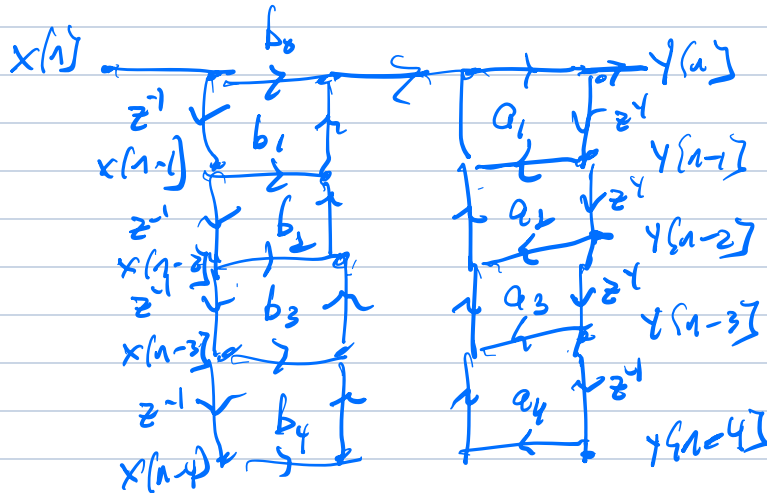


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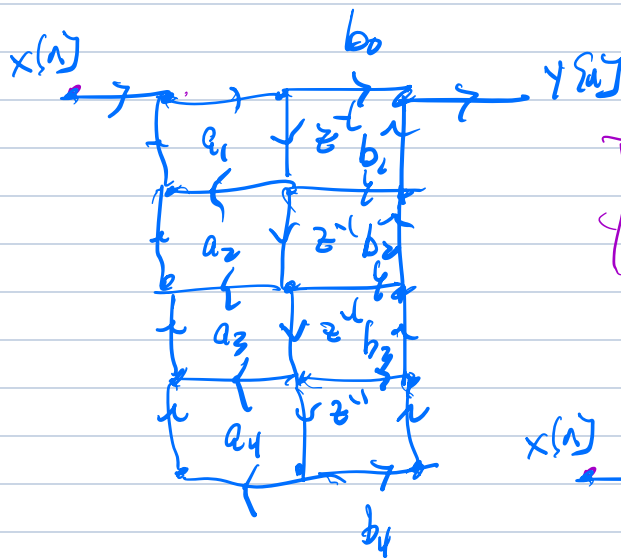
RECITATION 7B

IIR & FIR FILTER IMPLEMENTATION

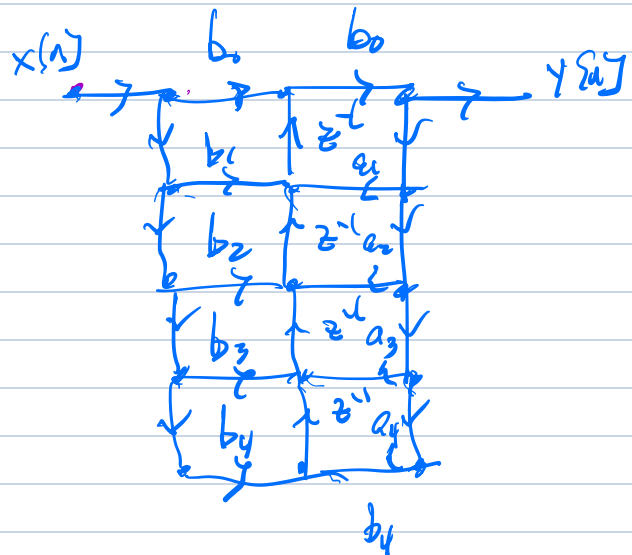
$$Y[n] = \sum_{k=1}^N a_k Y[n-k] + \sum_{l=0}^M b_l x[n-l]$$



