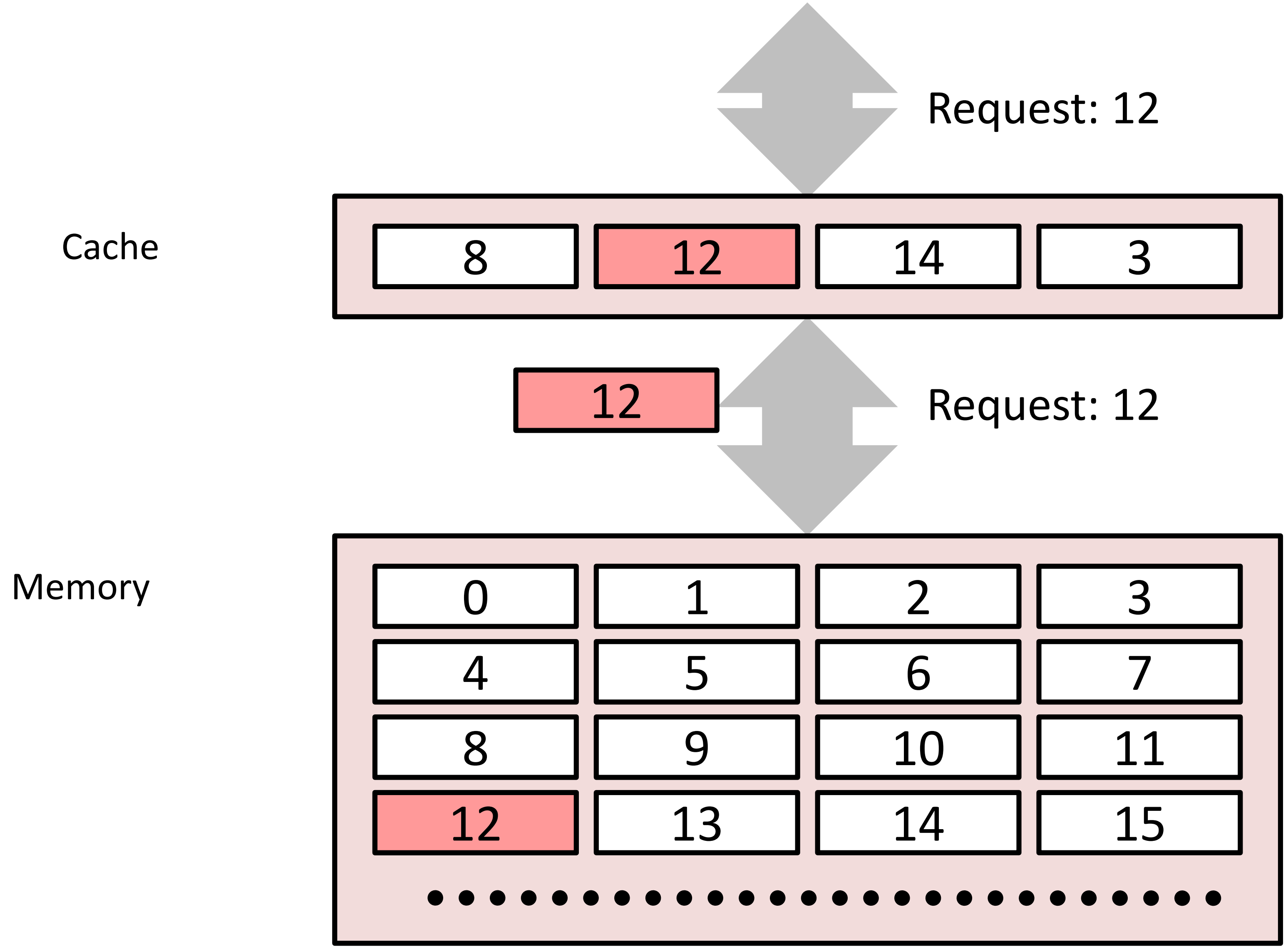
# General Cache Concepts: Hit




***Data in block b is needed***

***Block b is in cache:***
*Hit!*

# General Cache Concepts: Miss




***Data in block b is needed***

***Block b is not in cache:***
***Miss!***

***Block b is fetched from***
*memory*

***Block b is stored in cache***

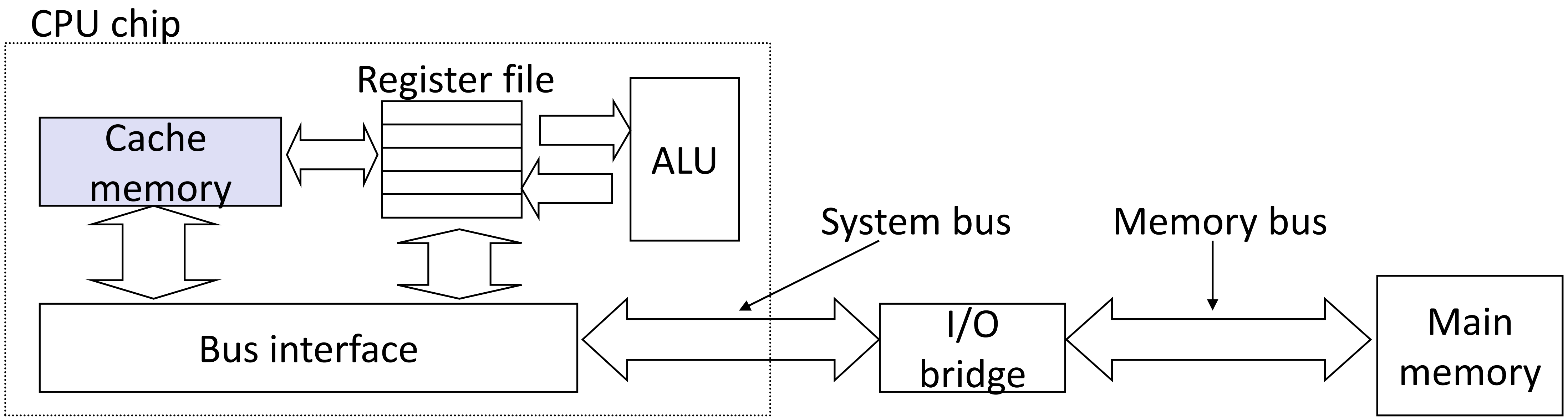
- Placement policy: determines where b goes
- Replacement policy: determines which block gets evicted (victim)

# General Caching Concepts: Types of Cache Misses

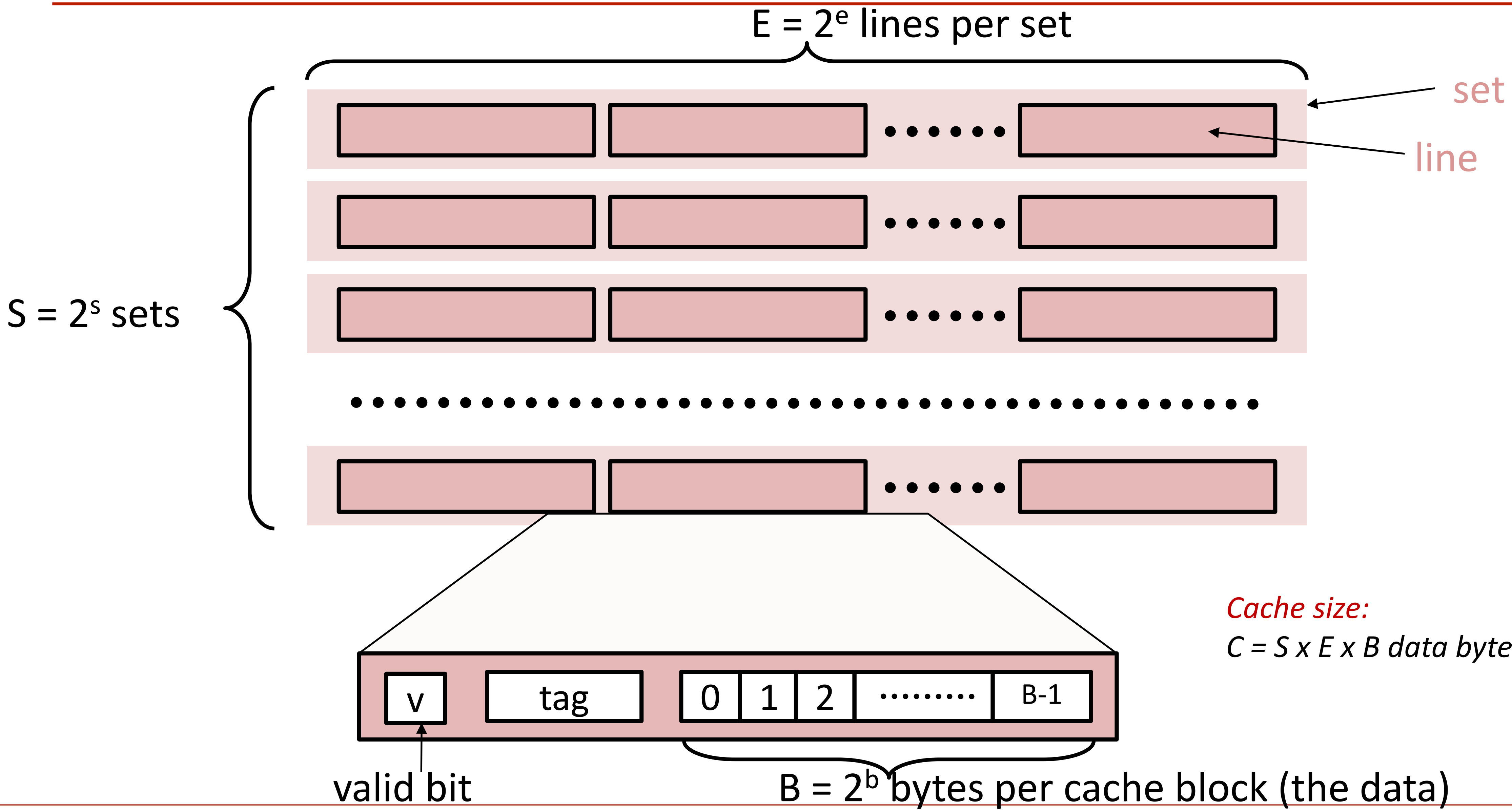
- Cold (compulsory) miss
  - Cold misses occur because the cache is empty.
- Capacity miss
  - Occurs when the set of active cache blocks (working set) is larger than the cache.
- Conflict miss
  - Occur when the level k cache is large enough, but multiple data objects all map to the same level k block.
    - E.g. Referencing blocks 0, 8, 0, 8, 0, 8, ... would miss every time.

