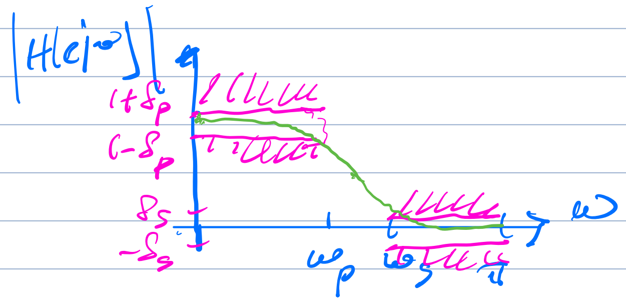
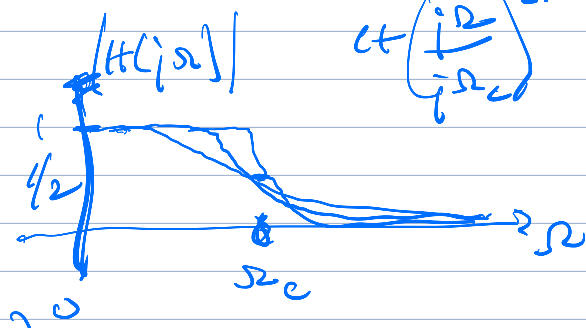
4/5/24 Recitation 8A IIR Filter Design

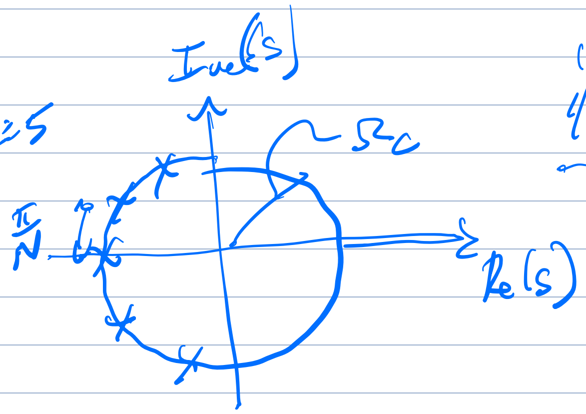


Butterworth Filter

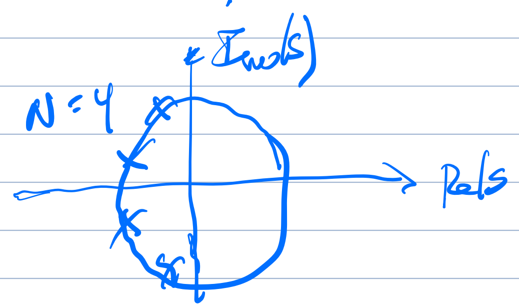
$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$$



$N=5$



$N=4$



Given $\delta_p, \delta_s, \Omega_p, \Omega_s$

Find N, Ω_c

$$k_1 = \frac{1}{(1-\delta_p)^2} - 1$$

$$k_2 = \frac{1}{\delta_s^2} - 1$$

$$N = \frac{\log(k_2/k_1)}{2\log(\Omega_s/\Omega_p)}$$

$\Omega_c = \dfrac{\Omega_p}{k_1^{1/2N}}$, meets spec @ $\Omega = \Omega_p$

$\Omega_c = \dfrac{\Omega_s}{k_2^{1/2N}}$, meets spec @ $\Omega = \Omega_s$

Amplitude $\quad dB = 20\log_{10}\left(\frac{x}{x_{REF}}\right)$

$$x = 10^{\frac{dB}{20}}$$

Energy or Power $\quad dB = 10\log_{10}\left(\frac{P}{P_{REF}}\right)$

DESIGN EXAMPLE

$\omega_p = .15\pi \qquad \delta_p = -3\,dB$

$\omega_s = .35\pi \qquad \delta_s = -20\,dB$

TRANSFORMATION METHODS

IMPULSE INVARIANCE

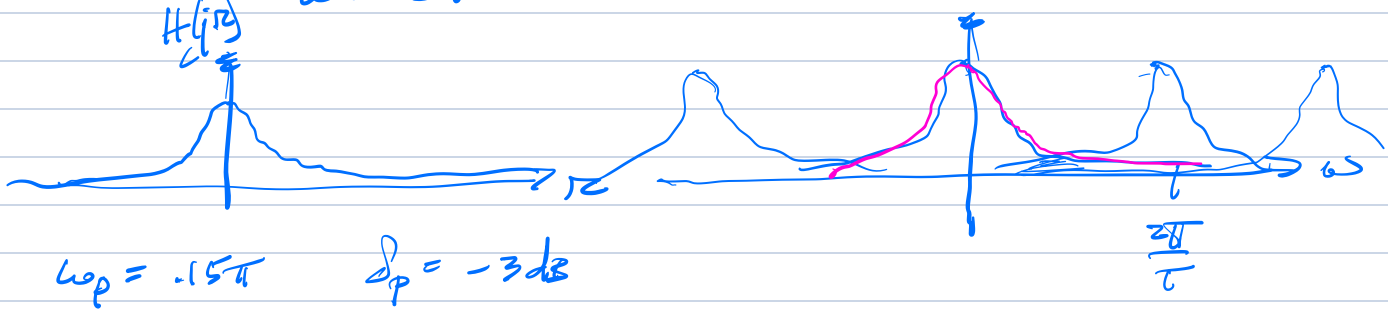
Given $h_c(t)$, let $h[n] = T\,h_c(nT)$

BILINEAR TRANSFORMATION

$H_c(s)$, let $H(z) = H_c(s)\Big|_{s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$