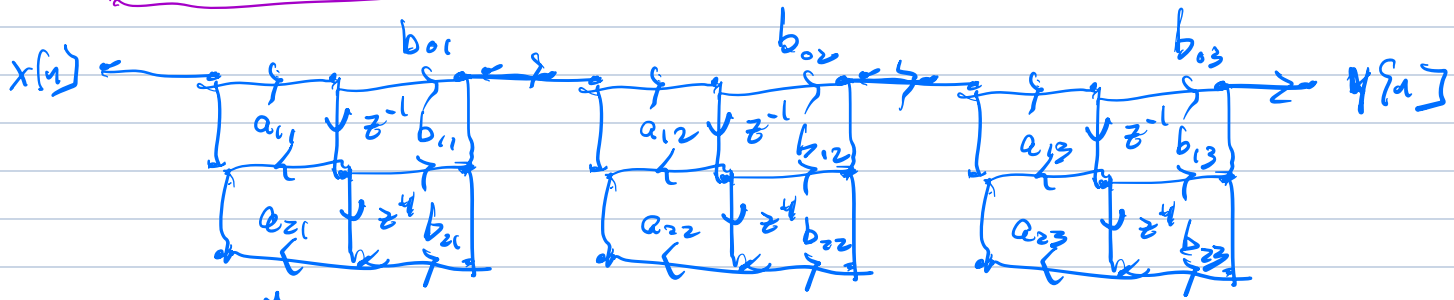
DIRECT FORM I



DIRECT FORM II



CASCADE FORM



$$H(z) = \frac{\sum_{l=0}^M b_l z^{-l}}{1 - \sum_{k=1}^N a_k z^{-k}} = \frac{b_0 \prod_{l=1}^M (1 - c_l z^{-1})}{\prod_{k=1}^N (1 - d_k z^{-k})}$$

2 REAL POLES d_1, d_2 $(1 - d_1 z^{-1})(1 - d_2 z^{-1}) = 1 - (d_1 + d_2)z^{-1} + d_1 d_2 z^{-2}$
 2 REAL POLES

2 COMPLEX-CONJ. POLES d, d^* $(1 - d z^{-1})(1 - d^* z^{-1}) = 1 - 2\text{Re}\{d\}z^{-1} + |d|^2 z^{-2}$

PARALLEL FORM

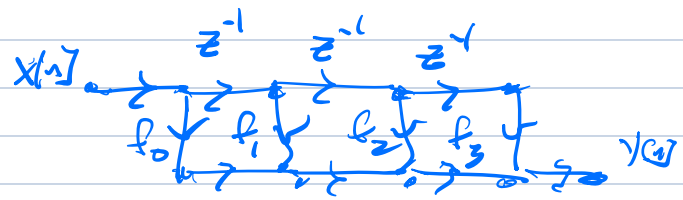
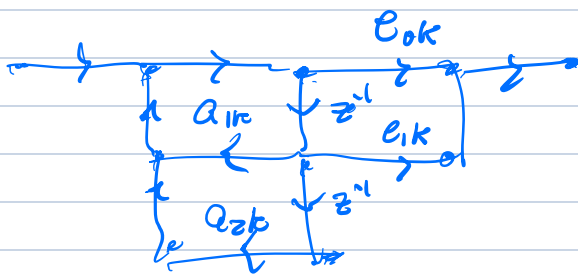
$$H(z) = \frac{\sum_{l=0}^M b_l z^{-l}}{1 - \sum_{k=1}^N a_k z^{-k}} = \sum_{k=1}^N \frac{A_k}{1 - d_k z^{-1}} + \sum_{l=0}^{M-N} f_l z^{-l}$$