# Cache Memories

- Cache memories are small, fast SRAM-based memories managed automatically in hardware
  - Hold frequently accessed blocks of main memory
- CPU looks first for data in cache
- Typical system structure:



# General Cache Organization (S, E, B)

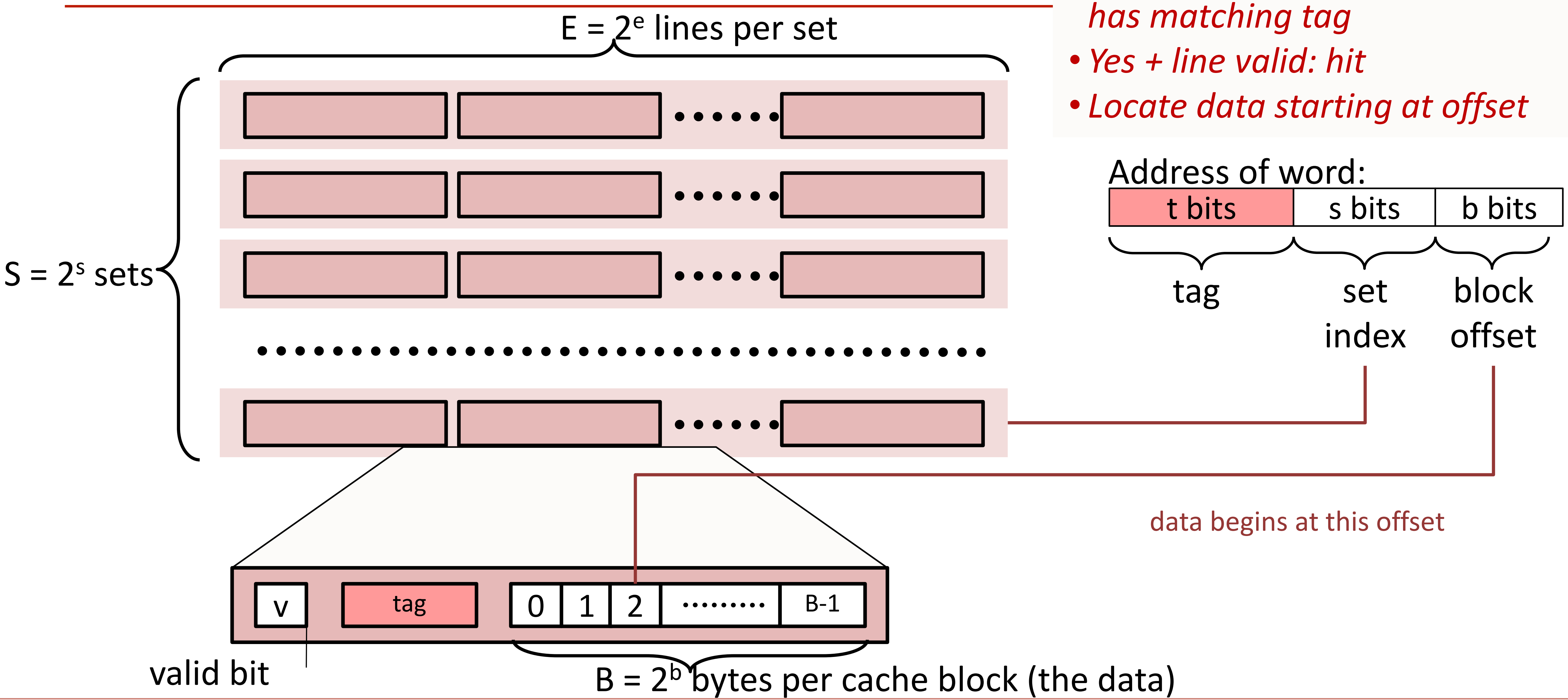



*Cache size:*

*C = S x E x B data bytes*

# Cache Read

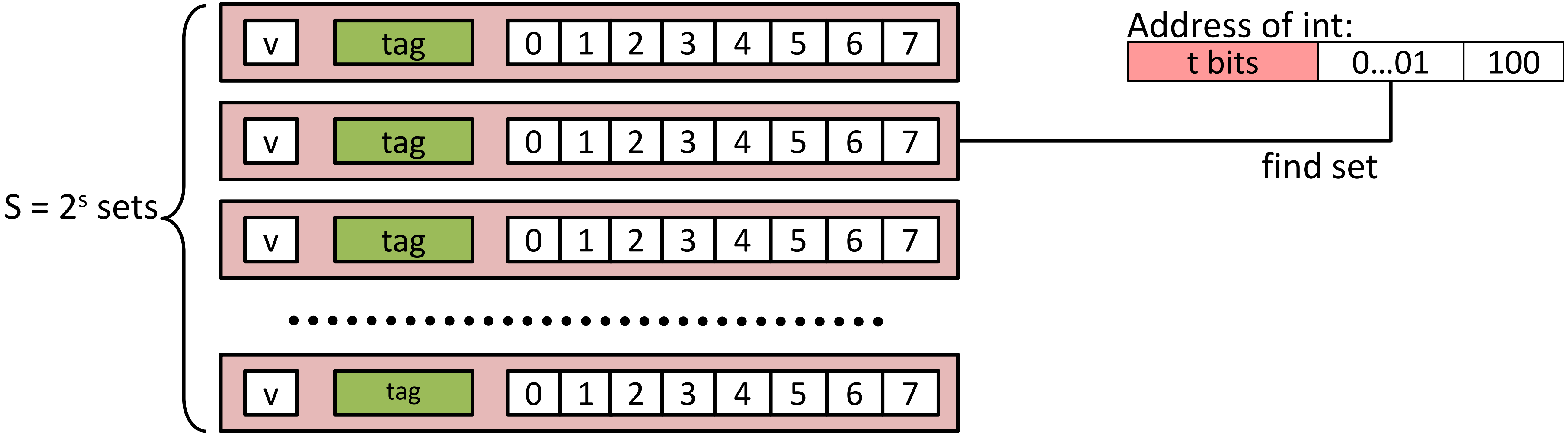
- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*



# Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

Assume: cache block size 8 bytes



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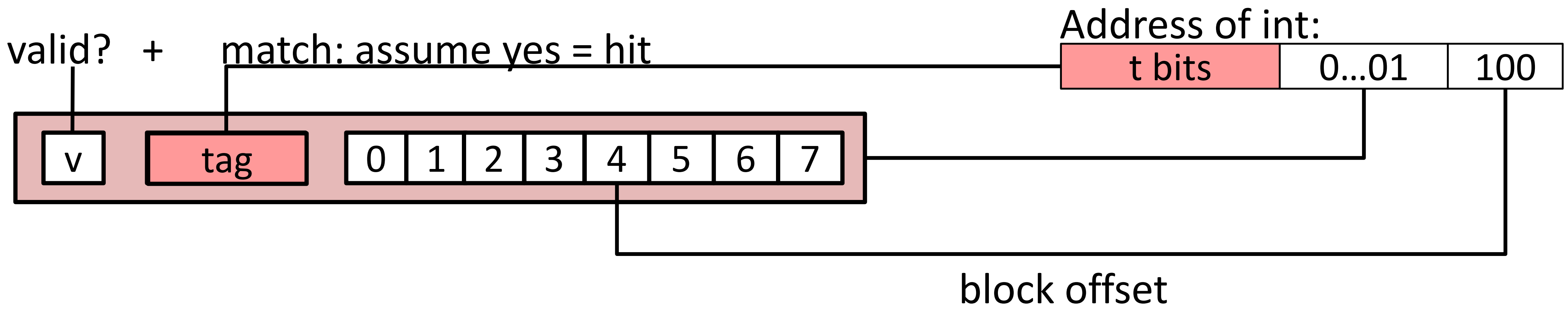
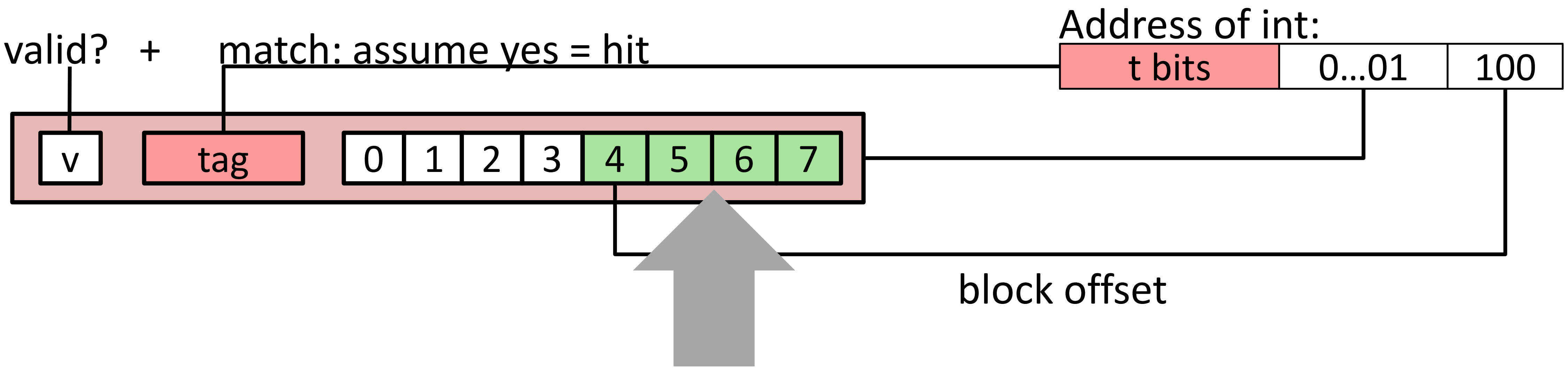


# Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set
Assume: cache block size 8 bytes




If tag doesn't match: old line is evicted and replaced

# Direct-Mapped Cache Simulation

| t=1 | s=2 | b=1 |
| --- | --- | --- |
| x | xx | x |

M=16 bytes (4-bit addresses), B=2 bytes/block,
S=4 sets, E=1 Blocks/set

Address trace (reads, one byte per read):

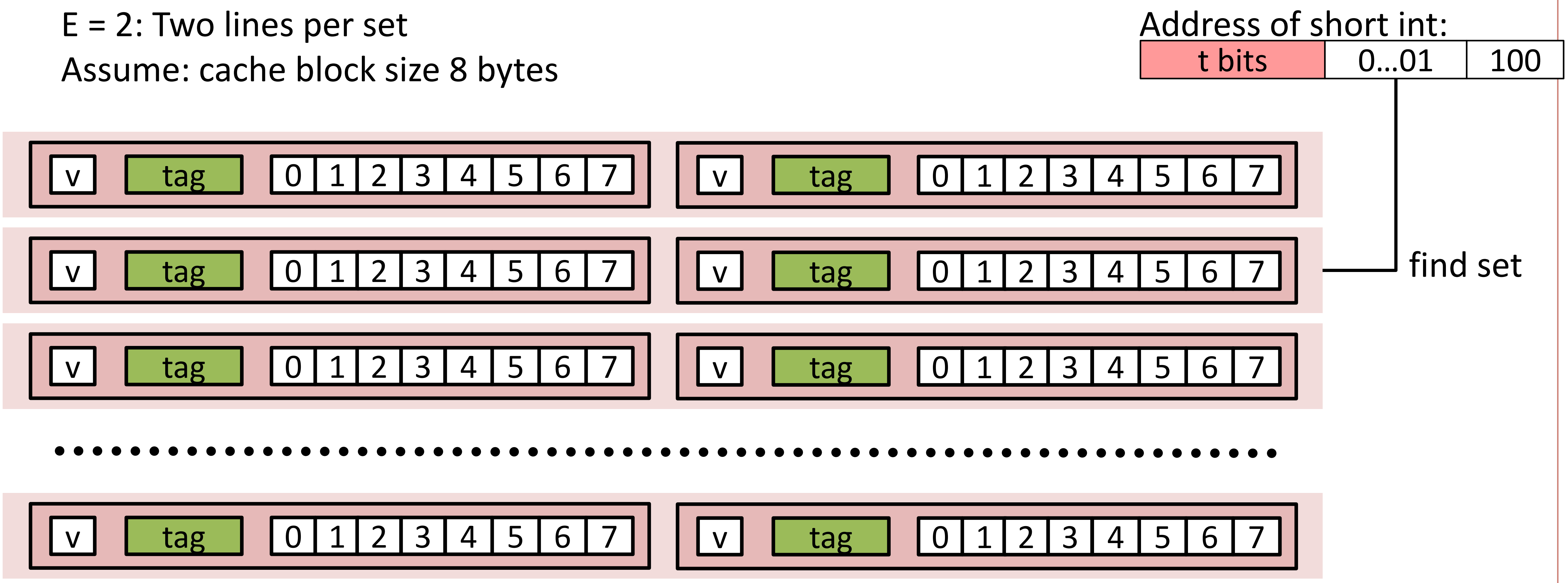
| | | |
| --- | --- | --- |
| 0 | $[0\underline{00}0_2]$, | miss |
| 1 | $[0\underline{00}1_2]$, | hit |
| 7 | $[0\underline{11}1_2]$, | miss |
| 8 | $[1\underline{00}0_2]$, | miss |
| 0 | $[0\underline{00}0_2]$ | miss |

| | v | Tag | Block |
| --- | --- | --- | --- |
| Set 0 | 1 | 0 | M[0-1] |
| Set 1 | | | |
| Set 2 | | | |
| Set 3 | 1 | 0 | M[6-7] |

# E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes



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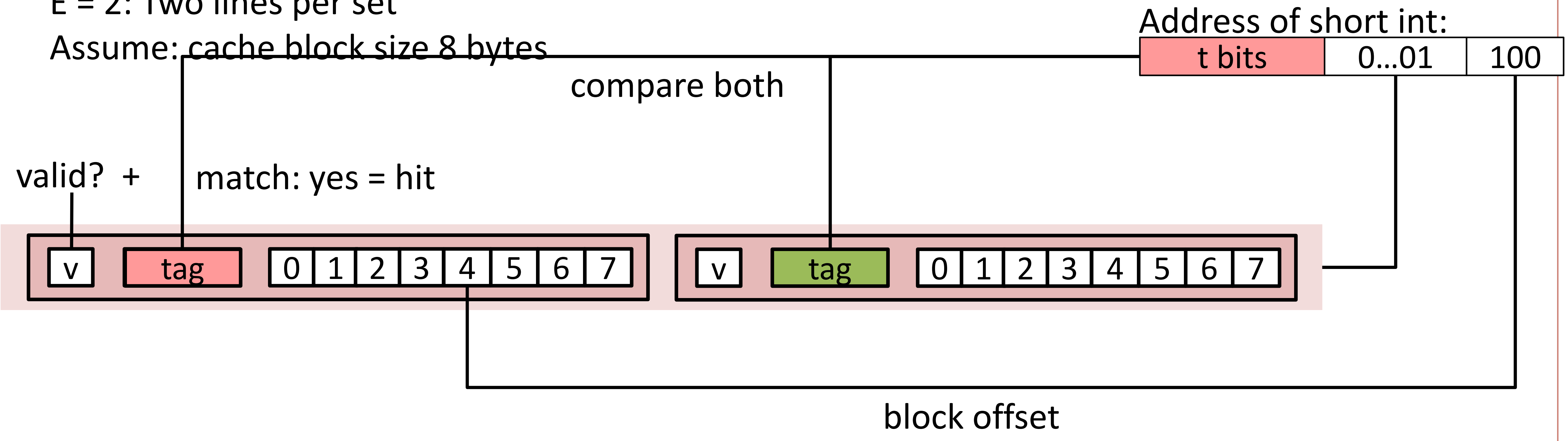
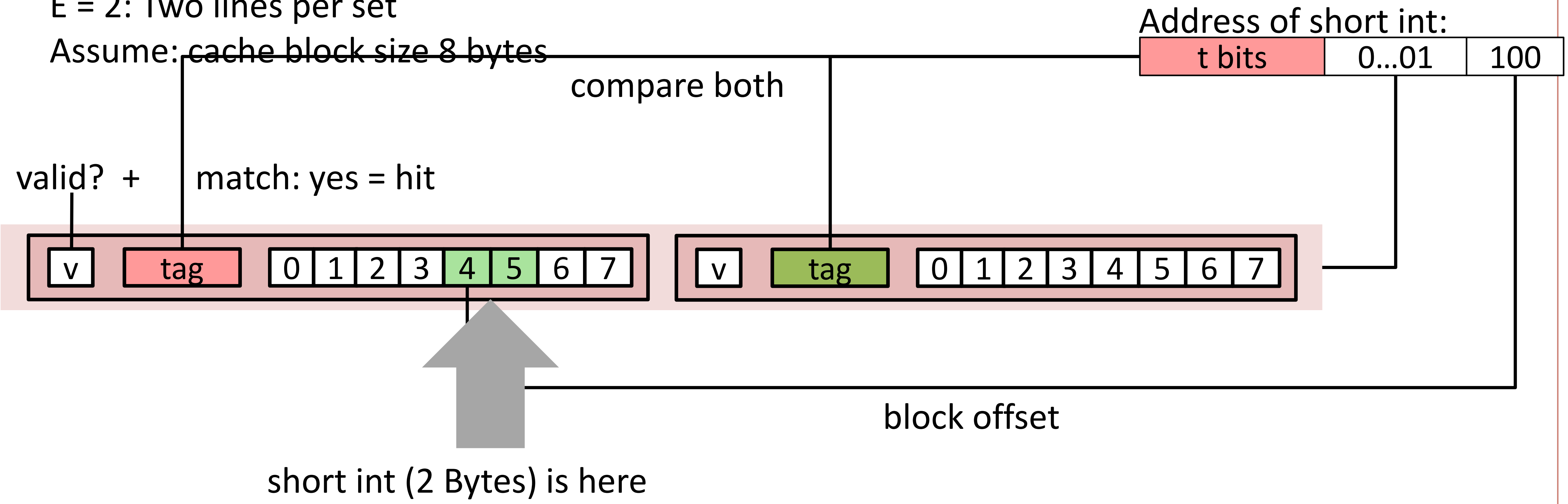