IMPULSE INVARIANCE

$\omega = \Omega T$



$\omega_p = .15\pi \qquad \delta_p = -3\,dB$

$\omega_s = .35\pi \qquad \delta_s = -20\,dB$

IMP. INV. SOLUTION

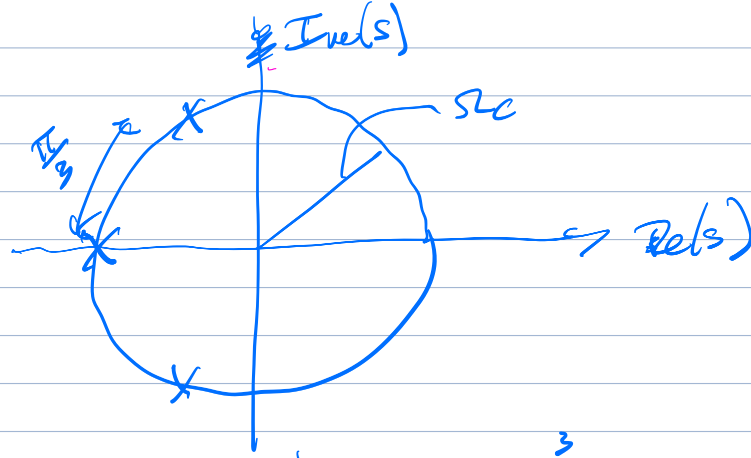
$\Omega_p = \frac{\omega_p}{T} = \frac{.4712}{T} \qquad \Omega_s = \frac{1.1}{T} \qquad k_1 = .9953,\ k_2 = 99$

$N = 2.71 \Rightarrow 3$

$\Omega_c = \frac{.472}{T}$, MEETING @ PASSBAND

$\omega_p = .15\pi \qquad \delta_p = -3\,dB$

$\Omega_c = \omega_s \frac{.511}{T}$ $.35\pi$ MEETING @ STOPBAND $\delta_s = -28\,dB$



$$H(s) = \frac{\Omega_c^3}{\left(s-(-\Omega_c)\right)\left(s-\Omega_c e^{j(\pi-\pi/3)}\right)\left(s-\Omega_c e^{j(\pi+\frac{\pi}{3})}\right)}$$

$$h(t) = e^{s_k t} u(t) \Longleftrightarrow H(s) = \frac{1}{s-s_k}, \quad Re(s) > Re(s_k)$$

$$h[n] = T e^{s_k T n} u[n] = T\left(e^{s_k T}\right)^n \Longleftrightarrow H(z) = \frac{T}{1-e^{s_k T} z^{-1}}$$

$$H(s) = \sum_{k=1}^{N} \frac{A_k}{s-s_k}, \quad A_k = H(s)(s-s_k)\Big|_{s=s_k}$$

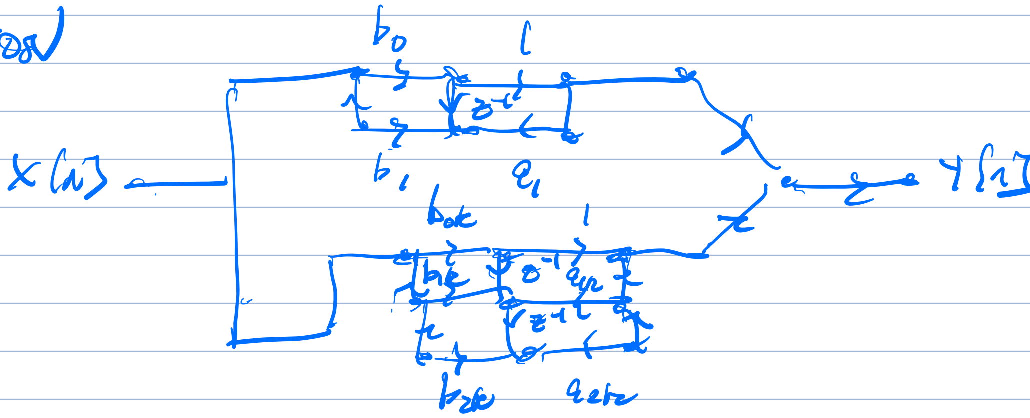
$$H(s) = \sum_{k=1}^{N} \frac{A_k}{s-s_k}$$

