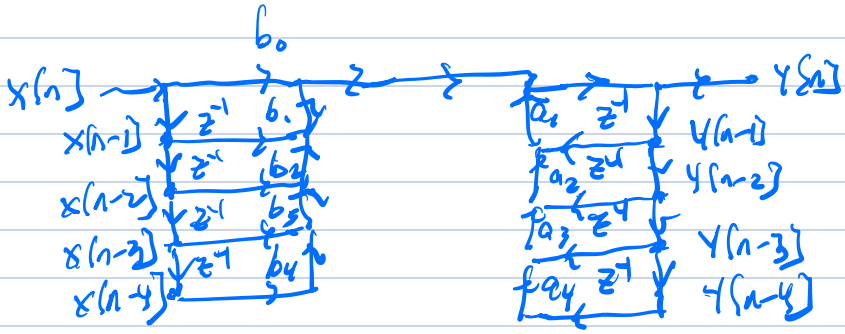
3/22/24 RECITATION 7.1 IIR, FIR IMPLEMENTATION

IIR FORMS

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{\ell=0}^{M} b_\ell x[n-\ell]$$

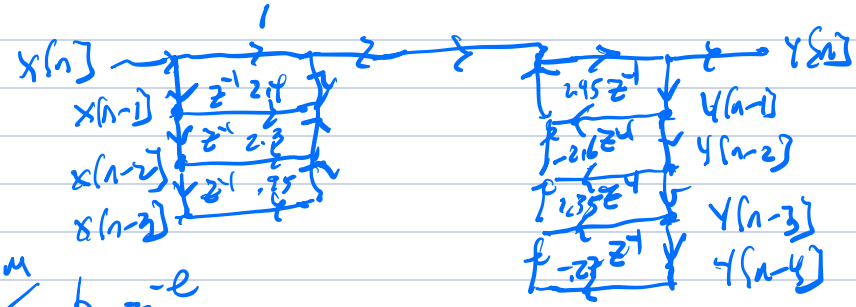
DIRECT FORM II

$M = N = 4$



ASSUME ZEROS @ $z = \left[-.95,\ \frac{-1+j}{\sqrt{2}},\ \frac{-1-j}{\sqrt{2}}\right]$

POLES @ $z = [.5\ \ .25\ \ .6(1+j)\ \ .6(1-j)]$

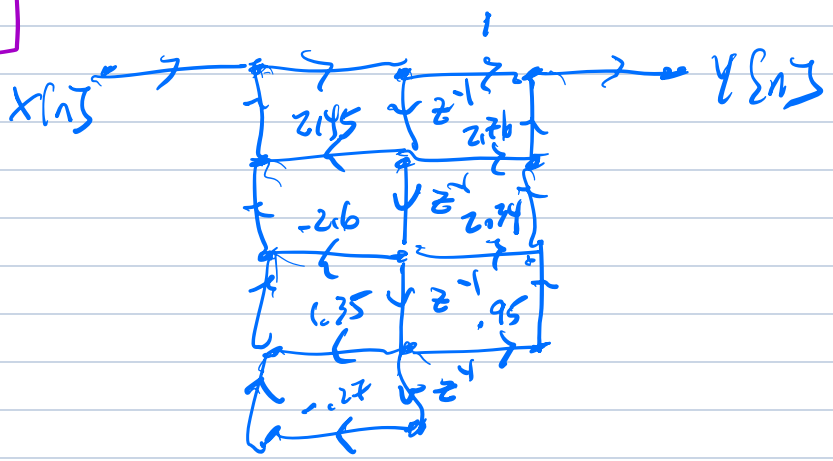


OSAP $$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

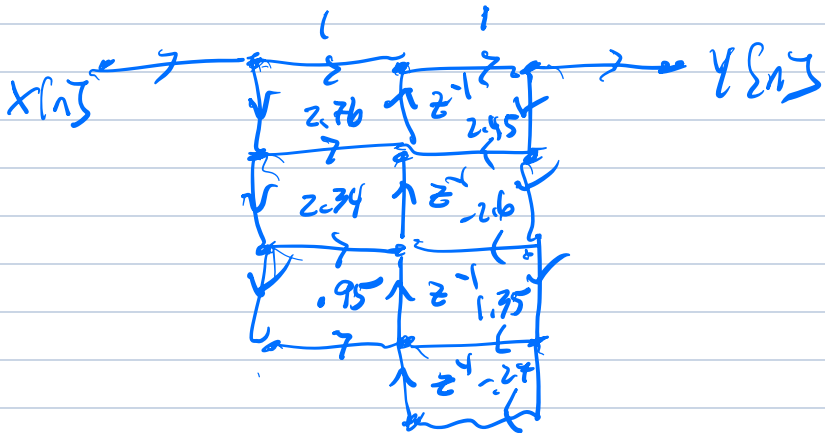
MATLAB $$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$y[n] = \sum_{\ell=0}^{M} b_\ell x[n-\ell] - \sum_{k=1}^{N} a_k y[n-k]$$

