

4/15/22

RECITATION 10 B

FIR FILTER DESIGN

WINDOWS DESIGNS

"CLASSICAL"

KAISER

PAPERS - McLELLAN ALGORITHM

FREQ. SAMPLING

$$h[n] = \underbrace{h_d[n - \frac{M}{2}]}_{\substack{\text{SHIFTED} \\ \text{IDEAL} \\ \text{FILTER} \\ \text{INFINITE} \\ \text{DURATION}}} \underbrace{w[n]}_{\substack{\text{FINITE,} \\ \text{SYMMETRIC} \\ \text{ABOUT } M=0}}$$

↑
FIR
LINEAR
PHASE

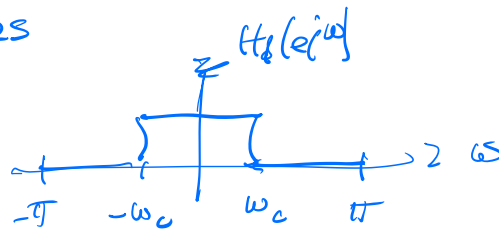
$N = \text{FIR FILTER LENGTH}$

$$M = N - 1$$

PROTOTYPE IDEAL FILTERS

LPF

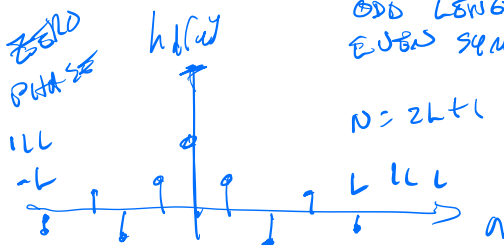
$$h_d[n] = \frac{\sin(\omega_c n)}{\pi n}$$



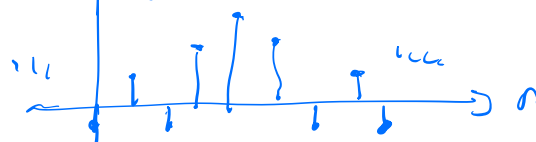
TYPE I FILTER

ODD LENGTH
EVEN SYMMETRY

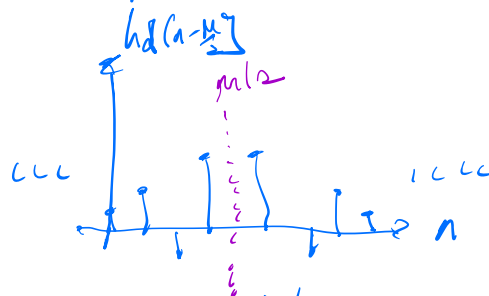
$$N = 2L + 1$$



$h_d[n - \frac{M}{2}]$
LINEAR PHASE



TYPE II



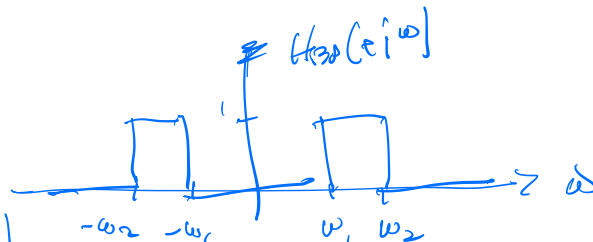
HPF

$$h_{HP}[n] = \delta[n] - \frac{\sin(\omega_c n)}{\pi n}$$


MUST BE TYPE II

BPF

$$H_{BP}(e^{j\omega}) = \frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n}$$

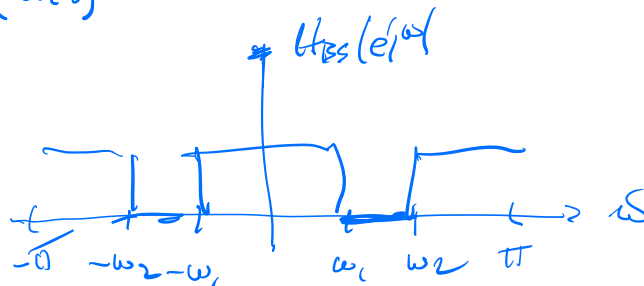


$$\text{let } \omega_c = \frac{\omega_1 + \omega_2}{2}$$

$$B = \frac{\omega_2 - \omega_1}{2}$$

$$= \frac{2 \sin(Bn)}{\pi n} \cdot \cos(\omega_c n)$$

$H_{BS}(e^{j\omega})$



$$h_{BS}[n] = \delta[n] - \left(\frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n} \right)$$

