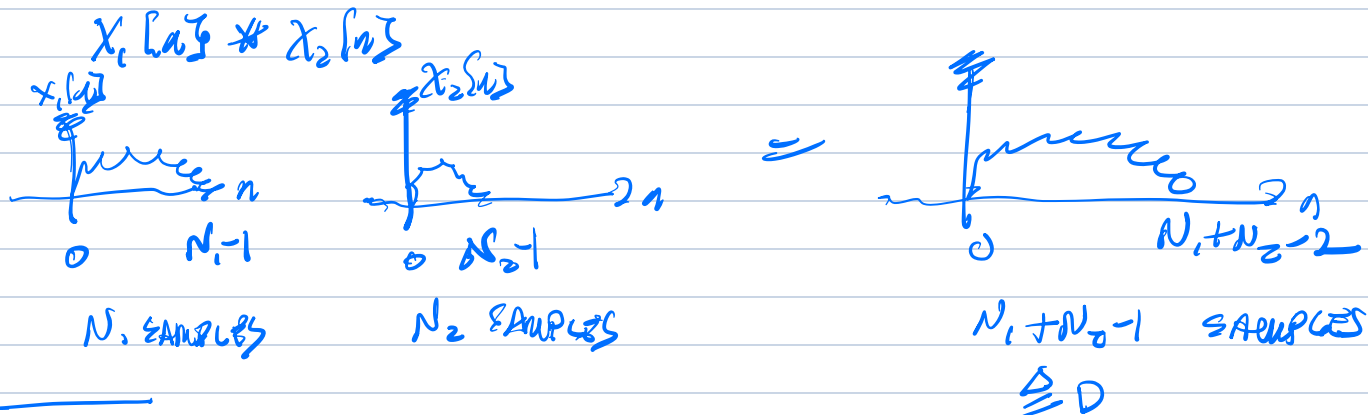


3/15/24

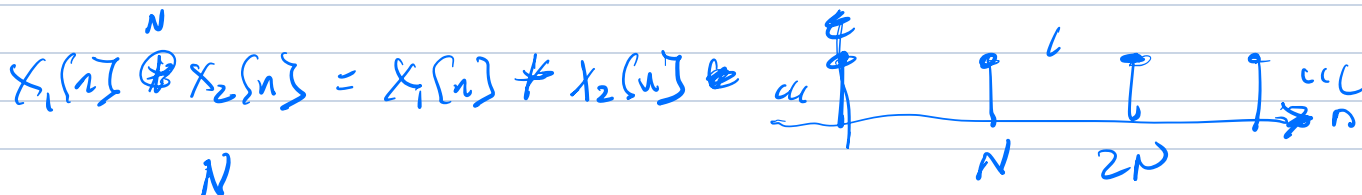
RECIPIROCAL 6A

LINEAR vs CIRCULAR CONVOLUTION FFT STRUCTURES

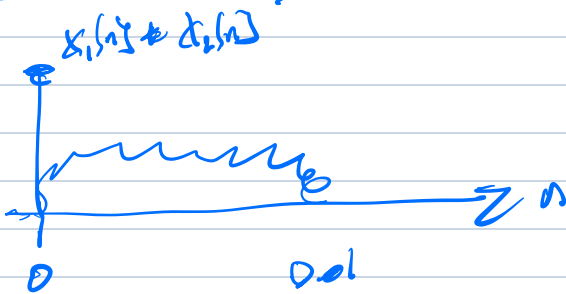
LINEAR vs CIRC CONV.



CIRC. CONV.