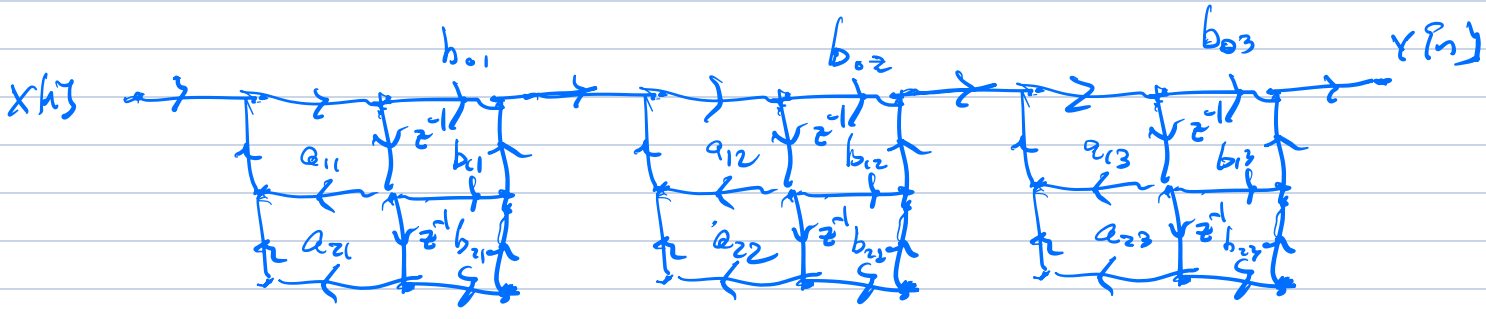
DIRECT FORM II



DIRECT FORM II TRANSPOSED



CASCADE FORM



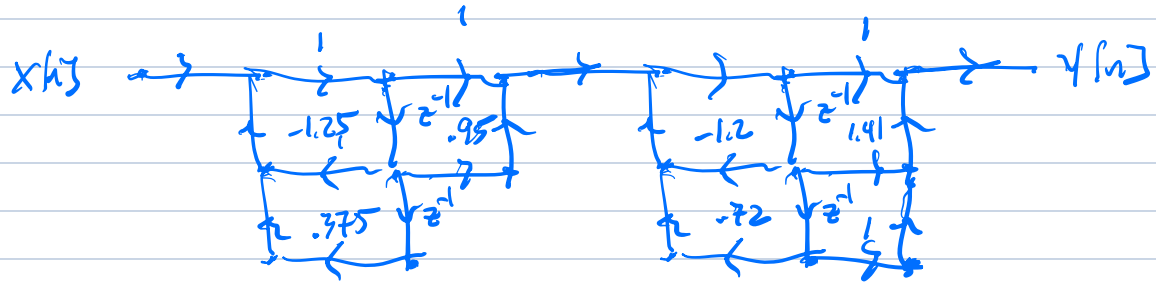
$$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 \prod_{\ell=1}^{M} (1 - c_\ell z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})}$$

