4/21/23 RECITATION 9A FIR FILTER DESIGN

APPROACHES TO FIR DESIGN

* DESIGN USING WINDOWS — CLASSICAL, KAISER
* FREQ SAMPLING DESIGN
* PARKS - McCLELLAN ALGORITHM

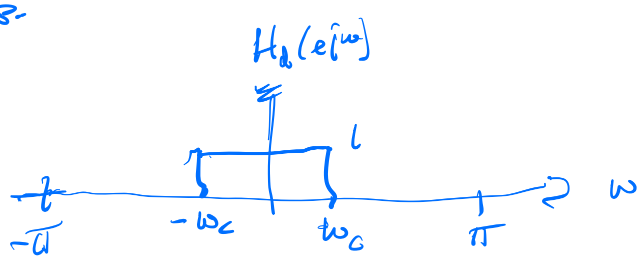
$$h[n] = h_d[n - \tfrac{M}{2}] \cdot w[n]$$

- $h[n]$: FIR LIN. PHASE
- $h_d[n - \tfrac{M}{2}]$: SHIFTED IDEAL SAMPLE RESP
- $w[n]$: FINITE DUR WINDOW SYMMETRIC ABOUT $M/2$

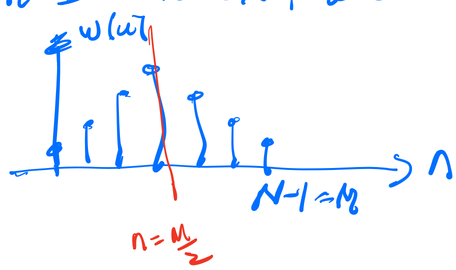
PROTOTYPE SAMPLE RESP.

LPF $\quad h_d[n] = \dfrac{\sin(\omega_c n)}{\pi n}$

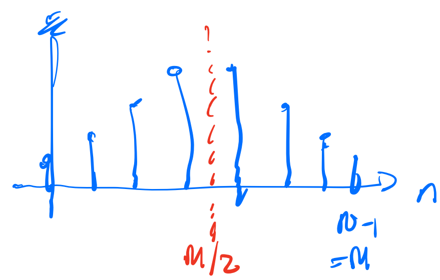
$n = 2L+1$



TYPE I $\quad N$ ODD, EVEN SYMM.

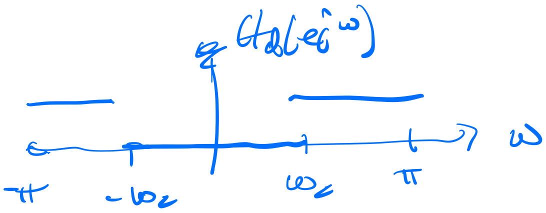


TYPE II $\quad N$ EVEN, EVEN SYMM.



HPF

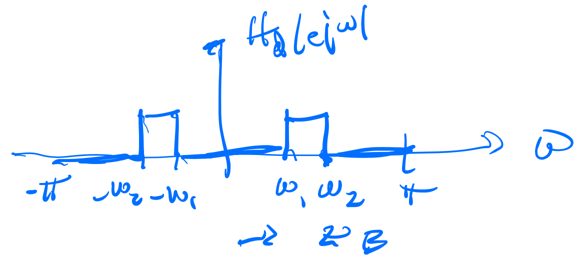
$$h_d[n] = \delta[n] - \frac{\sin(\omega_c n)}{\pi n}$$



MUST BE TYPE I

BPF

$$h_d[n] = \frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n}$$



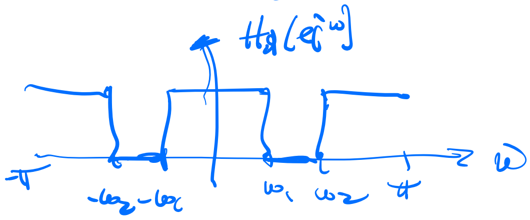
OR Let $\omega_0 = \frac{\omega_2 + \omega_1}{2}$, $B = \omega_2 - \omega_1$

$$h_d[n] = \frac{2\sin\left(\frac{B}{2} n\right)}{\pi n} \cdot \cos(\omega_0 n)$$

BAND STOP

$$h_d[n] = \delta[n] - \left(\frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n}\right)$$

TYPE I



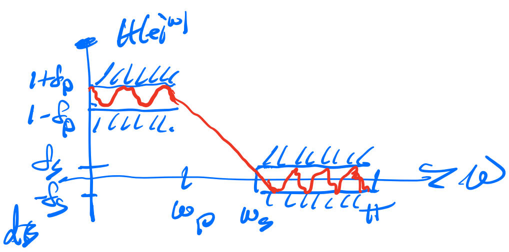
DESIGN EXAMPLE

Let $\omega_p = .25\pi$

$\omega_s = .3\pi$

PASSBAND RIPPLE ± 1 dB

STOPBAND RIPPLE ≤ -60 dB



$$20\log_{10}(1-\delta_p) = -1$$

$$20\log_{10}\delta_s = -60$$

$$h[n] = h_d[n - \tfrac{M}{2}]\, w[n] \longleftrightarrow H(e^{j\omega}) = \frac{1}{2\pi} H_d(e^{j\omega}) e^{-j\omega\frac{M}{2}} \circledast W(e^{j\omega})$$

DESIGN PROCEDURE:

1. CHOOSE SHAPE OF WINDOW BASED ON STOPBAND ATT.

2. CHOOSE LENGTH OF WINDOW BASED ON TRANSITION BW

-60 dB $\Rightarrow$ BLACKMAN WINDOW

MAIN LOBE $\frac{12\pi}{M} = .05\pi$ $\Delta\omega$

$$M = \frac{12}{.05} = 240$$

$$h[n] = h_d[n - \tfrac{M}{2}]\, w[n]$$

$$\omega_c = \frac{\omega_p + \omega_s}{2} = .275\pi$$

* $$h_d[n - \tfrac{M}{2}] = \frac{\sin(.275\pi(n-120))}{\pi(n-120)}$$

* $$w[n] = \begin{cases} .42 - .5\cos\left(\frac{2\pi n}{240}\right) + .08\cos\left(\frac{4\pi n}{240}\right) & 0 \le n \le 239 \\ 0, & \text{ELSE} \end{cases}$$

KAISER WINDOW DESIGN

$$w(n) = \frac{I_0\left[\beta\left(1-\left(\frac{n-\alpha}{\alpha}\right)^2\right)^{1/2}\right]}{I_0(\beta)}$$

$\alpha = \frac{M}{2}$

$\Delta\omega = .05\pi$

$A = -20\log_{10}(\delta_s) = 60$

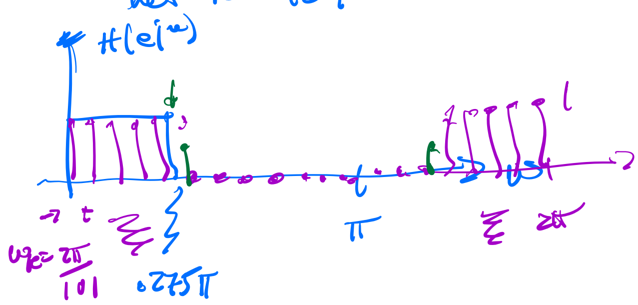
1. $\beta = \begin{cases} .1102(A-8.7), & A > 50 \\ .5842(A-21)^{.4} + .07886(A-21) & 21 \le A \le 50 \\ 0, & A \le 21 \end{cases}$

$\beta = (.1102)(60-8.7) = 5.6533$

2. $M = \dfrac{A-8}{(2.285)(\Delta\omega)} = 144.9 \Rightarrow 145$, with $\Delta\omega = .05\pi$

FREQUENCY SAMPLING APPROACH

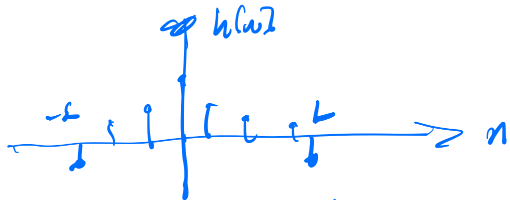
Let $N = 101$



Parks-McClellan Algorithm

TYPE I OSYP 7.7.1

TYPE II OSYP 7.7.2



$N = 2L+1$

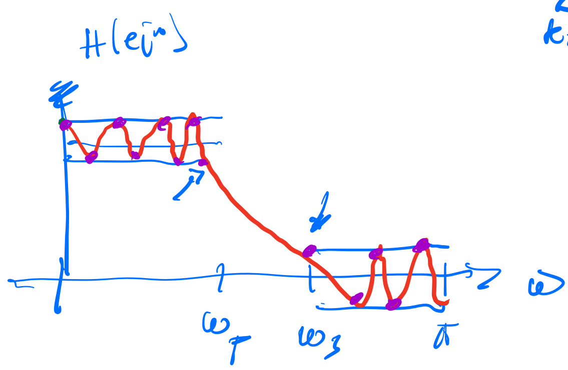
$$H(e^{j\omega}) = \sum_{n=-L}^{L} h[n]\, e^{-j\omega n}$$

$$h[n] = h[-n]$$

$$H(e^{j\omega}) = h[0] + \sum_{n=1}^{L} h[n]\, 2\cos(\omega n)$$

$$H(e^{j\omega}) = \sum_{k=0}^{L} b_k \left(\cos(\omega n)\right)^k$$

CHEBYSCHEV POLYNOMIALS



ALTERNATION THEOREM

FOR A FILTER TO BE OPTIMAL, NEED AT LEAST $\underline{L+2}$ ALTERNATIONS

$$\frac{dH(e^{j\omega})}{d\omega} = -n \sum_{k=1}^{L} k\, b_k \left(\cos(\omega n)\right)^{k-1} \cdot \sin(\omega n)$$

ZERO-SLOPE POINTS @ $\omega = 0, \pi$

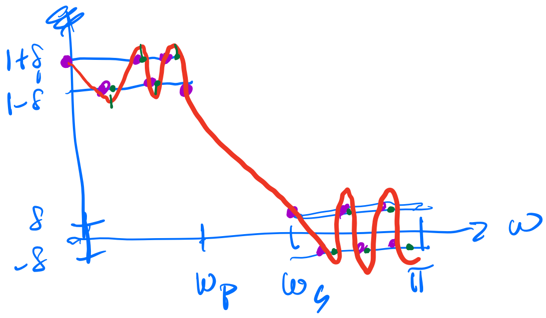
$+ \; L-1$, @ $0 < \omega < \pi$

TOTAL POSSIBLE ALTS $L-1+\underbrace{2}_{0,\pi}+\underbrace{2}_{\omega_s,\,\omega_p} = L+3$

IMPLEMENTATION of P. Mc

GIVEN N a L, ω_p, ω_s
δ_p / δ_s

$L = 10$



1. CHOOSE INITIAL ALT. FREQS, 0 a π, ω_s, ω_p $L-1$ MORE
2. THREAD CHEBYSCHEV POLYNOMIAL
3. REMEZ EXCHANGE: REPLACE ALT. FREQS BY OBSERVED MAX & MIN