IF POLES ARE IN UNIQUE LOCATIONS

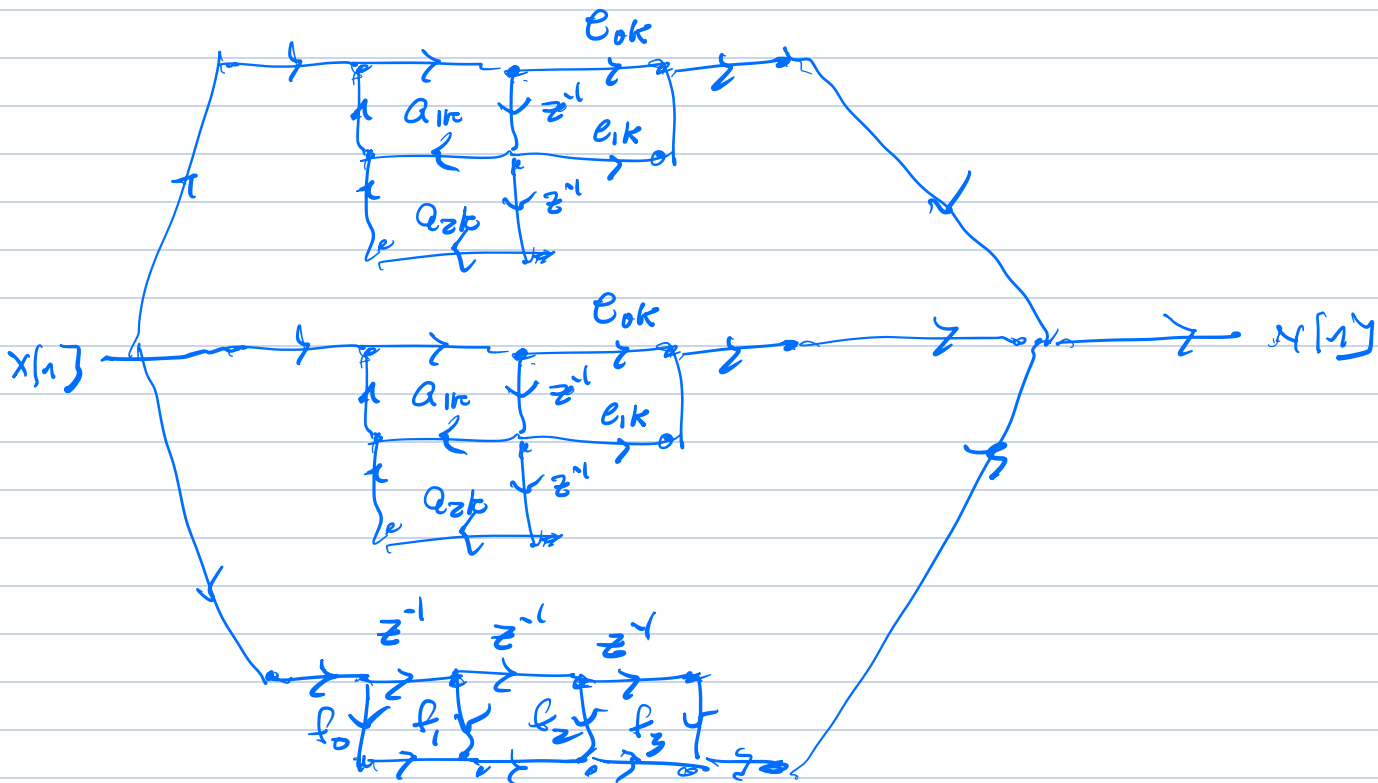
COMBINING INTO SECOND-ORDER SECTIONS

IF d_1, d_2 REAL $\frac{A_1}{1 - d_1 z^{-1}} + \frac{A_2}{1 - d_2 z^{-1}} = \frac{(A_1 + A_2) - z^{-1}(A_1 d_2 + A_2 d_1)}{1 - (d_1 + d_2)z^{-1} + d_1 d_2 z^{-2}}$

IF d, d^* COMPLEX CONJ. $\Rightarrow A, A^*$ $\frac{A}{1 - d z^{-1}} + \frac{A^*}{1 - d^* z^{-1}} = \frac{2\text{Re}\{A\} - z^{-1}(2\text{Re}\{A d\})}{1 - 2\text{Re}\{d\}z^{-1} + |d|^2 z^{-2}}$



$$H(z) = \frac{\sum_{l=0}^M b_l z^{-l}}{1 - \sum_{k=1}^N a_k z^{-k}} = \sum_{k=1}^N \frac{A_k}{1 - d_k z^{-1}} + \sum_{l=0}^{M-N} f_l z^{-l}$$