2 REAL POLES d_1, d_2 $(1 - d_1 z^{-1})(1 - d_2 z^{-1}) = 1 - (d_1 + d_2) z^{-1} + d_1 d_2 z^{-2}$

2 COMPLEX POLES d, d^* $(1 - d z^{-1})(1 - d^* z^{-1}) = 1 - 2\mathrm{Re}\{d\} z^{-1} + |d|^2 z^{-2}$

$$B(z) = (1 + .95 z^{-1})(1 + 1.41 z^{-1} + z^{-2})$$

$$A(z) = (1 - 1.25 z^{-1} + .375 z^{-2})(1 - 1.2 z^{-1} + .72 z^{-2})$$



PARALLEL FORM

USE PARTIAL FRACTIONS

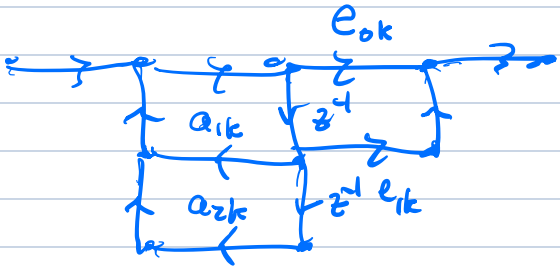
IF ALL POLES ARE IN UNIQUE LOCATIONS

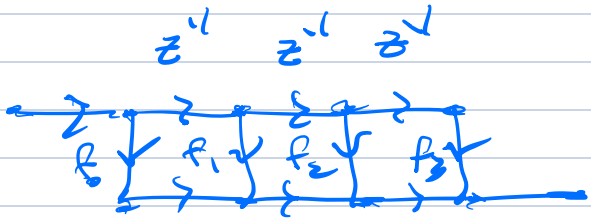
$$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \sum_{k=1}^{N} \frac{A_k}{1 - d_k z^{-1}} + \sum_{\ell=0}^{M-N} f_\ell z^{-\ell}$$

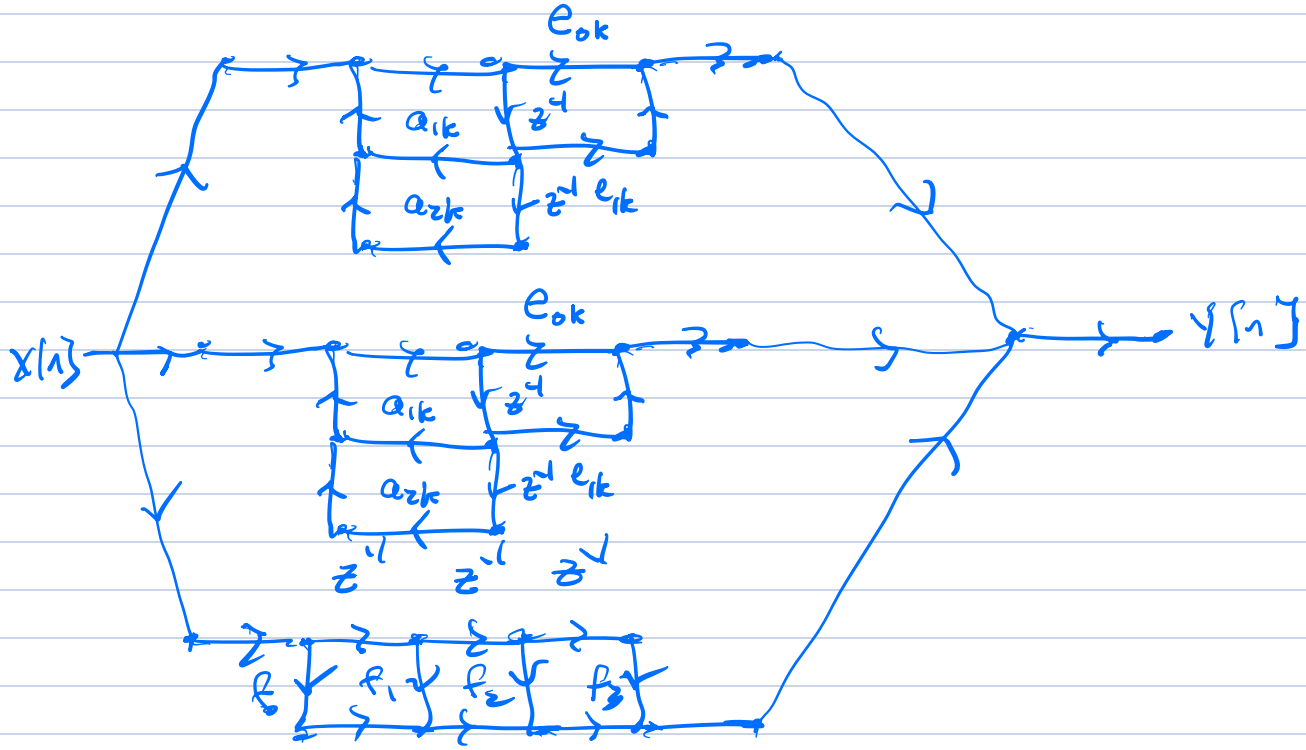
COMBINE INTO 2ND-ORDER SECTIONS

IF d_1, d_2 REAL $\quad \frac{A_1}{1 - d_1 z^{-1}} + \frac{A_2}{1 - d_2 z^{-1}} = \frac{(A_1 + A_2) - z^{-1}(A_1 d_2 + A_2 d_1)}{1 - (d_1 + d_2) z^{-1} + d_1 d_2 z^{-2}}$