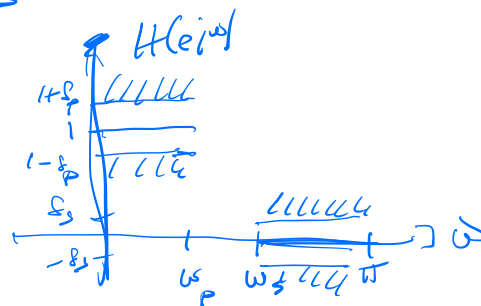
FIR DESIGNS USING WINDOWS

$$\text{let } \omega_p = 0.25\pi$$

$$\omega_s = 0.3\pi$$

$$\text{PASSBAND Ripples} \leq \pm 1\text{dB}$$

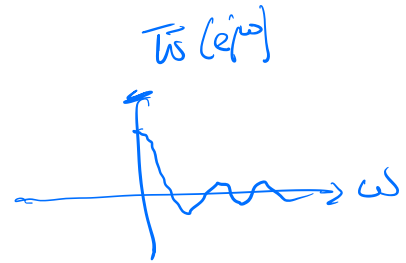
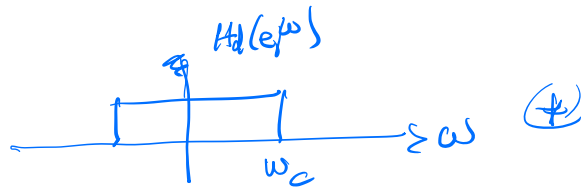
$$\text{STOPBAND Ripples} \leq -60\text{dB}$$



CLASSICAL WINDOW DESIGN

$$h[n] = h_d[n - \frac{M}{2}] \cdot w[n]$$

$$H(e^{j\omega}) = \frac{1}{2\pi} (H_d(e^{j\omega}) \otimes W(e^{j\omega}))$$



FIR DESIGN

1. CHOOSE WINDOW SHAPE BASED ON STOPBAND ATTENUATION

2. CHOOSE WINDOW LENGTH BASED ON TRANSITION BW

$$\text{Let } \omega_p = 0.25\pi$$

$$\omega_s = 0.3\pi$$

$$\text{PASSBAND RIPPLE} = \pm 1\text{dB}$$

$$\text{STOP BAND RIPPLE} < -60\text{dB}$$

CHOOSE BLACKMAN'S WINDOW

$$\Delta\omega = \omega_s - \omega_p = 0.05\pi$$

$$\omega_c = \frac{\omega_p + \omega_s}{2} = 0.275\pi$$

$$0.05\pi = \frac{12\pi}{M} \Rightarrow M = \frac{12\pi}{0.05\pi} = 240, \quad \frac{M}{2} = 120$$

$$h[n] = h_d\left[n - \frac{M}{2}\right] \cdot w\left[n\right]$$

$$= \frac{\sin\left(\omega_c\left(n - \frac{M}{2}\right)\right)}{\pi\left(n - \frac{M}{2}\right)} \cdot \left(0.42 - 0.5\cos\left(\frac{2\pi n}{M}\right) + 0.08\cos\left(\frac{4\pi n}{M}\right)\right)$$

Kaiser window

$$w(n) = \frac{I_0 \left[\beta \left(1 - \left[\frac{n-M}{2} \right]^2 \right)^{1/2} \right]}{I_0(\beta)}, \quad 0 \leq n \leq M$$

$$L = \frac{M}{2}$$

0, EUSF

$\beta \rightarrow$ SHAPE

NEED TO DETERMINE β , $M \rightarrow \alpha$

$$W_s = \leq -60 \text{ dB}$$

$$W_c = .275 \pi$$

$$\Delta W = .05 \pi$$

Kaiser DESIGN

$$1. \Delta W = W_s - W_p = .05 \pi$$

$$A = -20 \log_{10}(\delta_s) = 60$$

$$2. \beta = \begin{cases} .1102(A - 8.7), & A \geq 50 \\ .5842(A - 21)^4 + .07886(A - 21), & 21 \leq A < 50 \\ 0, & A \leq 21 \end{cases}$$

$$\beta = .1102(60 - 8.7) = 5.6533$$

$$M = \frac{A - 8}{\sqrt{2.285}(\Delta W)} = \frac{52}{\sqrt{2.285}(.05\pi)} = 144.87 \rightarrow 145$$

