4/15/22 Recitation 10 B FIR Filter Design

Window Design
- "Classical"
- Kaiser

Parks-McClellan Algorithm

Freq. Sampling

$$h[n] = h_d[n-\tfrac{M}{2}]\,w[n]$$

- $h[n]$: FIR linear phase
- $h_d[n-\frac{M}{2}]$: shifted ideal filter, infinite in duration
- $w[n]$: finite, symmetric about $M=$

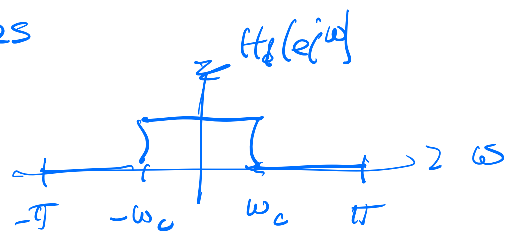
$N =$ FIR filter length

$M = N-1$

Prototype Ideal Filters

LPF

$$h_d[n] = \frac{\sin(\omega_c n)}{\pi n}$$

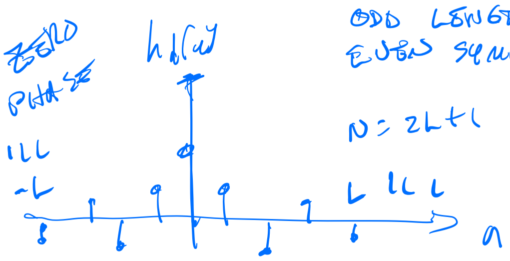


Type I Filter

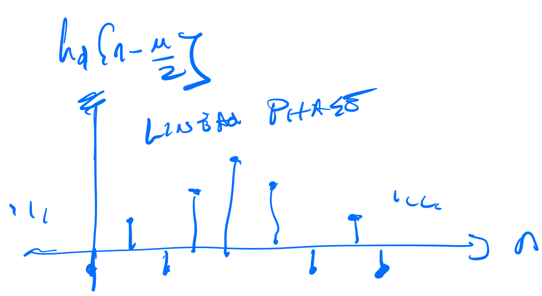
Odd length, even symmetry

$N = 2L+1$

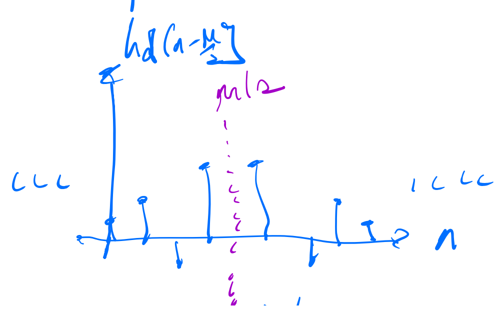
Zero phase



Linear phase

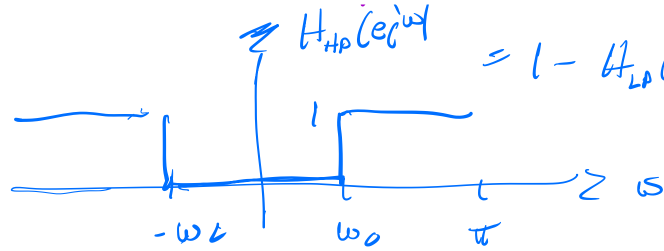


Type II



HPF

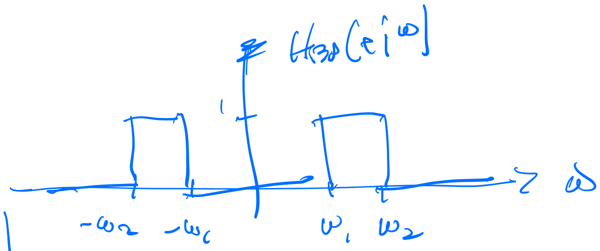
$h_{HP}[n] = \delta[n] - \frac{\sin(\omega_c n)}{\pi n}$



$= 1 - H_{LP}(e^{j\omega})$

MUST BE TYPE I

BPF



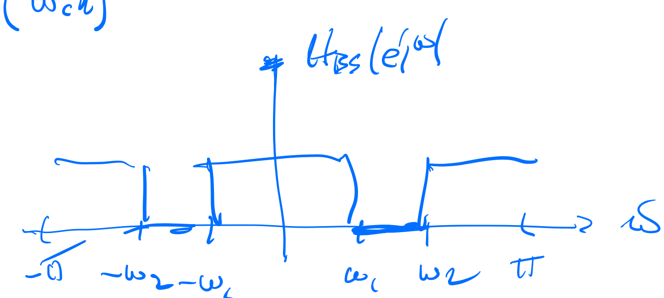
$H_{BP}(e^{j\omega}) = \frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n}$

