4/5/24 RECITATION 8B IIR FILTER DESIGN

IIR FILTER DESIGN APPROACHES

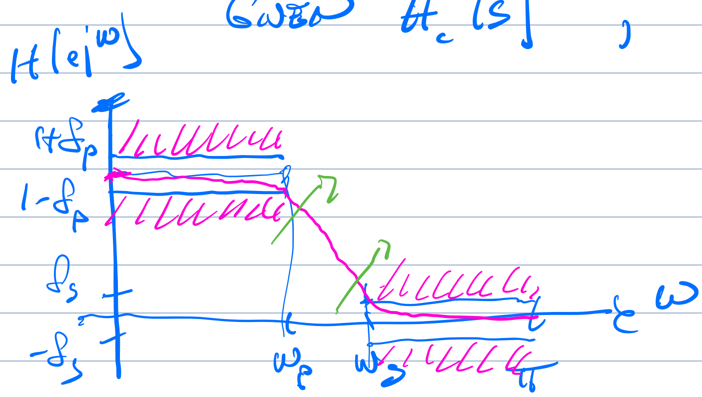
IMPULSE INVARIANCE

Given $h_c(t)$, let $h[n] = T\, h_c(nT)$

$$\omega = \Omega T$$

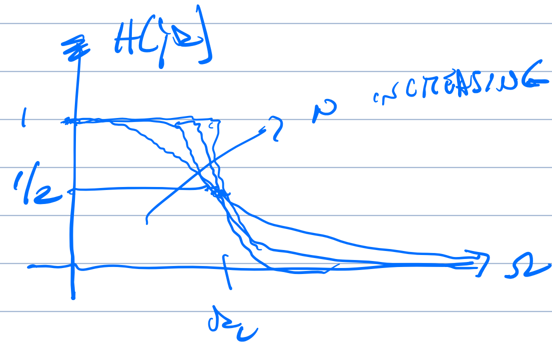
BILINEAR TRANSFORMATION

Given $H_c(s)$, let $H(z) = H_c(s)\Big|_{s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$

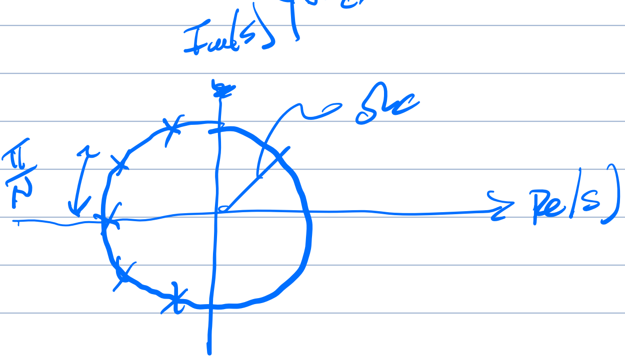


BUTTERWORTH DESIGN

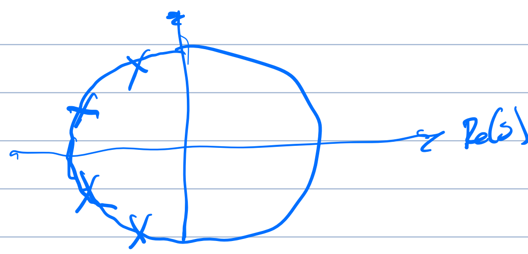
$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$$



$N=5$



$N=4$



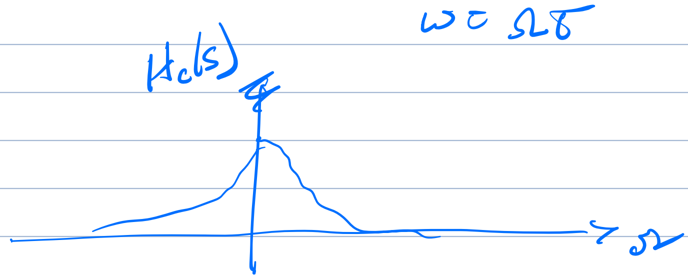
Given $\omega_p, \omega_s, \delta_p, \delta_s$ find smallest N, Ω_c to meet specs