IF d, d^* $\quad \frac{A}{1 - d z^{-1}} + \frac{A^*}{1 - d^* z^{-1}} = 2\mathrm{Re}[A] - z^{-1}(2\mathrm{Re}[A d^*])$





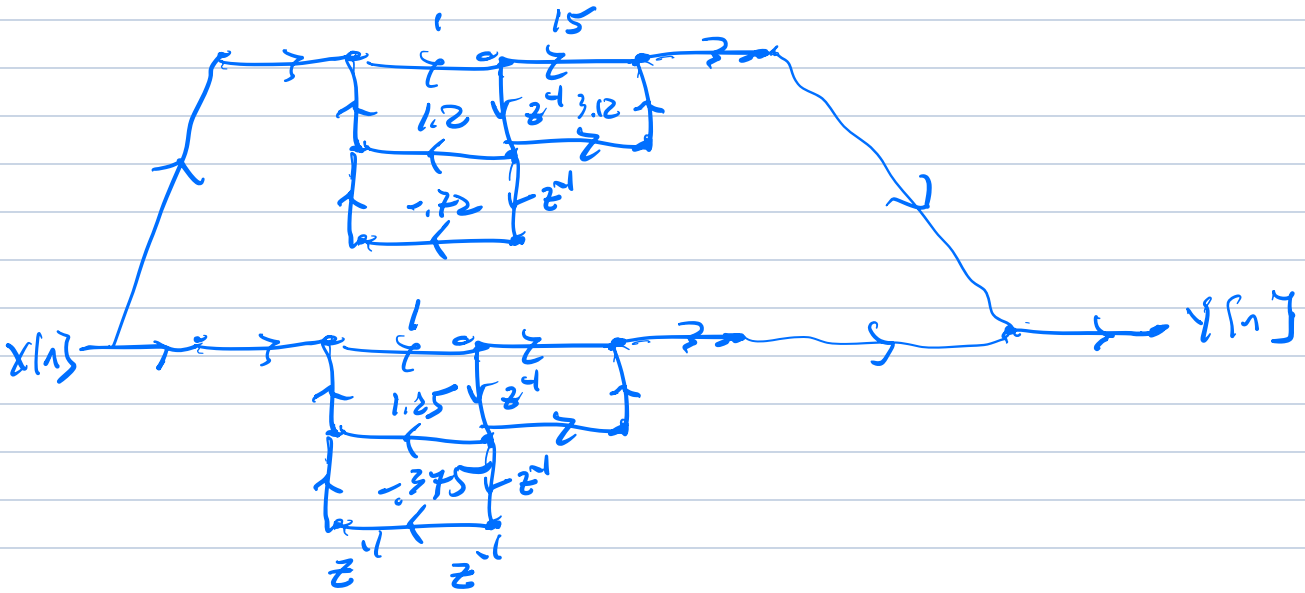


$2\,\mathrm{Re}[A] \cong 15$

$2\,\mathrm{Re}\{A\,d^{*}\} = 3.12$

$A_1 + A_2 \cong 16$

$A_1 d_2 + A_2 d_1 \varepsilon .305$



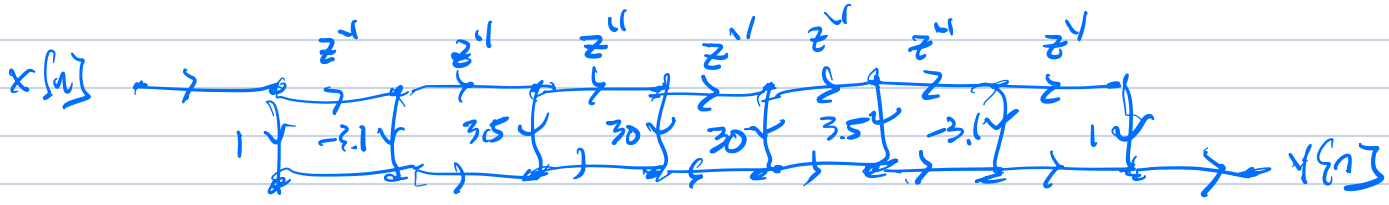
FIR FORMS — LINEAR PHASE

CONSIDER FIR FILTER WITH ZEROS AT

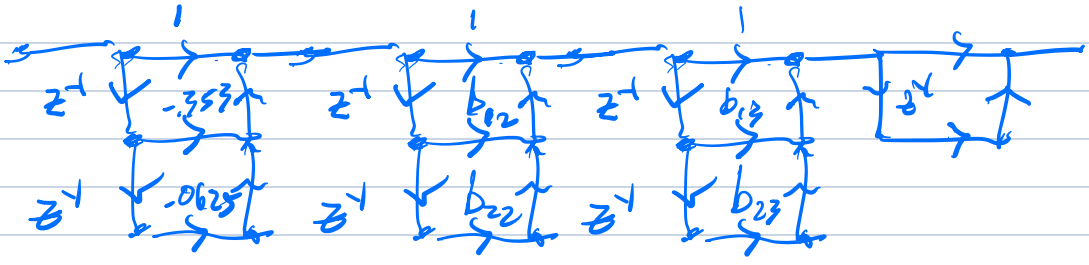
$z = \frac{1}{2\sqrt{2}(1+j)}, \frac{1}{2\sqrt{2}(1-j)}, 2\sqrt{2}(1+j), 2\sqrt{2}(1-j), -1, e^{j\cdot 0.8\pi}, e^{-j\cdot 0.8\pi}$

$$B(z) = 1 - 3.1z^{-1} + 3.52z^{-2} + 29.97z^{-3} + 29.97z^{-4} + 3.52z^{-5} - 3.1z^{-6} + z^{-7}$$

