4/21/23 RECITATION 9B FIR FILTER DESIGN

FIR DESIGN TECHNIQUES

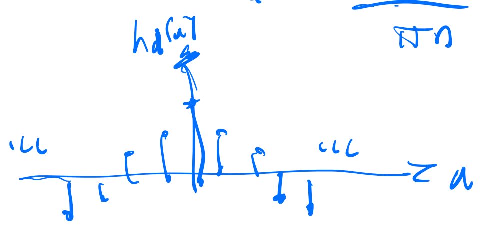
* DESIGN USING WINDOWS
* FREQUENCY SAMPLING DESIGN
* PARKS–McCLELLAN ALGORITHM

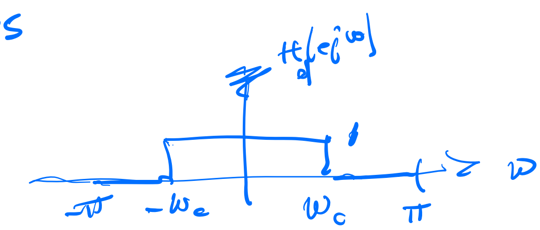
$$h[n] = h_d[n - \tfrac{M}{2}]\, w[n]$$

- $h[n]$: FIR LINEAR PHASE
- $h_d[n - \tfrac{M}{2}]$: SHIFTED IDEAL RESPONSE (ZERO PHASE, INFINITE)
- $w[n]$: FINITE WINDOW, SYMMETRIC ABOUT $M/2$

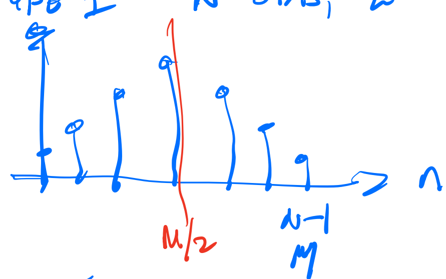
PROTOTYPE (IDEAL) FILTERS

LPF $h_d[n] = \dfrac{\sin(\omega_c n)}{\pi n}, \ \forall n$

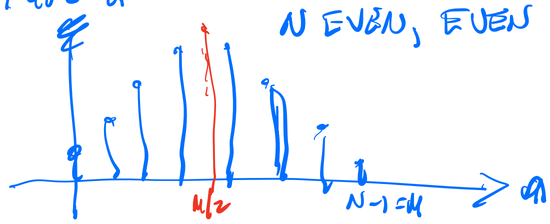




TYPE I N ODD, EVEN SYMM.

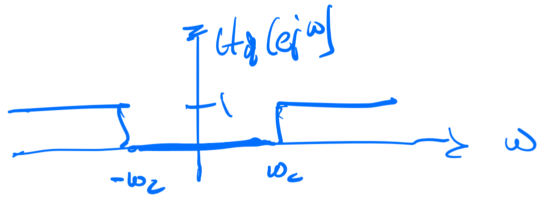


TYPE II N EVEN, EVEN SYMM.