# E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes




No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), …

# 2-Way Set Associative Cache Simulation

| t=2 | s=1 | b=1 |
|---|---|---|
| xx | x | x |

M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

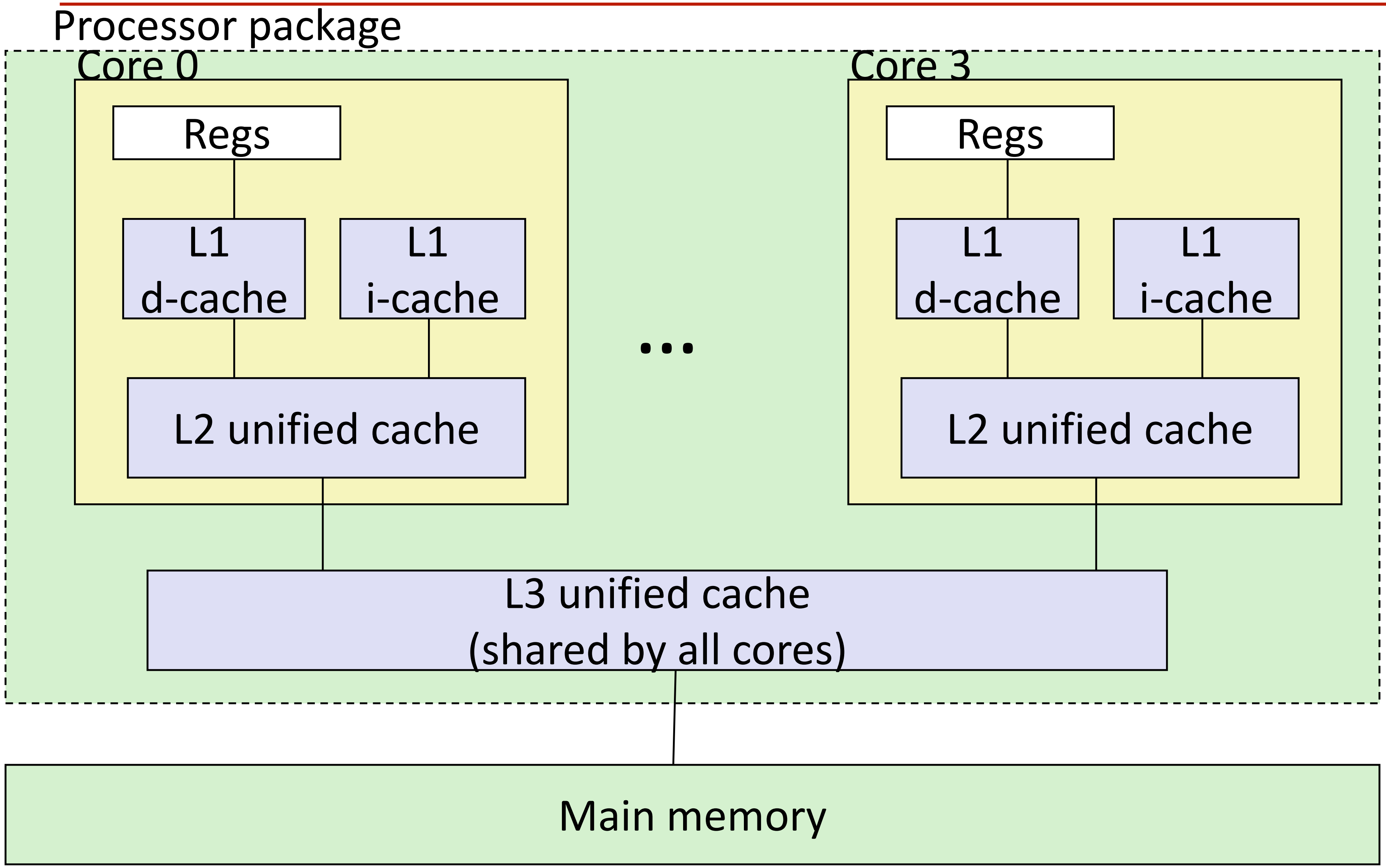
| | | |
|---|---|---|
| 0 | [00<u>0</u>0$_2$], | miss |
| 1 | [00<u>0</u>1$_2$], | hit |
| 7 | [01<u>1</u>1$_2$], | miss |
| 8 | [10<u>0</u>0$_2$], | miss |
| 0 | [00<u>0</u>0$_2$] | hit |

| | v | Tag | Block |
|---|---|---|---|
| Set 0 | 1 | 00 | M[0-1] |
| | 1 | 10 | M[8-9] |
| Set 1 | 1 | 01 | M[6-7] |
| | 0 | | |

# What about writes?

- Multiple copies of data exist:
  - L1, L2, L3, Main Memory, Disk
- What to do on a write-hit?
  - Write-through (write immediately to memory)
  - Write-back (defer write to memory until replacement of line)
    - Need a dirty bit (line different from memory or not)
- What to do on a write-miss?
  - Write-allocate (load into cache, update line in cache)
    - Good if more writes to the location follow
  - No-write-allocate (writes straight to memory, does not load into cache)
- Typical
  - Write-through + No-write-allocate
  - **Write-back + Write-allocate**

# Intel Core i7 Cache Hierarchy




L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for all caches.

# Cache Performance Metrics

- Miss Rate
  - Fraction of memory references not found in cache (misses / accesses) = 1 – hit rate
  - Typical numbers (in percentages):
    - 3-10% for L1
    - can be quite small (e.g., < 1%) for L2, depending on size, etc.
- Hit Time
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typical numbers:
    - 4 clock cycle for L1
    - 10 clock cycles for L2
- Miss Penalty
  - Additional time required because of a miss
    - typically 50-200 cycles for main memory (Trend: increasing!)

# Let's think about those numbers

- Huge difference between a hit and a miss
  - Could be 100x, if just L1 and main memory

- Would you believe 99% hits is twice as good as 97%?
  - Consider:
    cache hit time of 1 cycle
    miss penalty of 100 cycles

  - Average access time:
    97% hits: 1 cycle + 0.03 * 100 cycles = **4 cycles**
    99% hits: 1 cycle + 0.01 * 100 cycles = **2 cycles**

- This is why "miss rate" is used instead of "hit rate"

# Writing Cache Friendly Code

- Make the common case go fast
  - Focus on the inner loops of the core functions

- Minimize the misses in the inner loops
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories

# Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

# The Memory Mountain

- Read throughput (read bandwidth)
  - Number of bytes read from memory per second (MB/s)

- Memory mountain: Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

# Memory Mountain Test Function

```
long data[MAXELEMS];  /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *       array "data" with stride of "stride", using
 *       using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
```

*mountain/mountain.c*

Call `test()` with many combinations of `elems` and `stride`.

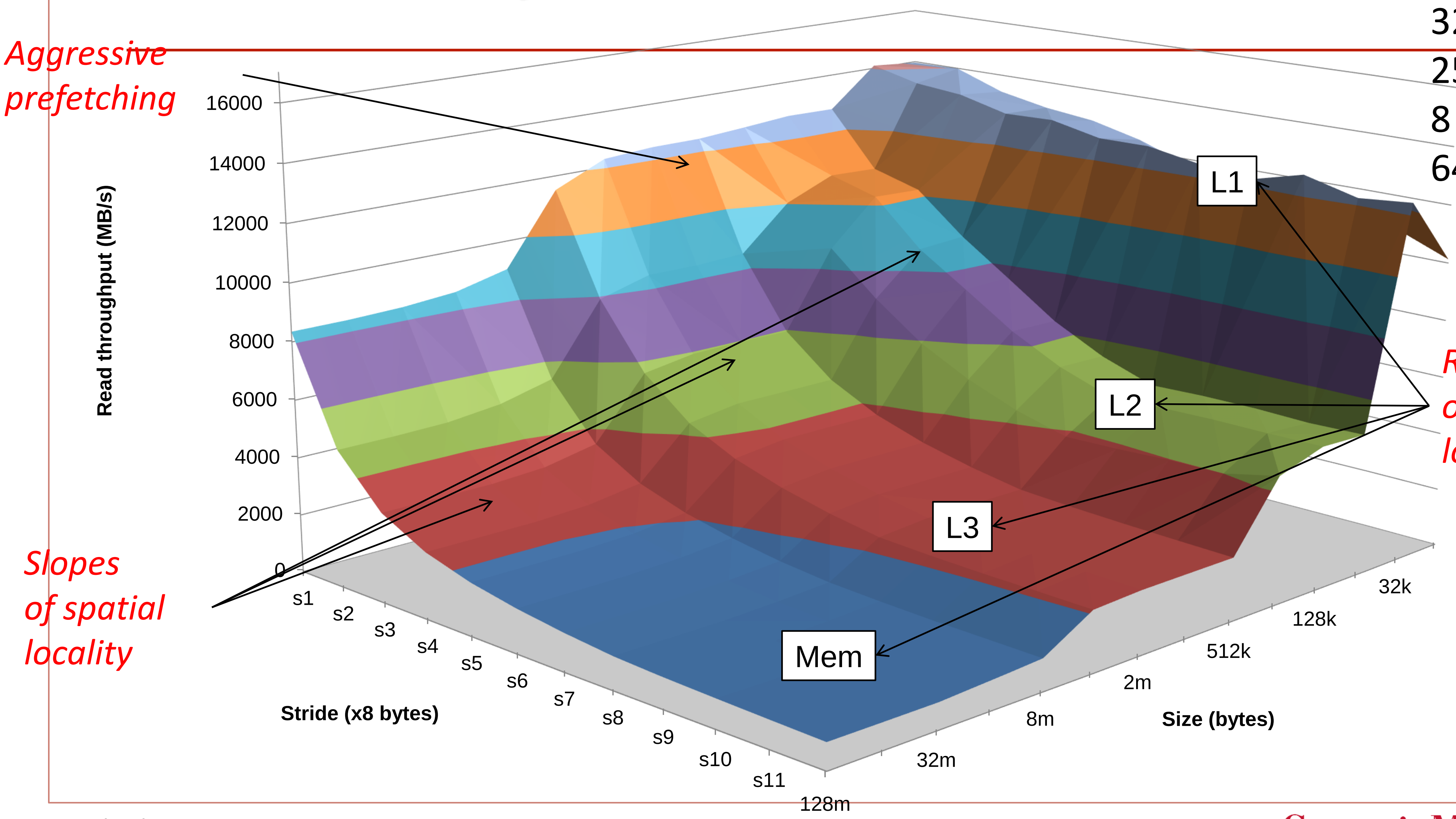
For each elems and stride:

1. Call test() once to warm up the caches.

2. Call test() again and measure the read throughput(MB/s)

# The Memory Mountain

Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



# Today

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  - Using blocking to improve temporal locality

# Matrix Multiplication Example

- Description:
  - Multiply N x N matrices
  - Matrix elements are doubles (8 bytes)
  - $O(N^3)$ total operations
  - N reads per source element
  - N values summed per destination
    - but may be able to hold in register

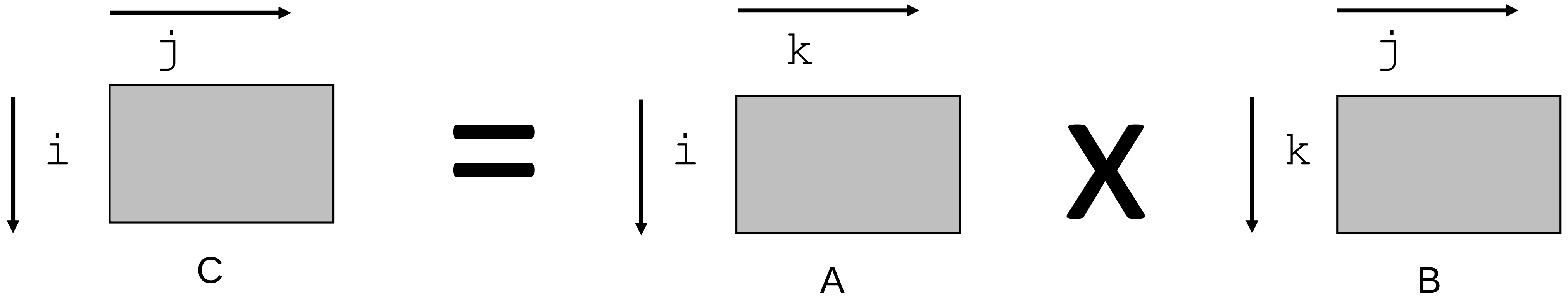
```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

*matmult/mm.c*

*Variable `sum` held in register*

# Miss Rate Analysis for Matrix Multiply

- Assume:
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop



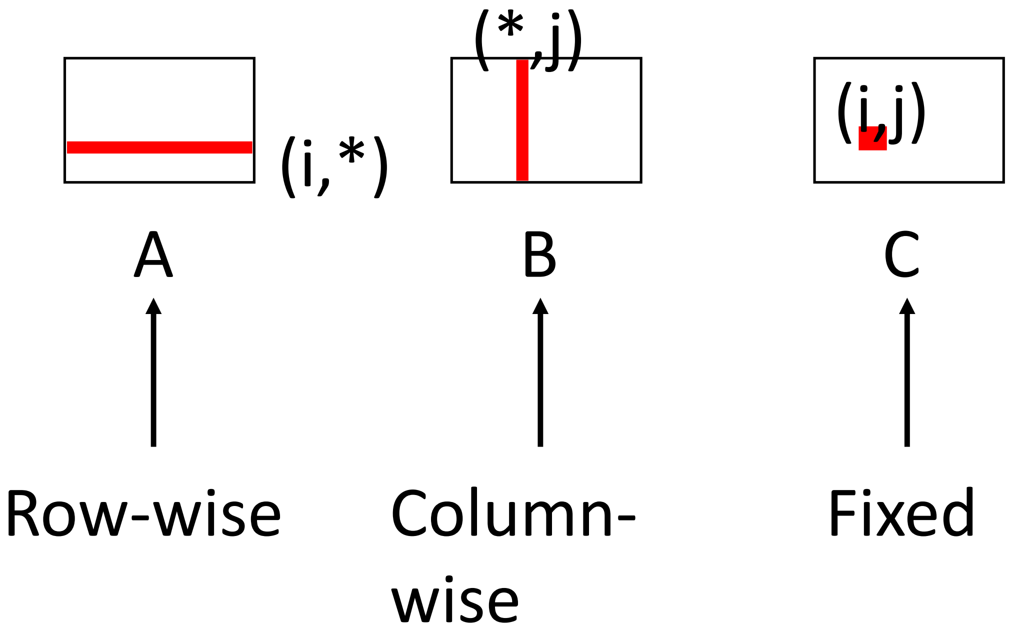
# Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations
- Stepping through columns in one row:
  - `for (i = 0; i < N; i++)`
    `sum += a[0][i];`
  - accesses successive elements
  - if block size (B) > sizeof($a_{ij}$) bytes, exploit spatial locality
    - miss rate = sizeof($a_{ij}$) / B
- Stepping through rows in one column:
  - `for (i = 0; i < n; i++)`
    `sum += a[i][0];`
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

# Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }                     matmult/mm.c
}
```

Inner loop:



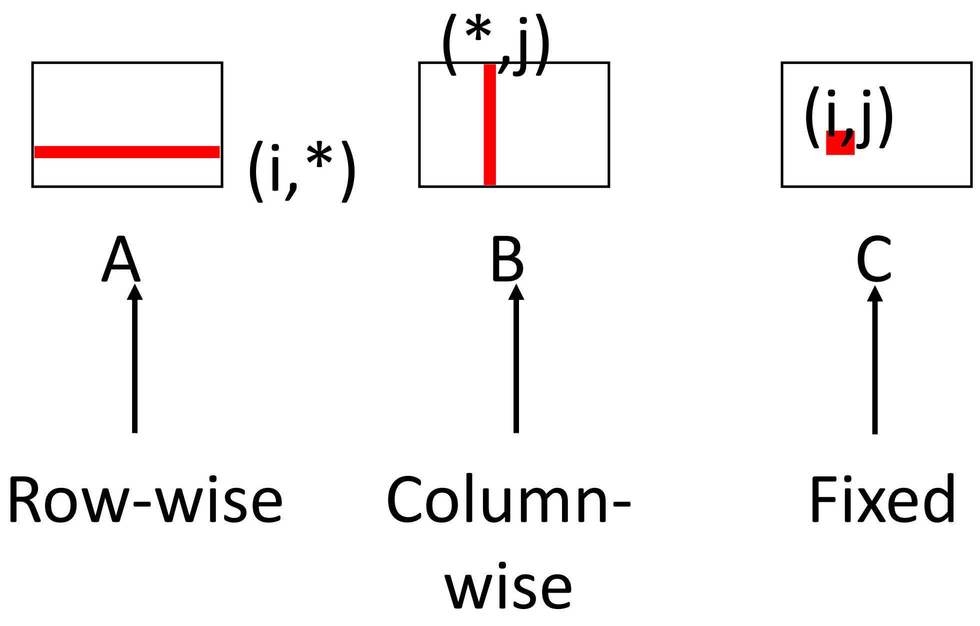

Misses per inner loop iteration:

| A | B | C |
|---|---|---|
| 0.25 | 1.0 | 0.0 |

# Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }                    matmult/mm.c
}
```

Inner loop:



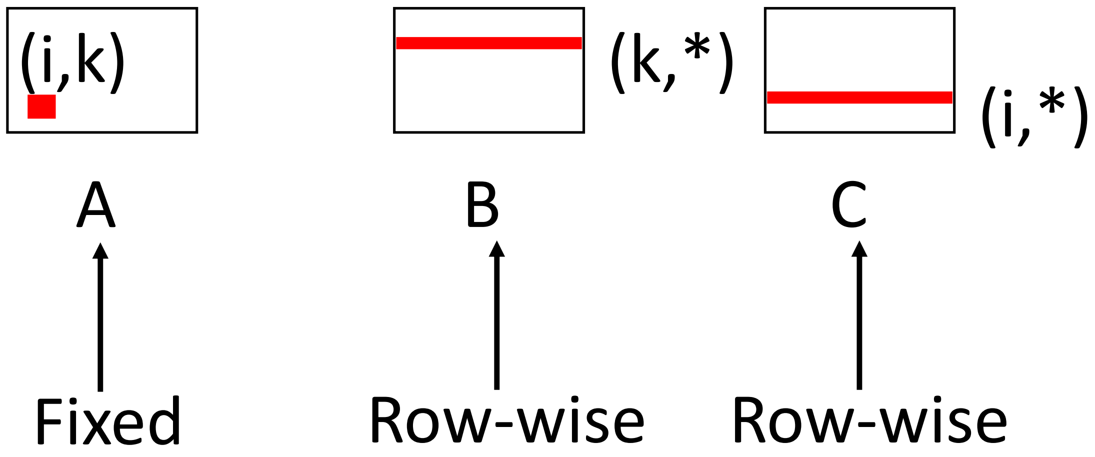

Misses per inner loop iteration:

| A | B | C |
|---|---|---|
| 0.25 | 1.0 | 0.0 |

# Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }                     matmult/mm.c
}
```