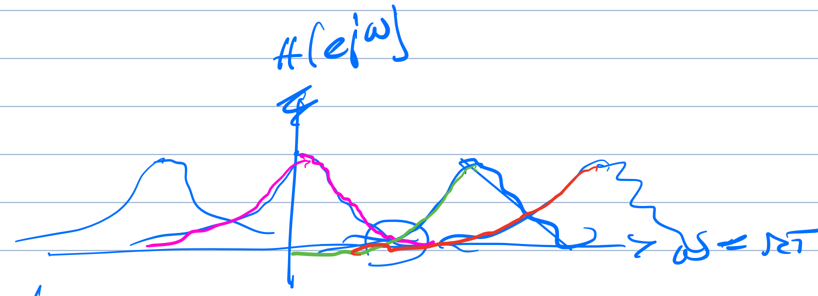
Let $k_1 = \dfrac{1}{(1-\delta_p)^2} - 1$, $k_2 = \dfrac{1}{\delta_s^2} - 1$

$$N = \frac{\log(k_2/k_1)}{2\log(\Omega_s/\Omega_p)}$$

DESIGN USING IMPULSE INVARIANCE

$\omega = \Omega T$





$\omega = \Omega T \; ; \; \Omega = \omega / T$

EX $\quad \omega_p = .15\pi, \; \omega_s = .35\pi, \; \delta_p = -3\text{dB}, \; \delta_s = -20\text{dB}$

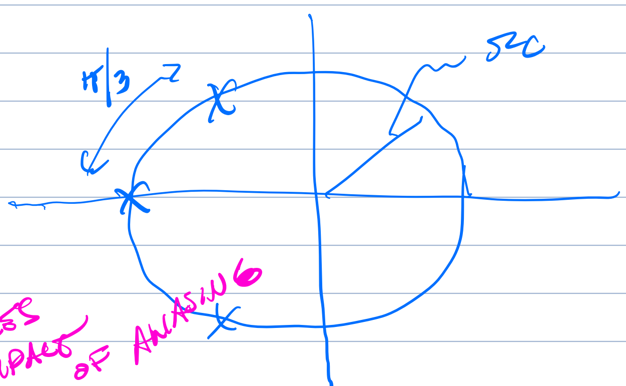
AMPLITUDE $\quad \text{dB} = 20 \log_{10}\left(\frac{X}{X_{REF}}\right) \; ; \; X = 10^{\text{dB}/20}$

ENERGY, POWER $\quad \text{dB} = 10 \log_{10}\left(\frac{P}{P_{REF}}\right) \; ; \; P = 10^{\text{dB}/10}$

$\Omega_p = \frac{\omega_p}{T} = \frac{.4712}{T}, \quad \Omega_s = \frac{\omega_s}{T} = \frac{1.1}{T}$

$K_1 = \frac{1}{(1-\delta_p)^2} - 1 = .9953, \quad K_2 = \frac{1}{\delta_s^2} - 1 = 99$

$N = \frac{\log(K_2/K_1)}{2\log(\Omega_s/\Omega_p)} = 2.71 \Longrightarrow 3$



POLES @ $-\Omega_c = \Omega_c e^{j\pi}$

$\Omega_c e^{j(\pi - \frac{\pi}{3})}$

$\Omega_c e^{j(\pi + \frac{\pi}{3})}$

REDUCES IMPACT OF ALIASING

$\star$ MEET SPECS @ PASSBAND $\longrightarrow \Omega_c = \frac{\Omega_p}{K_1^{1/2N}} = \frac{.472}{T}$

MEET SPECS @ STOPBAND $\longrightarrow \Omega_c = \frac{\Omega_s}{K_2^{1/2N}} = \frac{.511}{T}$

$$H_c(s) = \frac{\Omega_c^3}{(s-(-\Omega_c))(s-\Omega_c e^{j(\pi-\pi/3)})(s-\Omega_c e^{j(\pi+\pi/3)})}$$

$$h(t) = e^{s_k t} u(t) \Longleftrightarrow \frac{1}{s-s_k}$$

$$\Updownarrow$$

$$h[n] = T e^{s_k nT} u[n] = T(e^{s_k T})^n$$

$$\Longleftrightarrow H(z) = \frac{T}{1-e^{s_k T} z^{-1}}$$

Let $H_c(s) = \sum_{k=1}^{N} \frac{A_k}{(s-s_k)}$, $A_k = H(s)(s-s_k)\Big|_{s=s_k}$