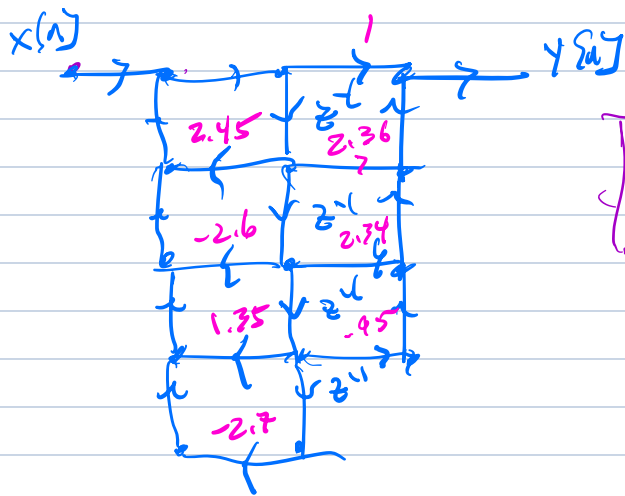


ASSUME ZEROS @ $z = \left[\begin{array}{ccc} -0.95 & \frac{-1+j}{\sqrt{2}} & \frac{-1-j}{\sqrt{2}} \end{array} \right]$

POLES @ $z = \left[\begin{array}{ccc} 0.5 & 0.75 & 0.6(1+j) \quad 0.6(1-j) \end{array} \right]$