Let $\omega_c = \frac{\omega_1 + \omega_2}{2}$

$B = \frac{\omega_2 - \omega_1}{2}$

$= 2\frac{\sin(Bn)}{\pi n} \cdot \cos(\omega_c n)$

$H_{BS}(e^{j\omega})$



$h_{BS}[n] = \delta[n] - \left(\frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n}\right)$

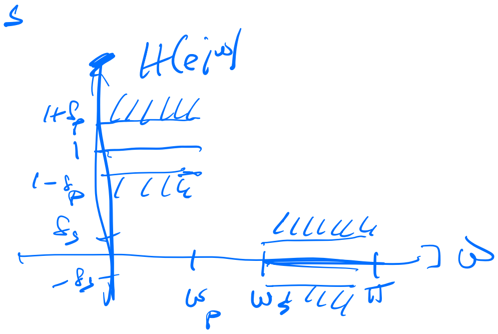
FIR DESIGN USING WINDOWS

Let $\omega_p = .25\pi$

$\omega_s = .3\pi$

PASSBAND RIPPLE $\leq \pm 1$dB

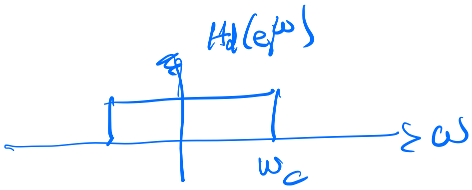
STOPBAND RIPPLE < -60dB



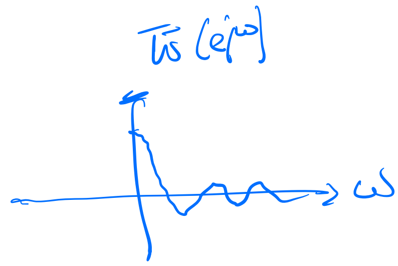
CLASSICAL WINDOW DESIGN

$h[n] = h_d[n - \frac{M}{2}] \cdot w[n]$

$$H(e^{j\omega}) = \frac{1}{2\pi} H_d(e^{j\omega}) \circledast W(e^{j\omega})$$



$\circledast$



FIR DESIGN

1. CHOOSE WINDOW SHAPE BASED ON STOPBAND ATTENUATION

2. CHOOSE WINDOW LENGTH BASED ON TRANSITION BW

Let $\omega_p = .25\pi$

$\omega_s = .3\pi$

PASSBAND RIPPLE $= \pm 1$ dB

STOPBAND RIPPLE < -60 dB

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CHOOSE BLACKMAN WINDOW

$$\Delta\omega = \omega_s - \omega_p = .05\pi$$

$$\omega_c = \frac{\omega_p + \omega_s}{2} = .275\pi$$

$$.05\pi = \frac{12\pi}{M} \; ; \quad M = \frac{12\pi}{.05\pi} = 240, \quad \frac{M}{2} = 120$$

$$h[n] = h_d[n - \tfrac{M}{2}] \cdot w(n)$$

$$= \frac{\sin\left(\omega_c\left(n - \frac{M}{2}\right)\right)}{\pi\left(n - \frac{M}{2}\right)} \cdot \left(.42 - .5\cos\left(\frac{2\pi n}{M}\right) + .08\cos\left(\frac{4\pi n}{M}\right)\right)$$

KAISER WINDOW

$$w[n] = \frac{I_0\left[\beta\left(1-\left[\frac{n-\alpha}{\alpha}\right]^2\right)^{1/2}\right]}{I_0(\beta)}, \quad 0 \le n \le M$$

0, ELSE

$\alpha = \frac{M}{2}$

$\beta \rightarrow$ SHAPE

NEED TO DETERMINE β, $M \rightarrow \alpha$

$\omega_s = \le -60\text{dB}$

$\omega_c = .275\pi$

$\Delta\omega = .05\pi$

KAISER DESIGN...