$$h[n] = \sum_{k=1}^{N} T A_k \left(e^{s_k T}\right)^n$$

$$H(z) = \sum_{k=1}^{N} \frac{T}{1-e^{s_k T} z^{-1}}$$

IMPLEMENTATION

PARALLEL FORM



BILINEAR TRANSFORMATION

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

Let $z = \rho e^{j\omega}$

$s = \sigma + j\Omega$

$$\sigma + j\Omega = \frac{2}{T}\,\frac{1-\rho e^{j\omega}}{1+\rho e^{j\omega}}$$

$$\sigma + j\Omega = \frac{2}{T} \cdot \frac{1-\rho\left(\cos(\omega) - j\sin(\omega)\right)}{1+\rho\left(\cos(\omega) - j\sin(\omega)\right)}$$

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \; ; \quad sT + sTz^{-1} = 2 - 2z^{-1}$$

$$sTz + sT = 2z - 2$$

$$sTz - 2z = -sT - 2$$

$$z(sT-2) = -sT - 2$$

$$z = \frac{2+sT}{2-sT} = \frac{\frac{2}{T}+s}{\frac{2}{T}-s} = \frac{\frac{2}{T}+(\sigma+j\Omega)}{\frac{2}{T}-(\sigma+j\Omega)}$$

IF $\sigma = 0$, $z = \dfrac{\frac{2}{T}+j\Omega}{\frac{2}{T}-j\Omega}$

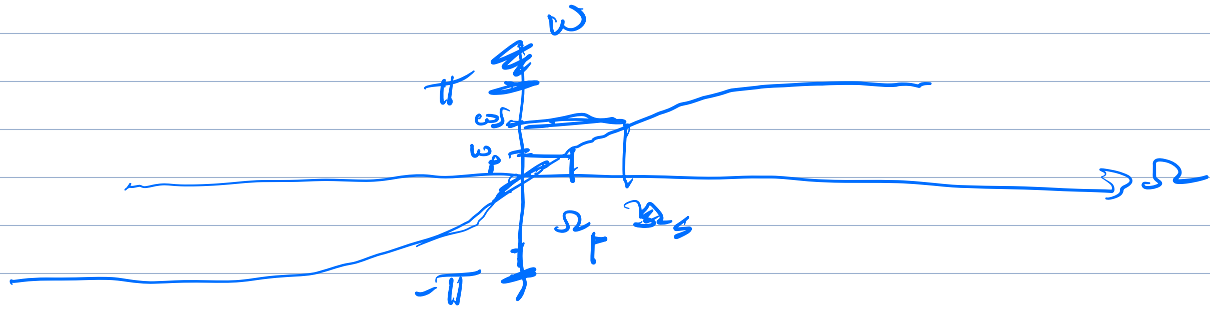
$$|z| = \frac{\left(\frac{2}{T}\right)^2 + (\Omega)^2}{\left(\frac{2}{T}\right)^2 + (\Omega)^2}$$

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \qquad \text{For } s = j\Omega,\; z = e^{j\omega}$$

$$j\Omega = \frac{2}{T}\cdot\frac{1-e^{-j\omega}}{1+e^{-j\omega}} = \frac{2}{T}\,\frac{\cancel{e^{-j\frac{\omega}{2}}}\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)}{\cancel{e^{-j\frac{\omega}{2}}}\left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right)}$$

$$\cancel{j}\Omega = \frac{2}{T}\,\frac{\cancel{2j}\sin\left(\frac{\omega}{2}\right)}{\cancel{2}\cos\left(\frac{\omega}{2}\right)}$$

$$\boxed{\begin{aligned}\Omega &= \frac{2}{T}\tan\left(\frac{\omega}{2}\right)\\ \omega &= 2\tan^{-1}\left(\frac{\Omega T}{2}\right)\end{aligned}}$$



FILTER DESIGN USING BLT

$\omega_p = .15\pi \qquad \delta_p = -3\text{dB}$

$\omega_s = .35\pi \qquad \delta_s = -20\text{dB}$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{.48}{T}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{1.23}{T}$$

$$N = \frac{\log(k_2/k_1)}{2\log(\Omega_s/\Omega_p)} = 2.45 \Rightarrow 3$$

$\Omega_c = \frac{.48}{T}$ MATCHING @ Ω_p

$\Omega_c = \frac{.57}{T}$ MATCHING @ Ω_s

$$H(s) = \frac{\Omega_c^3}{\left(s - (-\Omega_c)\right)\left(s - \Omega_c e^{j(\pi - \pi/3)}\right)\left(s - \Omega_c e^{j(\pi + \frac{\pi}{3})}\right)}$$

$$H(s) = \frac{.1850 / T^3}{s^3 + \frac{1.14}{T} s^2 + \frac{.649}{T^2} s + \frac{.185}{T^3}}$$

$$H(s) = \frac{.1850 / T^3}{\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^3 + \frac{1.14}{T}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \frac{.649}{T^2}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right) \frac{.185}{T^3}}$$