MATLAB RESIDUE

To convert,

1. Find $\{A_k\}$ using partial fractions
2. Let $H(z) = \sum_{k=1}^{N} A_k \frac{1}{1-e^{s_k T} z^{-1}}$

BILINEAR TRANSFORM

$$s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}$$

$$sT(1+z^{-1}) = 2(1-z^{-1})$$

$$sT(z+1) = 2(z-1)$$

$$sTz - 2z = -2 - sT$$

$$z(sT-2) = -(sT+2)$$

$$z\left(s-\frac{2}{T}\right) = -\left(s+\frac{2}{T}\right)$$

$$z = \frac{\frac{2}{T}+s}{\frac{2}{T}-s}$$

$s = \sigma + j\Omega$

$$z = \frac{\frac{2}{T} + (\sigma + j\Omega)}{\frac{2}{T} - (\sigma + j\Omega)}$$

For $\sigma = 0$, $$z = \frac{\frac{2}{T} + j\Omega}{\frac{2}{T} - j\Omega}$$

$$|z|^2 = \frac{\left(\frac{2}{T}\right)^2 + \Omega^2}{\left(\frac{2}{T}\right)^2 + \Omega^2} = 1$$

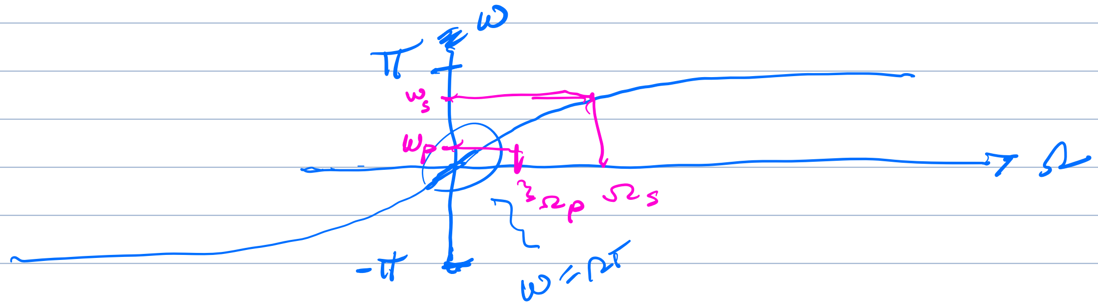
$$s = \frac{2}{T} \frac{(1 - z^{-1})}{(1 + z^{-1})}$$

For $s = j\Omega$
$z = e^{j\omega}$

$$j\Omega = \frac{2}{T} \frac{(1 - e^{-j\omega})}{(1 + e^{-j\omega})} = \frac{2}{T} \frac{\cancel{e^{-j\frac{\omega}{2}}}\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)}{\cancel{e^{-j\frac{\omega}{2}}}\left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right)}$$

$$j\Omega = \frac{2}{T} \frac{\cancel{2}\cancel{j} \sin\left(\frac{\omega}{2}\right)}{\cancel{2} \cos\left(\frac{\omega}{2}\right)}$$

$$\boxed{\Omega = \frac{2}{T} \tan\left(\frac{\omega}{2}\right) \qquad \omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)}$$



$\omega_p = .15\pi \qquad \delta_p = -3\,dB \Rightarrow k_1 = .9953$

$\omega_s = .35\pi \qquad \delta_s = -20\,dB \Rightarrow k_2 = 99$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{.48}{T}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{.57}{T}$$

$$N = \frac{\log(k_2/k_1)}{2\log(\Omega_s/\Omega_p)} = 2.45 \Rightarrow 3$$

$\Omega_c = \dfrac{.98}{T}$ MATCHING @ Ω_p

$\Omega_c = \dfrac{.57}{T}$ MATCHING @ Ω_s

$$H_c(s) = \frac{\Omega_c^3}{(s-\Omega_c e^{j\pi})(s-\Omega_c e^{j(\pi-\pi/6)})(s-\Omega_c e^{j(\pi+\pi/6)})}$$

$$H_c(s) = \frac{.1850/T^3}{s^3 + \frac{1.14}{T}s^2 + \frac{.699}{T^2}s + \frac{.185}{T^3}}$$

$$H(z) = \frac{.1850/T^3}{\left(\frac{2}{T}\frac{(1-z^{-1})}{1+z^{-1}}\right)^3 + \frac{1.14}{T}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \frac{.699}{T^2}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right) + \frac{.185}{T^3}}$$