$$\{B\} = [1 \quad 2.36 \quad 2.34 \quad .95]$$

$$\{A\} = [1 \quad -2.45 \quad 2.6 \quad -1.35 \quad .27]$$



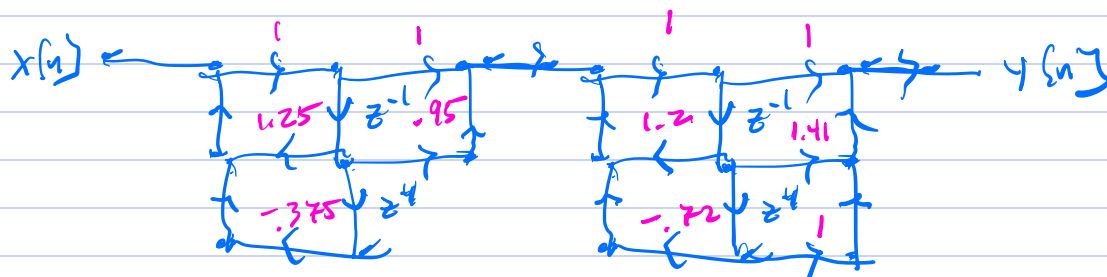
DIRECT FORM II

OSYP
$$H(z) = \frac{\sum_{l=1}^M b_l z^{-l}}{1 - \sum_{k=1}^P a_k z^{-k}}$$

MATLAB
$$H(z) = \frac{\sum_{l=1}^M b_l z^{-l}}{1 + \sum_{k=1}^P a_k z^{-k}}$$

CASCADE FORM

$$y[n] = - \sum_{k=1}^P a_k y[n-k] + \sum_{l=0}^M b_l x[n-l]$$



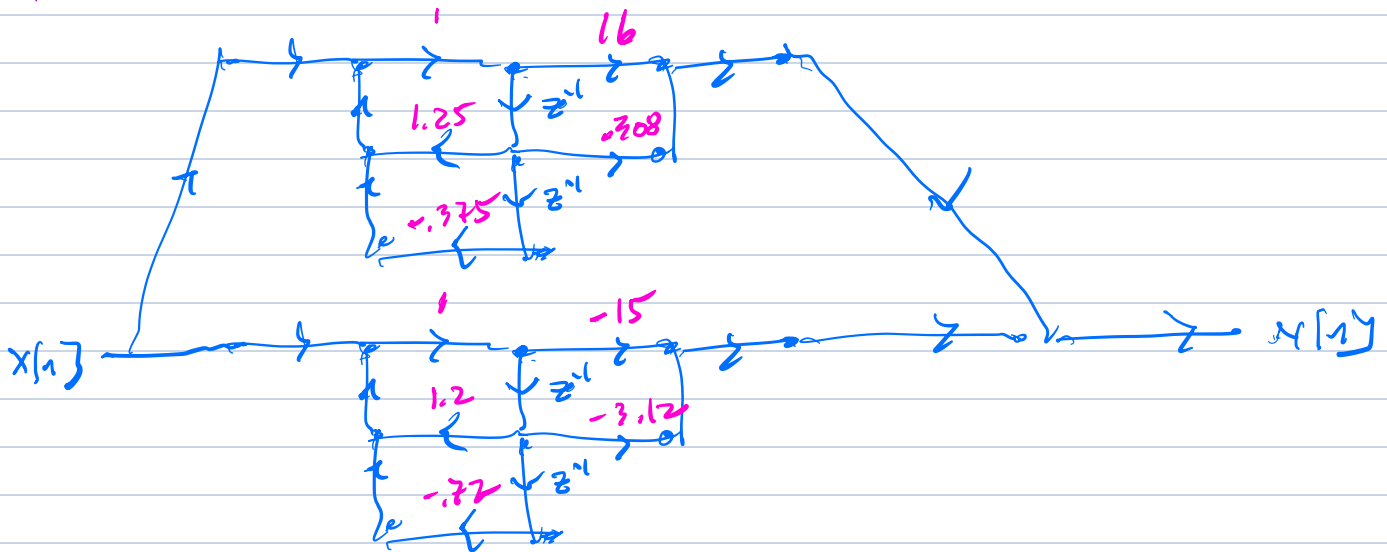
$$b_1 = [1 \quad 0.95]$$

$$a_1 = [1 \quad -1.25 \quad 0.375]$$

$$b_2 = [1 \quad 1.414 \quad 1]$$

$$a_2 = [1 \quad -1.2 \quad 0.72]$$

PARALLEL FORM



$$A_1 + A_2 = 16$$

$$A_1 d_2 + A_2 d_1 = -3.08$$

$$2 \operatorname{Re}(A) d = -15$$

$$2 \operatorname{Re}(A d^*) = -3.12$$

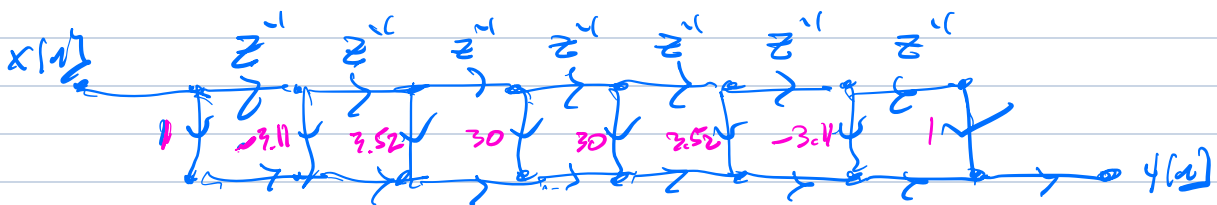
FIR FILTER

EX: LOWER PASS, ZEROS AT

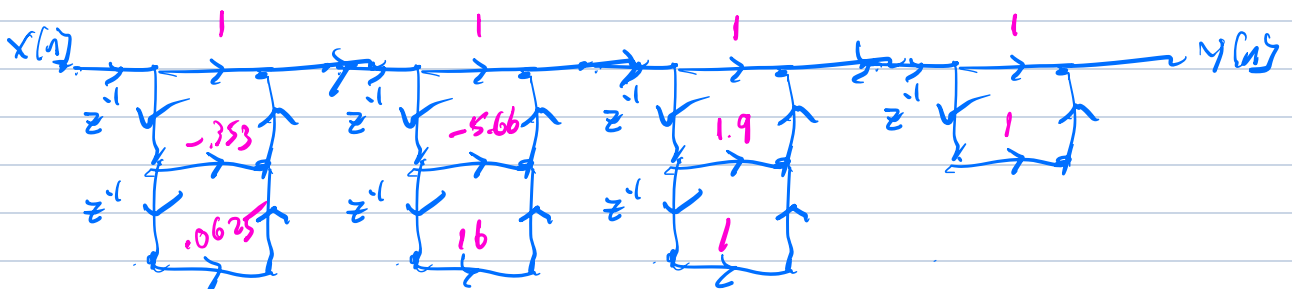
$$Z = \left[\frac{1}{2\sqrt{2}(1+j)}, \frac{1}{2\sqrt{2}(1-j)}, 2\sqrt{2}(1+j), 2\sqrt{2}(1-j), -1, e^{i \cdot 9\pi}, e^{-i \cdot 9\pi} \right]$$

$$b = [1 \quad -3.11 \quad 3.52 \quad 30 \quad 30 \quad 3.52 \quad -3.11 \quad 1]$$