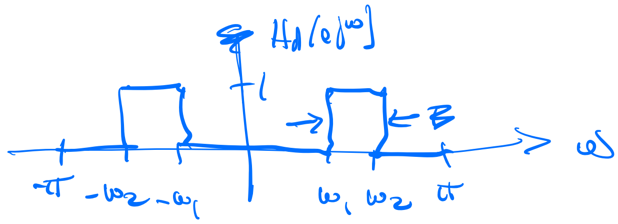
HPF

$$h_d[n] = \delta[n] - \frac{\sin(\omega_c n)}{\pi n}$$



TYPE I ONLY

BPF

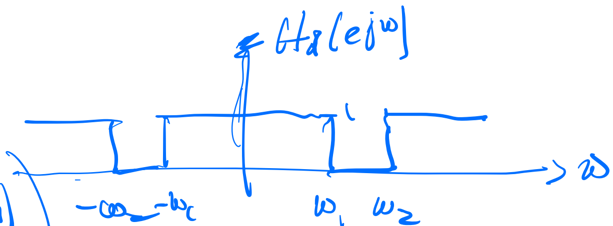


$$h_d[n] = \frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n}$$

Let $\omega_c = \frac{\omega_2 + \omega_1}{2}$, $B = \omega_2 - \omega_1$

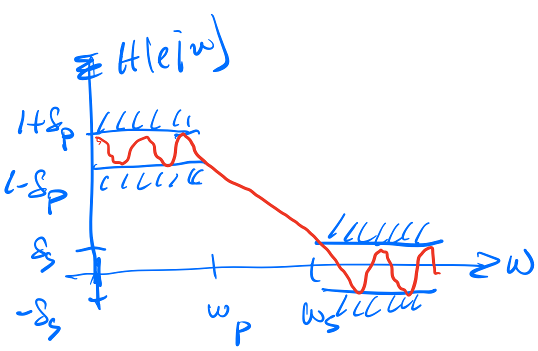
$$h_d[n] = \frac{2\sin\left(\frac{B}{2} n\right)}{\pi n} \cos(\omega_c n)$$

BAND STOP



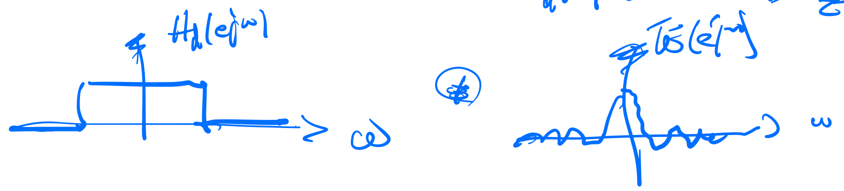
$$h_d[n] = \delta[n] - \left(\frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n}\right)$$

TYPE I ONLY



WINDOW DESIGN

$$h[n] = h_d[n - \tfrac{M}{2}]\, w[n] \Longleftrightarrow H(e^{j\omega}) = H_d(e^{j\omega}) \cdot e^{-j\frac{\omega M}{2}} \circledast \frac{1}{2\pi} W(e^{j\omega})$$



PROCEDURE

1. CHOOSE WINDOW SHAPE BASED ON DESIRED STOPBAND ATTENUATION

2. CHOOSE WINDOW LENGTH BASED ON TRANSITION BANDWIDTH

Ex. $\left.\begin{array}{l}\omega_p = .25\pi \\ \omega_s = .3\pi\end{array}\right\}\ \Delta\omega = .05\pi$

PASSBAND RIPPLE $= \pm 1\,dB$

STOPBAND RIPPLE $< -60\,dB$

$20\log_{10}(1-\delta_p) < -1$

$20\log_{10}(\delta_s) < -60$

60 dB STOPBAND ATTENUATION $\rightarrow$ BLACKMAN WINDOW

$\Delta\omega = .05\pi = \frac{12\pi}{M}$

$M = \frac{12}{.05} = 240$, $\omega_c = \frac{\omega_p + \omega_s}{2} = .275\pi$

$$h[n] = h_d[n - \tfrac{M}{2}] \cdot w[n]$$

$$= \frac{\sin(.275\pi(n-140))}{\pi(n-140)} \cdot \left(.42 - .5\cos\left(\frac{2\pi n}{240}\right) + .08\cos\left(\frac{4\pi n}{240}\right)\right)$$

KAISER WINDOW DESIGN

$$w[n] = \frac{I_0\left[\beta\left(1-\left(\frac{n-\alpha}{\alpha}\right)^2\right)^{1/2}\right]}{I_0(\beta)} \qquad 0 \le n \le M$$