DIRECT FORM

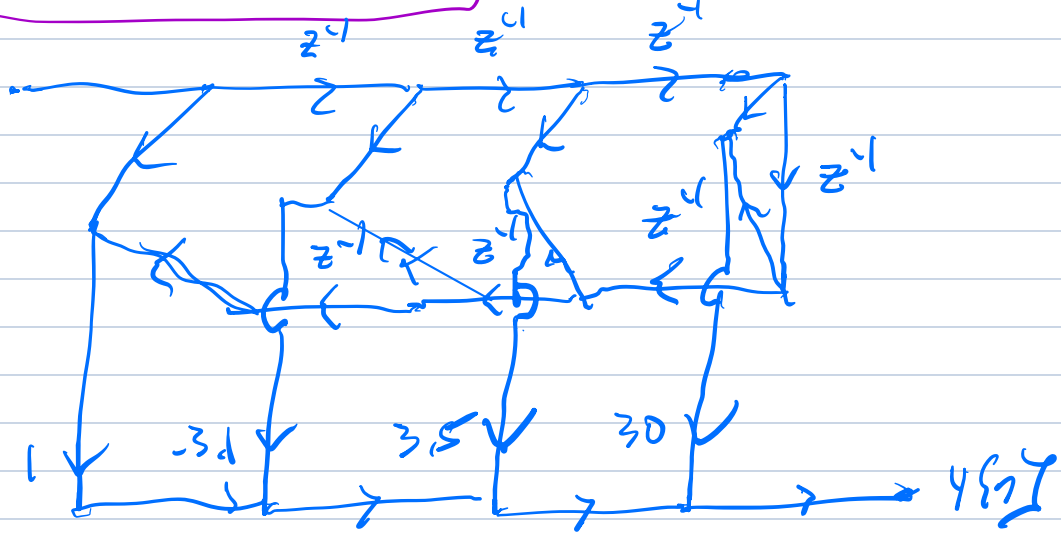


CASCADE FORM



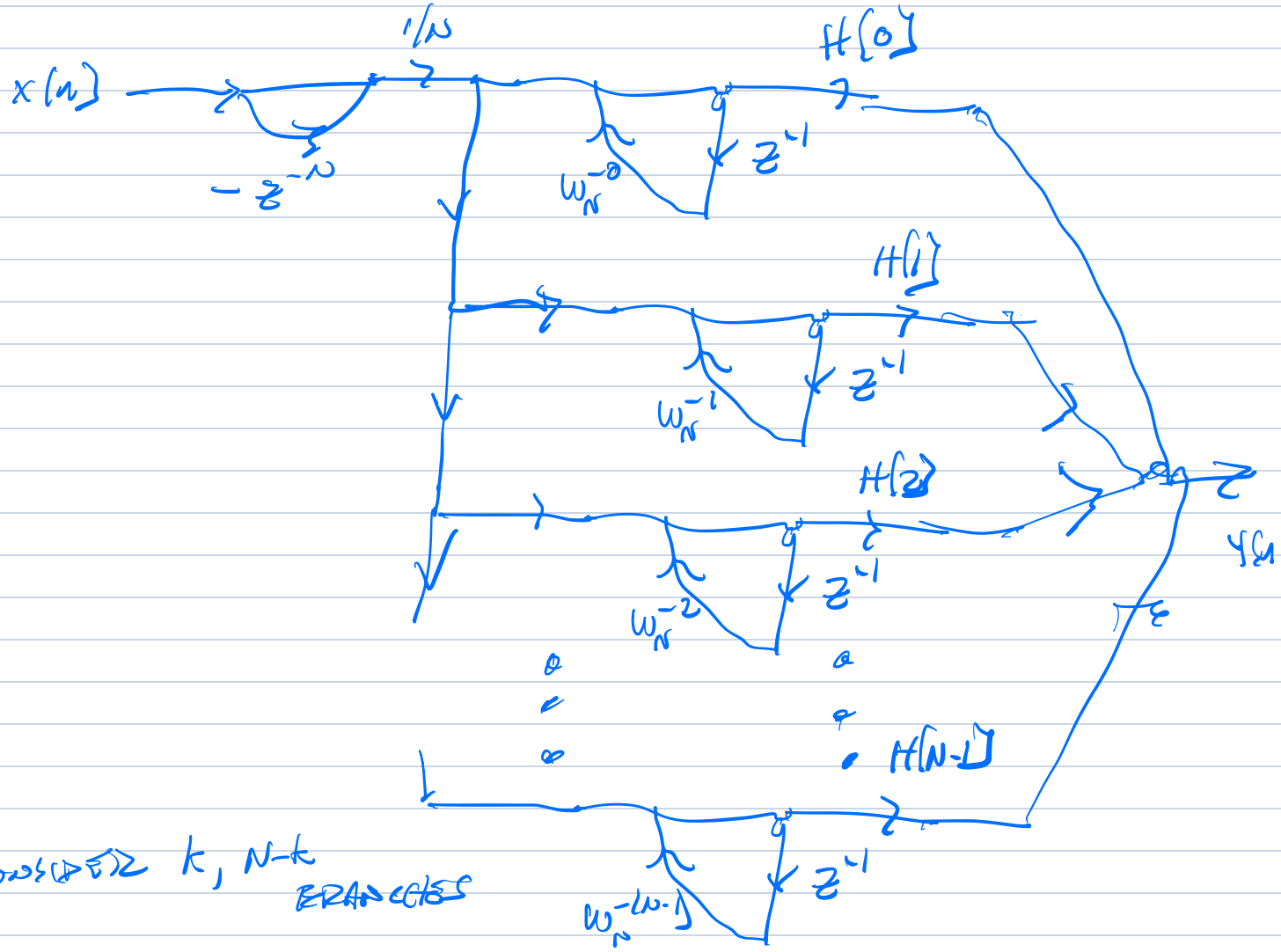
$$B(z) = 1 - 3.1z^{-1} + 3.52z^{-2} + 29.97z^{-3} + 29.97z^{-4} + 3.52z^{-5} - 3.1z^{-6} + z^{-7}$$

LINEAR PHASE FORM



FREQUENCY-SAMPLING FORM

$$H(z) = \frac{1-z^{-N}}{N} \sum_{K=0}^{N-1} \frac{H[k]}{1-W_N^{-k} z^{-1}}$$



Consider k, N-k branches

$$\frac{H[k]}{1-W_N^{-k} z^{-1}} + \frac{H[N-k]}{1-W_N^{-(N-k)} z^{-1}} \qquad H[N-k] = H^*[k]$$

$$W_N^{-(N-k)} = W_N^{-N} \cdot W_N^{k} = \left(W_N^{-k}\right)^*$$