1. $\Delta\omega = \omega_s - \omega_p = .05\pi$

$A = -20\log_{10}(\delta_s) = 60$

2. $$\beta = \begin{cases} .1102(A-8.7), & A \ge 50 \\ .5842(A-21)^{.4} + .07886(A-21) \\ 0, & A \le 21 \end{cases}$$

$\beta = .1102(60-8.7) = 5.6533$

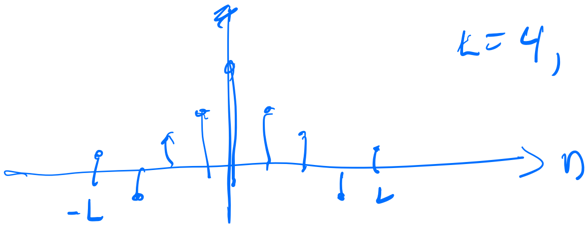
$M = \frac{A-8}{(2.285)(\Delta\omega)} = \frac{52}{(2.285)(.05\pi)} = 144.87 \rightarrow 145$

PARKS-McCLELLAN ALGORITHM

TYPE I OS4P 7.7.1

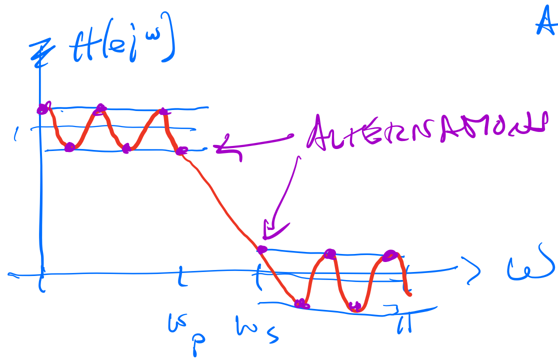
TYPE II OS4P 7.7.2

TYPE I LPF



$L = 4, \; N = 2L+1 = 9$

TYPE I FILT OPTIMAL IF IT HAS AT LEAST L+2 ALTERNATIONS



11 ALTS, $11 = L+2 \Rightarrow L = 9, N = 19$

FOR A SAMPLE RESP. of LENGTH $N = 2L+1$

$$H(e^{j\omega}) = \sum_{n=-L}^{L} h[n]\, e^{-j\omega n} = h[0] + \sum_{n=1}^{L} h[n]\; 2\cos(\omega n)$$

$$= \sum_{\ell=0}^{L} a_\ell (\cos\omega)^\ell$$

ZERO-SLOPE PTS. @ $\omega = 0, \pi$, $L-1$ POINTS IN $0 < \omega < \pi$

MAX # of ALTS. $0, \pi, \omega_p, \omega_s$, $L-1$ MORE $= L+3$

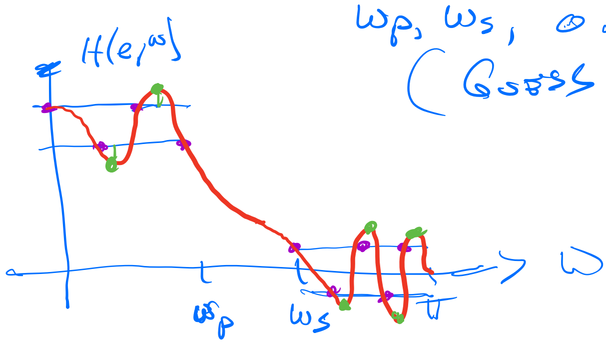
MAX # of ALTS. $= L+3$

PARKS-MCCLELLAN ALGORITHM

GIVEN N, ω_p, ω_s, δ_p/δ_s

1. CHOOSE $L+2$ ALT POINTS INCLUDING ω_p, ω_s, 0 or π (NOT BOTH)

(GUESS RIPPLE SIZE)



2. THREAD CHEBYSCHEV POLYNOMIAL THROUGH PTS.

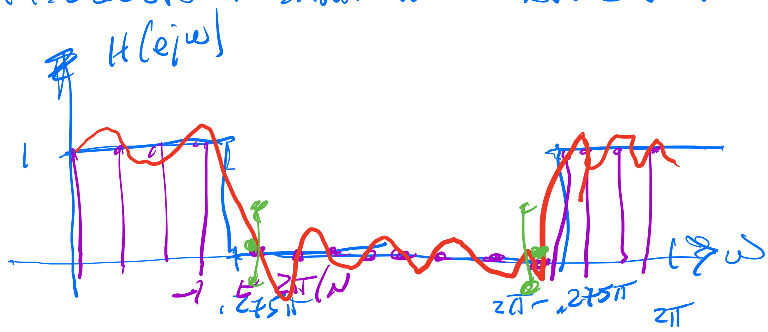
3. IDENTIFY ACTUAL EXTREMAL FREQUENCIES, USE THEM FOR NEXT ALT. POINTS.

(REMEZ EXCHANGE)

WHEN RIPPLE SIZE CONVERGES, DONE

FREQUENCY SAMPLING APPROACH

$\omega_c = .275\pi$



$$\frac{2\pi k}{N} = .275\pi$$

Let $N = 101$

$$k = \frac{.275\pi N}{2\pi} = 13.38$$

$H[k] = 1$, $0 \le k \le 13$, $(N-1-13) \le k \le N-1$