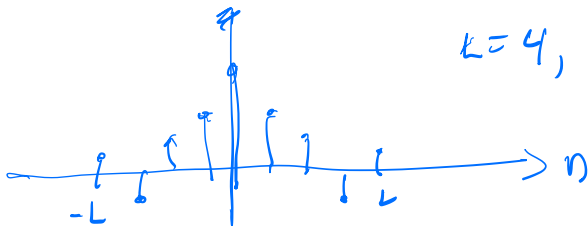
Parks-McClellan Algorithm

TYPE I OSFP 7.7.1

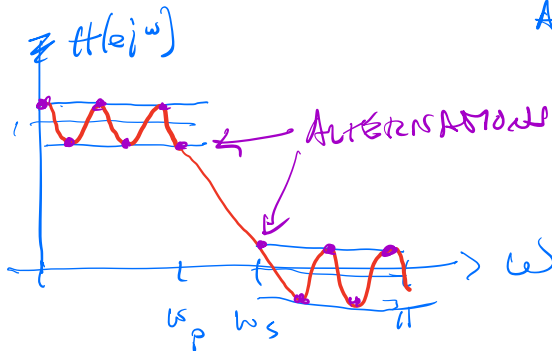
TYPE II OSFP 7.7.2

TYPE I LDF

$$L=4, N=2L+1=9$$



TYPE I RCT OPTIMAL IF IT HAS AT LEAST $L+2$ ALTERNATIONS



$$11 \text{ ALTS, } 11 = L+2 \Rightarrow L=9, N=19$$

FOR A SAMPLE DOSP. of LENGTH $N=2L+1$

$$\begin{aligned} H(e^{jw}) &= \sum_{n=-L}^L h[n] e^{-jwn} = h[0] + \sum_{n=1}^L h[n] 2 \cos(wn) \\ &= \sum_{l=0}^L b_l (\cos w)^l \end{aligned}$$

ZERO-SLOPE PTS. @ $w=0, \pi, L-1$ POINTS IN $0 < w < \pi$

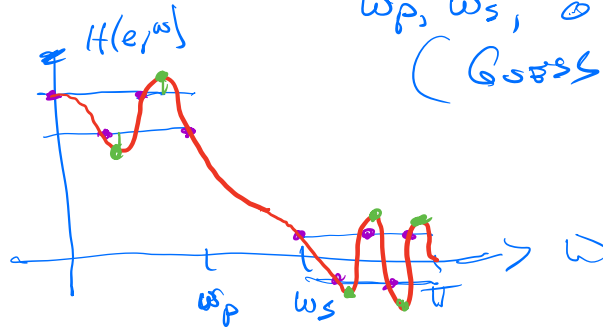
MAX # of ALTS. $0, \pi, w_p, w_s, L-1$ MORE = $L+3$

$$\text{MAX \# of ALTS.} = L+3$$

Parks-McClellan Algorithm

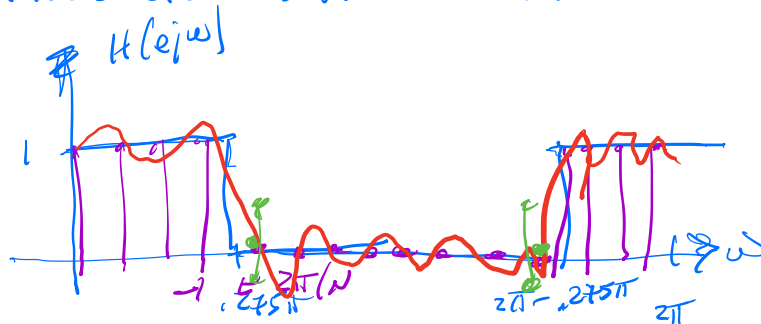
Given $N, \omega_p, \omega_s, \delta_p/\delta_s$

1. CHOOSE $M+2$ ALT POINTS INCLUDING $\omega_p, \omega_s, 0, 2\pi$ (NOT BOTH)
(GROSS RIPPLE SIZE)



2. THREAD CHEBYSHEV POLYNOMIAL THROUGH PTS.
 3. IDENTIFY ACTUAL EXTREMAL FREQUENCIES, USE THEM FOR NEXT ALT. POINTS.
(Remez Exchange)
- WHEN RIPPLE SIZE CONVERGES, DONE

FREQUENCY SAMPLING APPROACH



$$\omega_c = .275\pi$$

$$\frac{2\pi k}{N} = .275\pi$$

$$\text{Let } N=101$$

$$k = \frac{.275\pi N}{2\pi} = 13.38$$

$$\text{Apt } \downarrow 1. \quad n < k \leq 13 \quad (N-13) \quad k \leq N-1$$

11 10 5 4 3 2 1
20 25 30 35 40 45 50
55 60 65 70 75 80 85
90 95 100 105 110 115 120
125 130 135 140 145 150 155
160 165 170 175 180 185 190
195 200 205 210 215 220 225
230 235 240 245 250 255 260
265 270 275 280 285 290 295
300 305 310 315 320 325 330
335 340 345 350 355 360 365
370 375 380 385 390 395 400
405 410 415 420 425 430 435
440 445 450 455 460 465 470
475 480 485 490 495 500 505
510 515 520 525 530 535 540
545 550 555 560 565 570 575
580 585 590 595 600 605 610
615 620 625 630 635 640 645
650 655 660 665 670 675 680
685 690 695 700 705 710 715
720 725 730 735 740 745 750
755 760 765 770 775 780 785
790 795 800 805 810 815 820
825 830 835 840 845 850 855
860 865 870 875 880 885 890
895 900 905 910 915 920 925
930 935 940 945 950 955 960
965 970 975 980 985 990 995
1000