Inner loop:



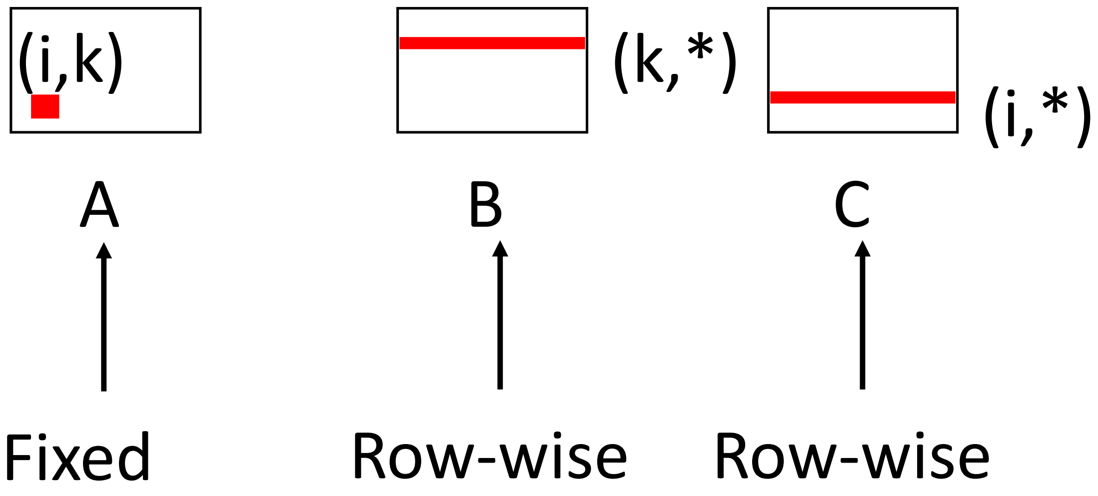

Misses per inner loop iteration:

| A | B | C |
|---|---|---|
| 0.0 | 0.25 | 0.25 |

# Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }                    matmult/mm.c
}
```

Inner loop:



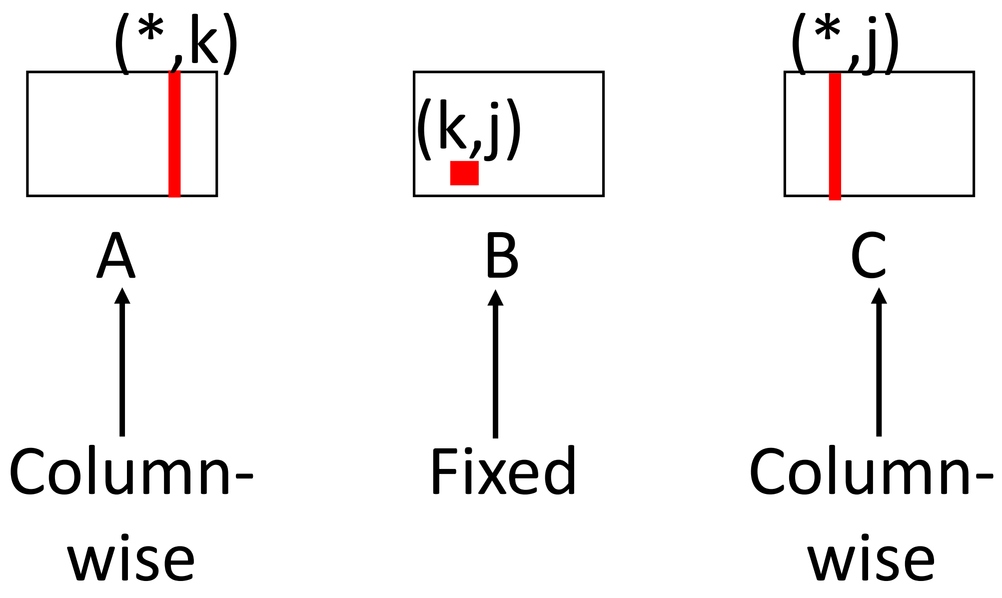

## Misses per inner loop iteration:

| A | B | C |
| --- | --- | --- |
| 0.0 | 0.25 | 0.25 |

# Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }                    matmult/mm.c
}
```

Inner loop:



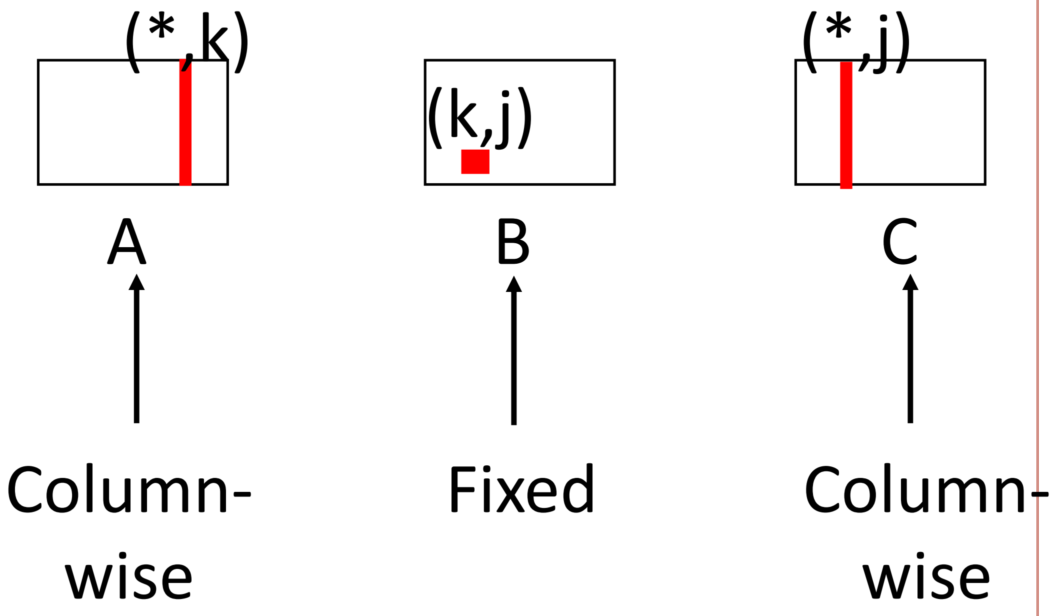

Misses per inner loop iteration:

| A | B | C |
|---|---|---|
| 1.0 | 0.0 | 1.0 |

# Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }                          matmult/mm.c
}
```

Inner loop:




Misses per inner loop iteration:

| A | B | C |
|---|---|---|
| 1.0 | 0.0 | 1.0 |

# Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
  }
}
```