DIRECT FORM

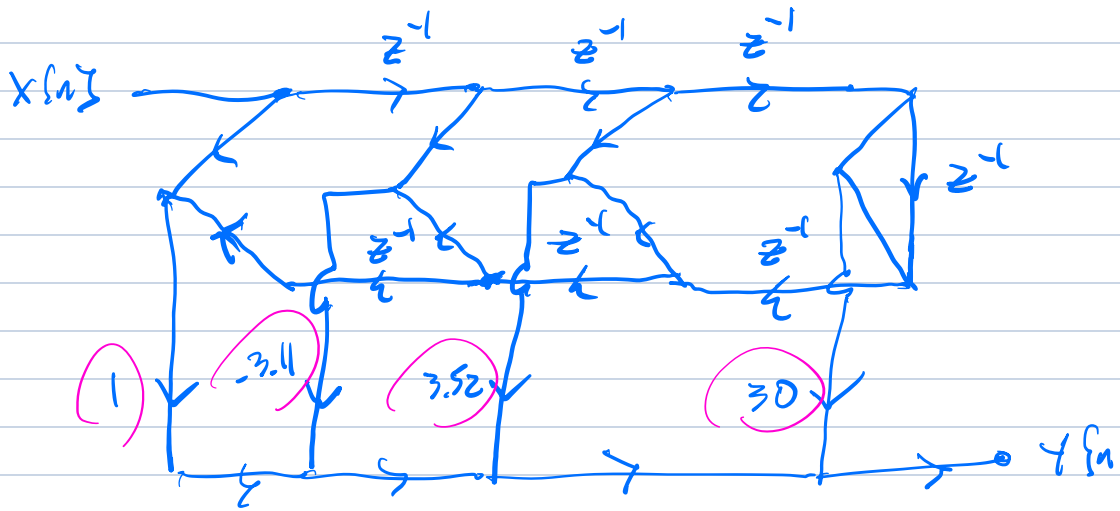


CASCADE FORM



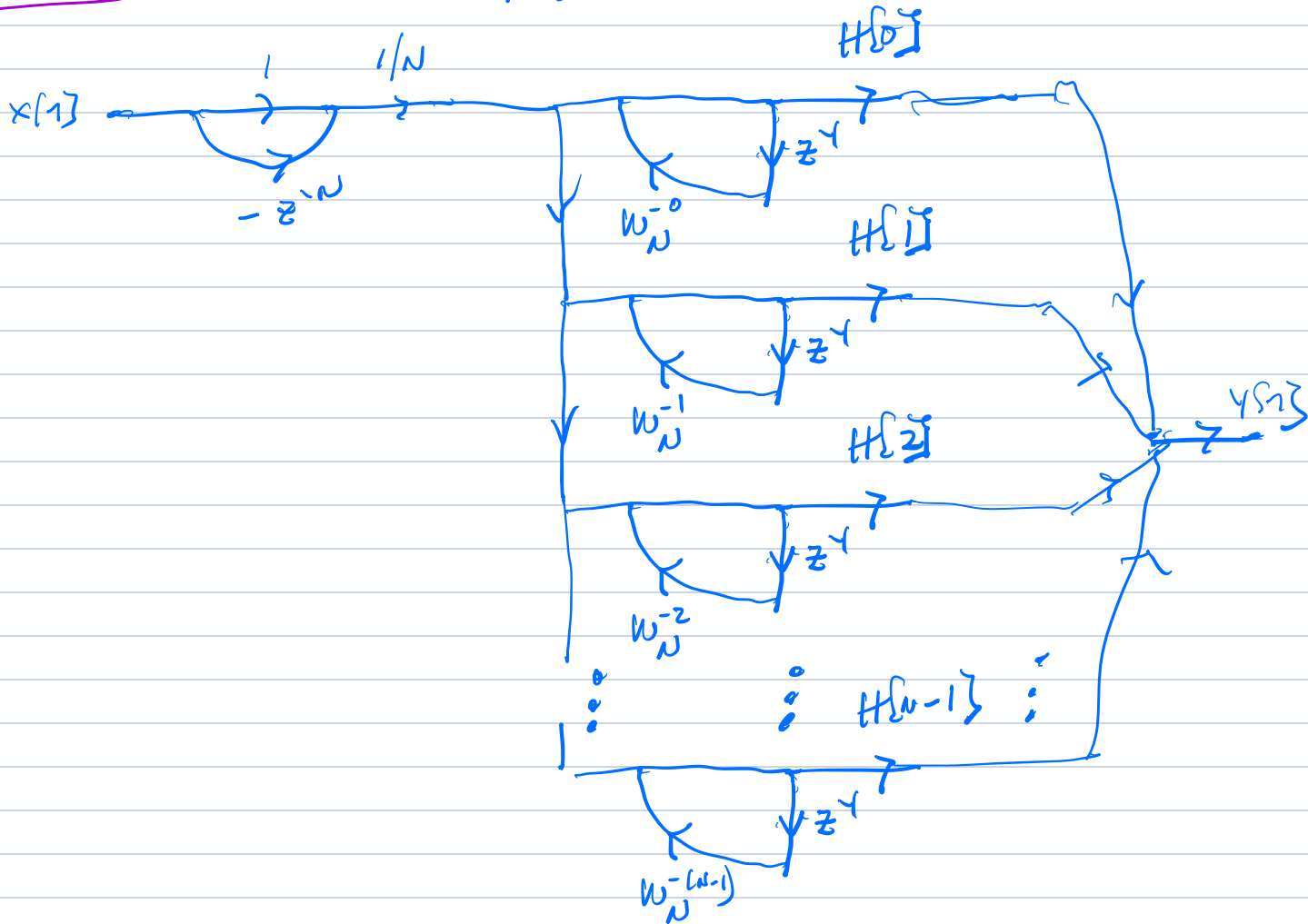
LINEAR PHASE FORM

$$b = [1 \quad -3.11 \quad 3.52 \quad 30 \quad 30 \quad 3.52 \quad -3.11 \quad 1]$$

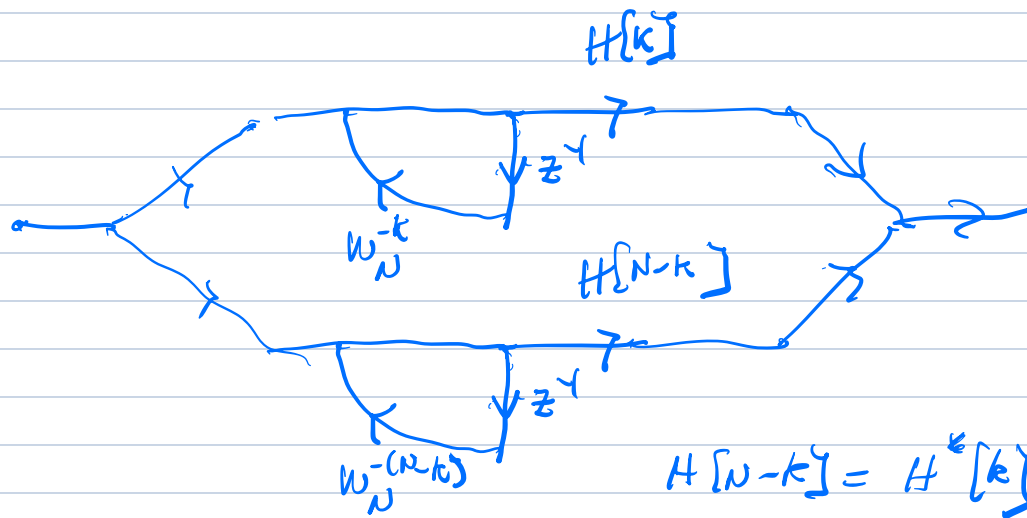


FREQUENCY SAMPLING FORM

$$H(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H[k]}{1 - \omega_N^{-k} z^{-1}}$$



COMBINE CHANNELS $k, N-k$



$$H[N-k] = H^*[k]$$

$$w_N^{-(N-k)} = \cancel{w_N^{-N}} \cdot w_N^k = \left(w_N^{-k}\right)^*$$