$$0, \text{ ELSE}$$

β SHAPE PARAM

$\alpha = M/2$

DESIGN PROCEDURE

$$\Delta\omega = \omega_s - \omega_p = .05\pi$$

$$A = -20\log_{10}(\delta_s) = 60$$

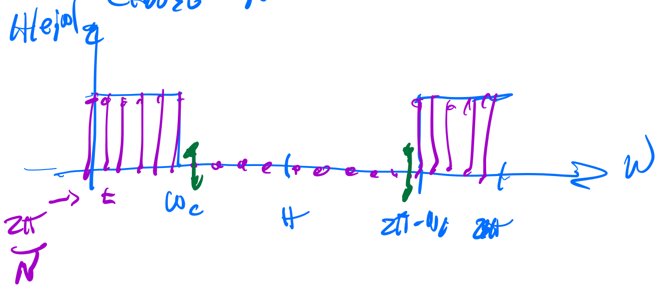
1. $$\beta = \begin{cases} .1102(A-8.7), & A \ge 50 \\ .5842(A-21)^{.4} + .07886(A-21), & 21 \le A \le 50 \\ 0, & A \le 21 \end{cases}$$

$$\beta = (.1102)(51.3) = 5.6533$$

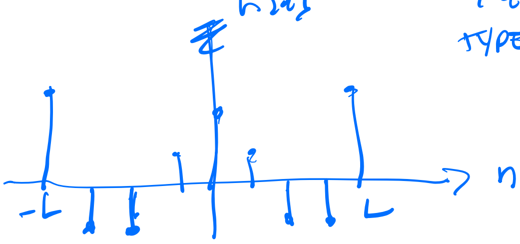
2. $$M = \frac{A-8}{(2.285)(\Delta\omega)} = 144.9 \Rightarrow 145$$

FREQUENCY - SAMPLING DESIGN

CHOOSE N



PARKS-McCLELLAN ALGORITHM



$N = 2L+1$

TYPE I	OE/YP	7.7.1
TYPE II	OS/YP	7.7.2

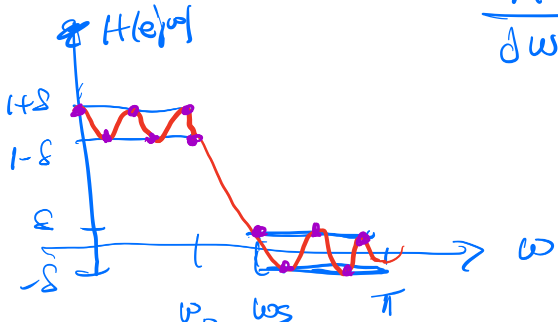
$$H(e^{j\omega}) = \sum_{n=-L}^{L} h[n] e^{-j\omega n}$$

$$= h[0] + \sum_{n=1}^{L} h[n]\, 2\cos(\omega n)$$

$$H(e^{j\omega}) = \sum_{\ell=0}^{L} b_\ell \left(\cos(\omega)\right)^\ell$$

ALTERNATION THM.

FOR A TYPE I TO BE OPTIMAL, NEED AT LEAST $L+2$ ALTERNATIONS



$$\frac{d H(e^{j\omega})}{d\omega} = -n \sum_{\ell=1}^{L} b_\ell\, \ell \left(\cos(\omega)\right)^{\ell-1} \sin(\omega n)$$

$L-1$ ZEROS FOR $0<\omega<\pi$

0 FOR $\omega = 0, \pi$

MAX ALTS WOULD BE

$$L-1 + 2 + 2 < L+3$$

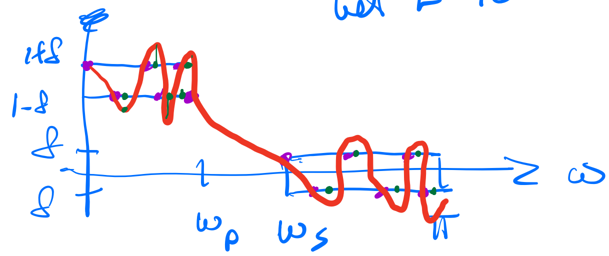
ω_p, ω_s $0, \pi$

Parks-McClellan Algorithm

Given L, ω_p, ω_s, δ_p/δ_s

1. ~~Choose~~ Guess $L+2$ freqs. Include ω_p, ω_s, 0 and π; $L-1$ more

Let $L=10$



2. Thread Chebyshev polynomial through alt points

3. Remez exchange: Replace alt freqs by observed alt freqs.