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = 1.25

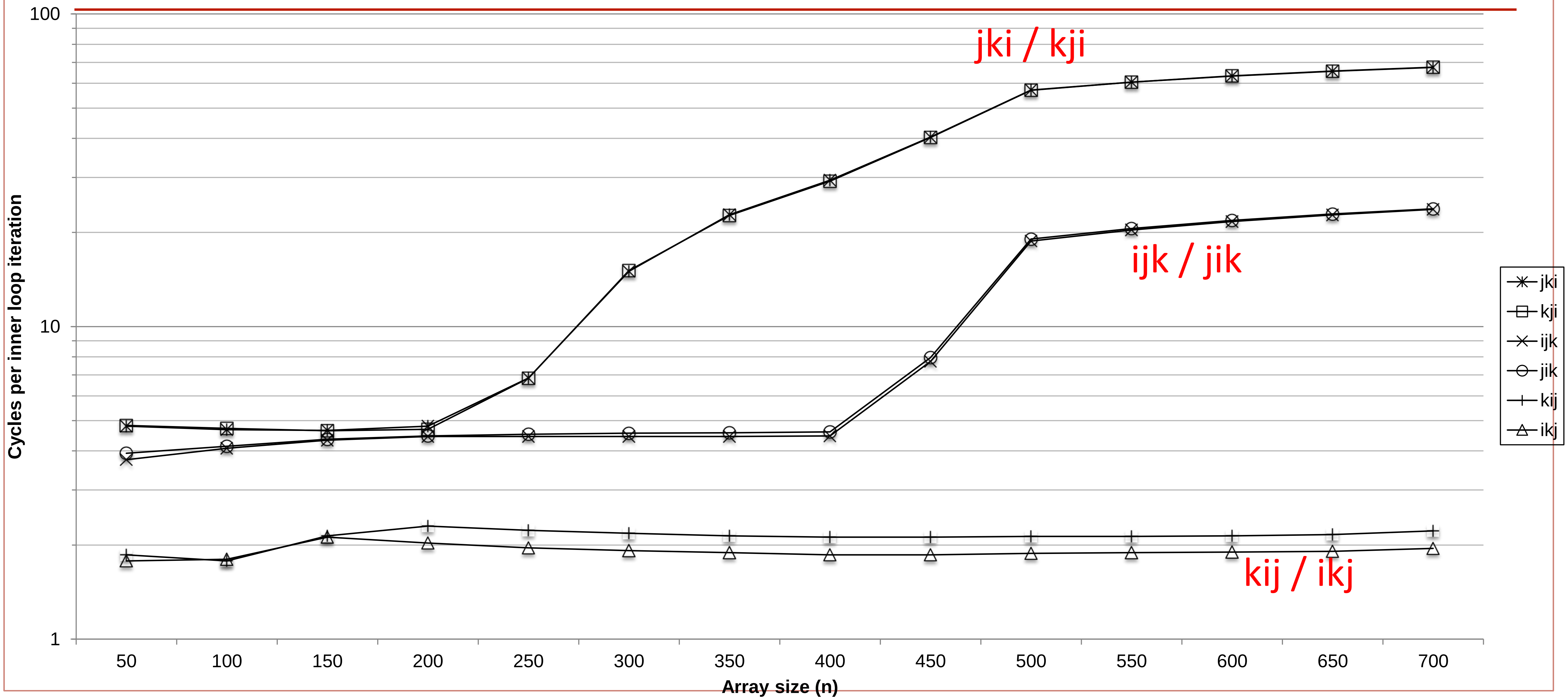
```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = 0.5

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = 2.0

# Core i7 Matrix Multiply Performance



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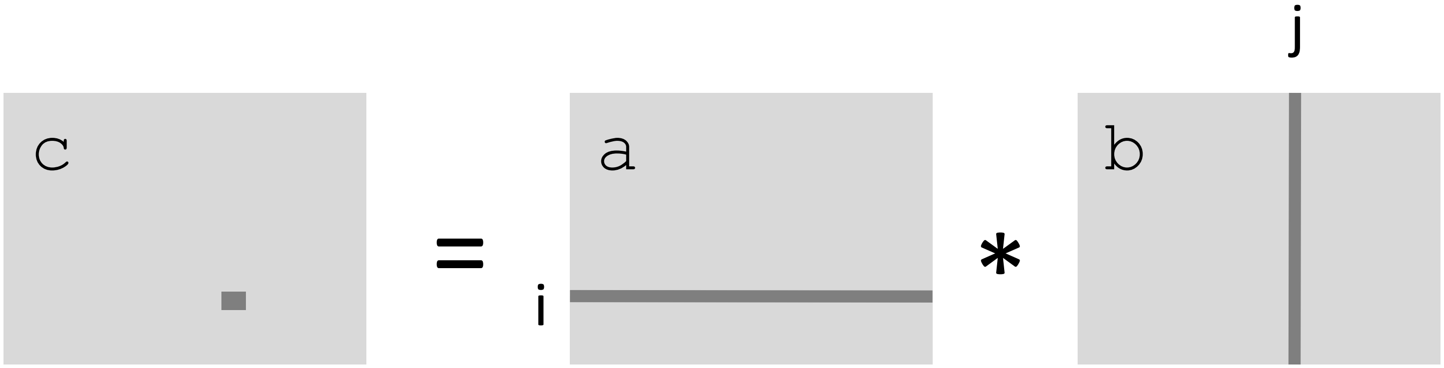
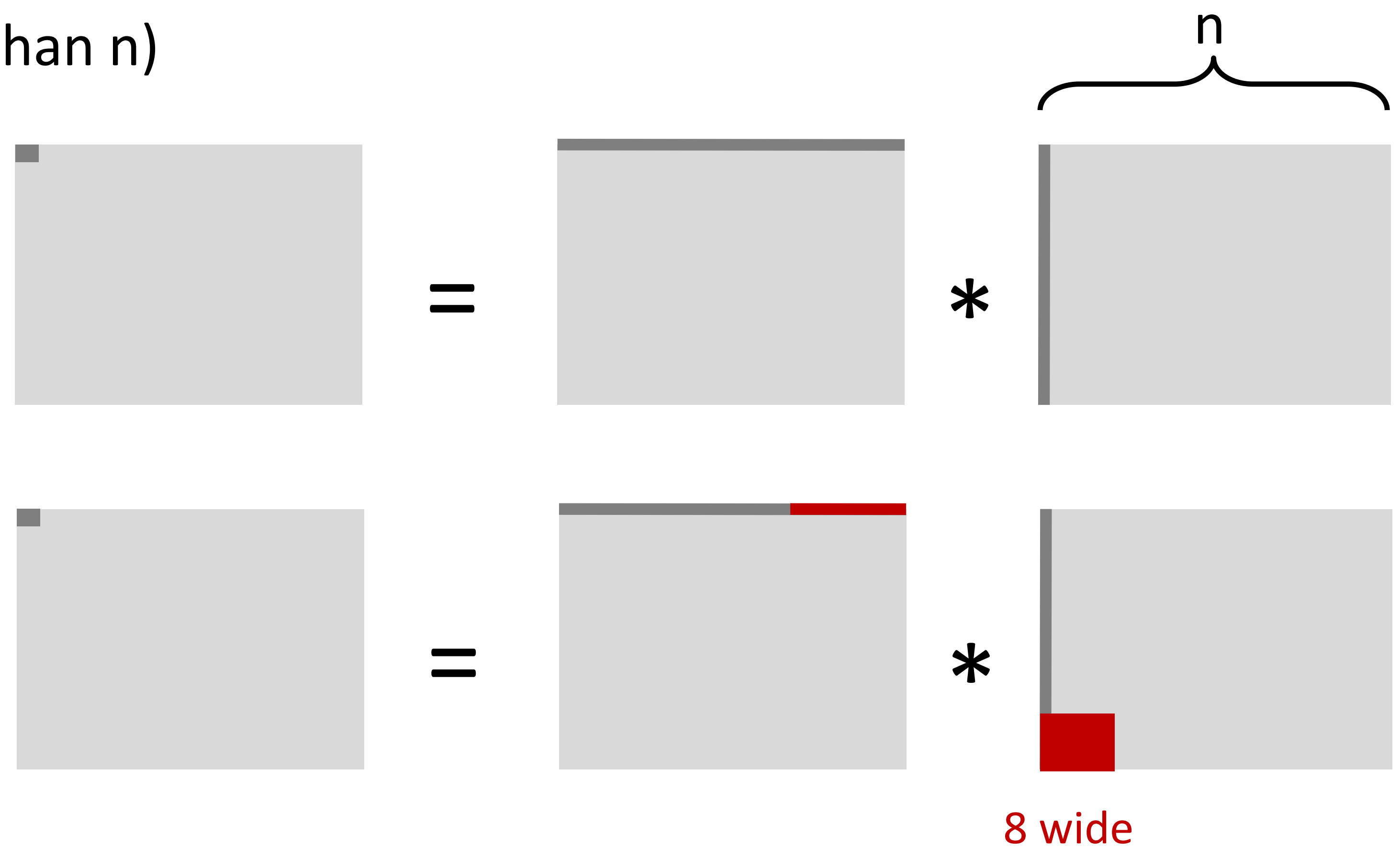
# Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
      for (j = 0; j < n; j++)
         for (k = 0; k < n; k++)
           c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



# Cache Miss Analysis

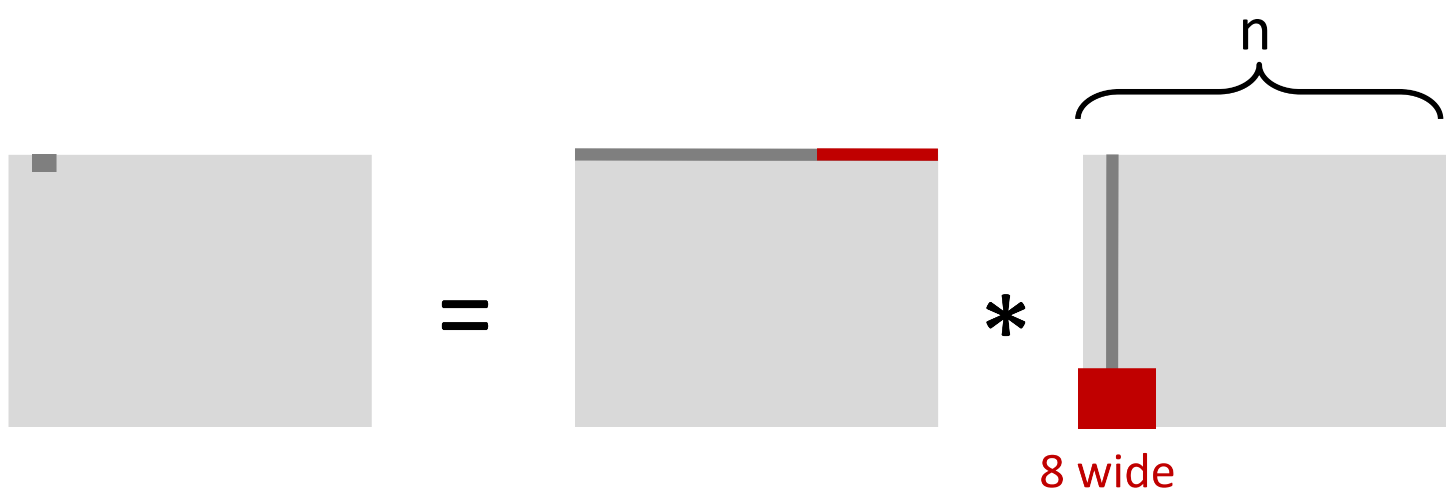
- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
- First iteration:
  - n/8 + n = 9n/8 misses
  - Afterwards in cache: (schematic)



# Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)

- Second iteration:
  - Again:
    n/8 + n = 9n/8 misses

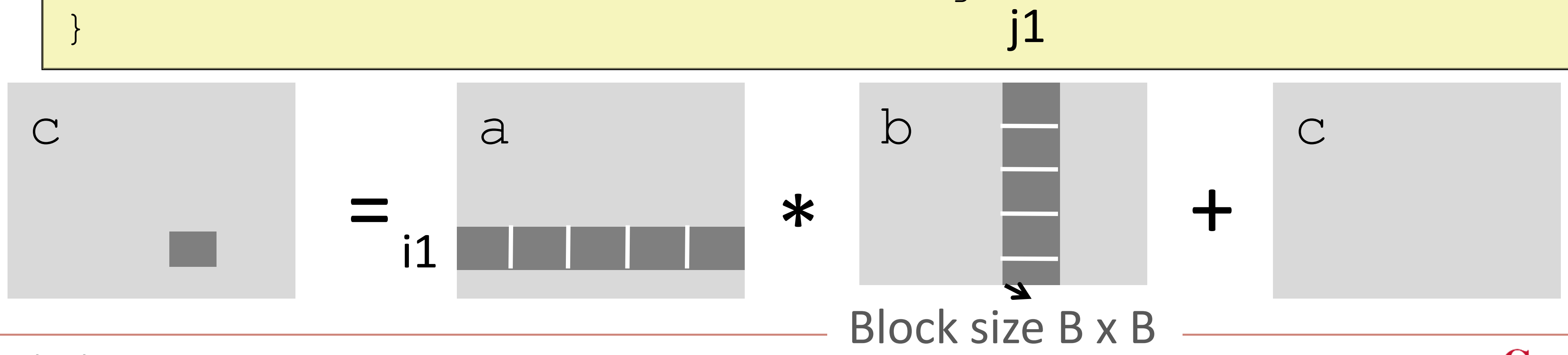



- Total misses:
  - $9n/8 * n^2 = (9/8) * n^3$

# Blocked Matrix Multiplication

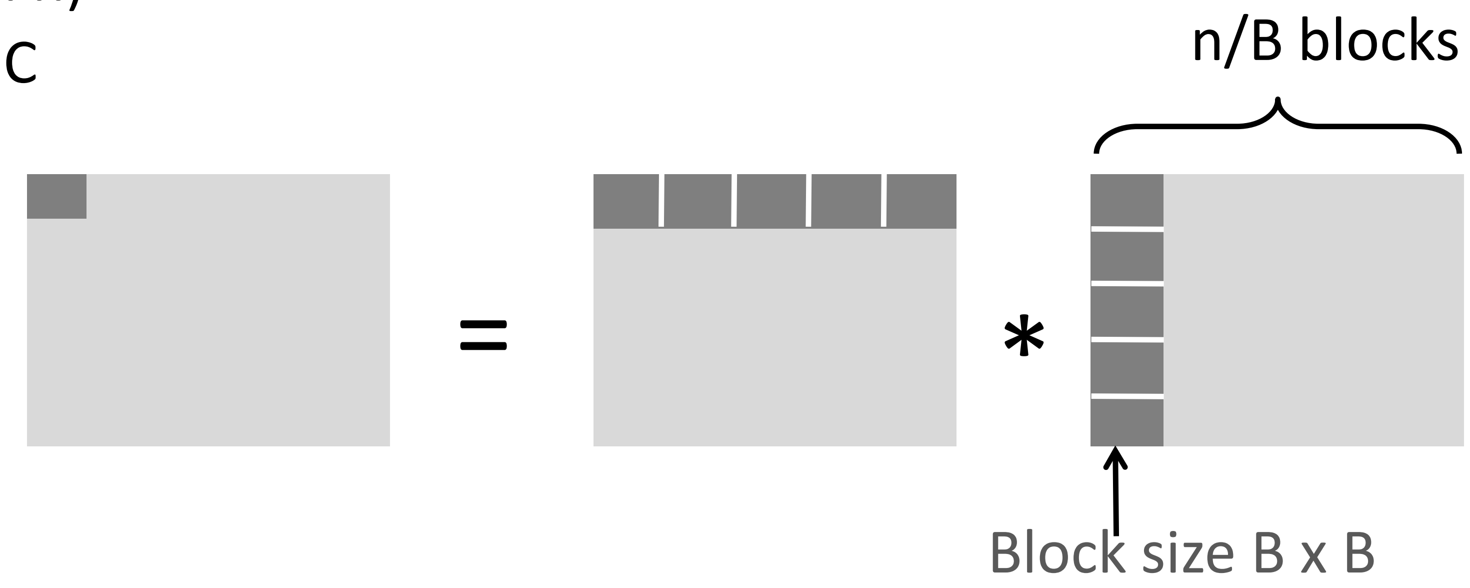
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
      for (j = 0; j < n; j+=B)
             for (k = 0; k < n; k+=B)
             /* B x B mini matrix multiplications */
                  for (i1 = i; i1 < i+B; i++)
                      for (j1 = j; j1 < j+B; j++)
                          for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

matmult/bmm.c



# Cache Miss Analysis

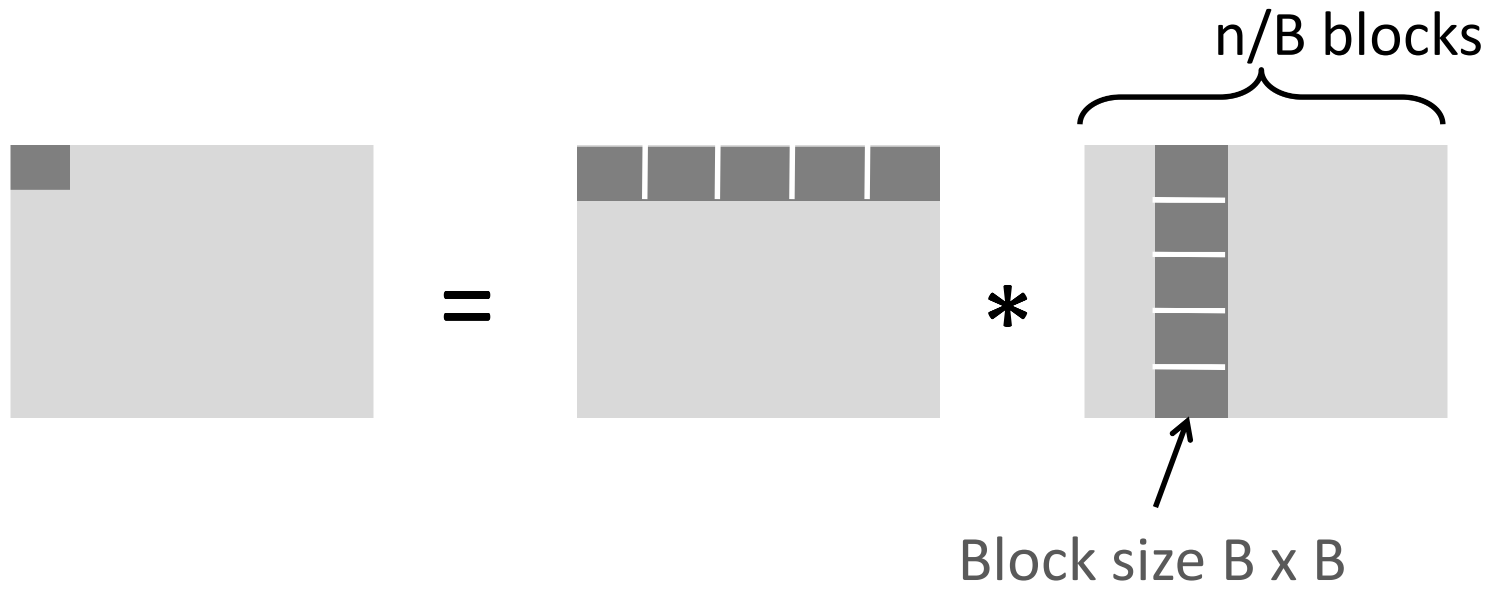
- Assume:
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$
- First (block) iteration:
  - $B^2/8$ misses for each block
  - $2n/B * B^2/8 = nB/4$ (omitting matrix c)




- Afterwards in cache (schematic)

# Cache Miss Analysis

- Assume:
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$
- Second (block) iteration:
  - Same as first iteration
  - $2n/B * B^2/8 = nB/4$
- Total misses:
  - $nB/4 * (n/B)^2 = n^3/(4B)$



# Blocking Summary

- No blocking: (9/8) * $n^3$
- Blocking: 1/(4B) * $n^3$

- Suggest largest possible block size B, but limit $3B^2 < C$!

- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

# Cache Summary

- Cache memories can have significant performance impact

- You can write your programs to exploit this!
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

# 18-600 Foundations of Computer Systems

## Lecture 14:
## "Program Performance Optimization"

John P. Shen & Zhiyi Yu
October 17, 2016

# Next Time …

- Required Reading Assignment:
  - **Chapter 5 of CS:APP (3rd edition) by Randy Bryant & Dave O'Hallaron.**

Electrical & Computer ENGINEERING