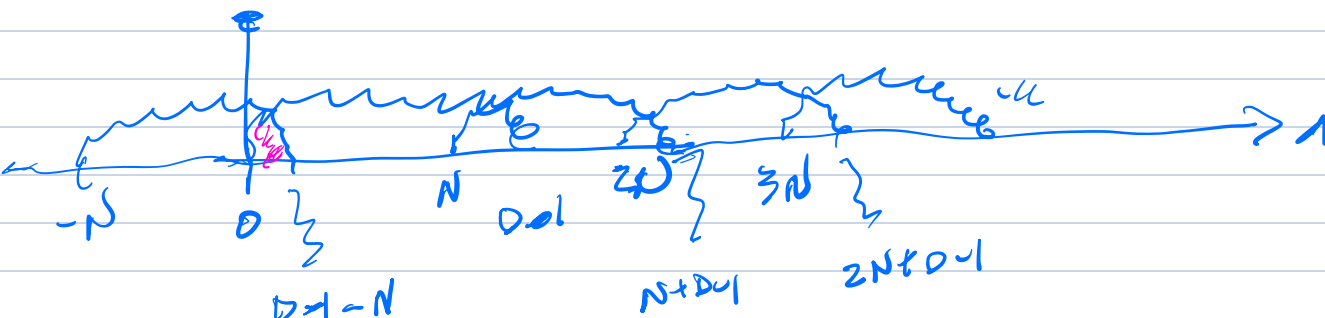


Let $N_1 + N_2 - 1 \leq D$

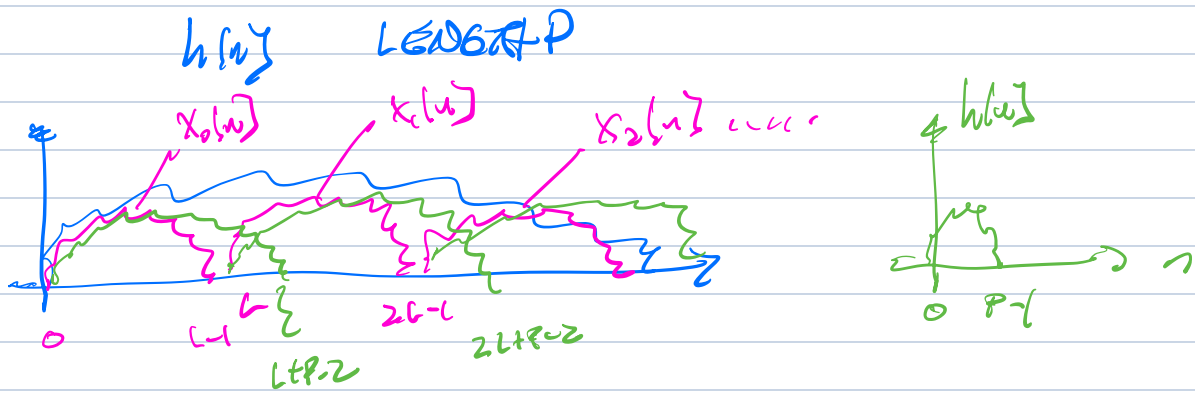


FOR $x_1[n] \circledast x_2[n]$

NEED $N \geq N_1 + N_2 - 1$



OLA $X[n]$ LENGTH



$$\text{let } X_i[n] = \begin{cases} X[n + iL], & 0 \leq n \leq L-1 \\ 0, & \text{ELSE} \end{cases}$$

$$\text{let } Y_i[n] = X_i[n] * h[n] \quad \text{DURATION } L+P-1$$

NON ZERO, $0 \leq n \leq L+P-2$

$$Y[n] = \sum_{i=0}^{\infty} Y_i[n - iL]$$

Ex. OLA DFT SIZE 512
 $h[n]$ of LENGTH 100 = P

$$L+P-1 = L + 100 - 1 = 512; \quad L = 512 - 99 = 413$$

i	INPUT		OUTPUT	
	BE6	END	B86	END
0	0	412	0	511
1	413	825	413	924
2	826	1238	826	1337

$$X[n] = \sum_{i=0}^{\infty} X_i[n]$$

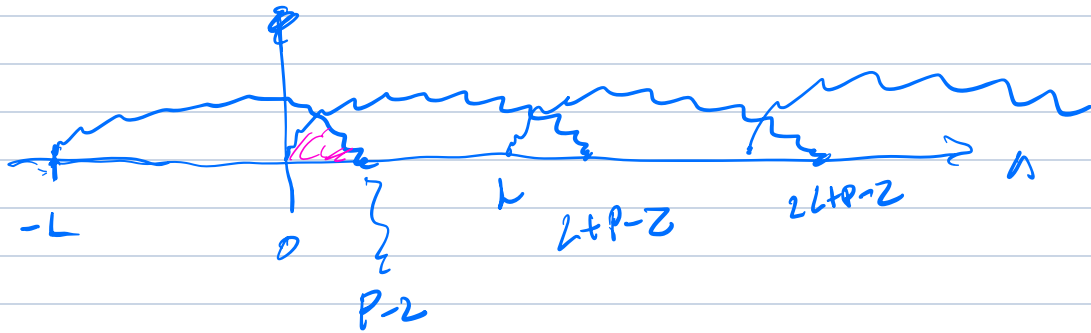
$$Y_i[n] = X_i[n] * h[n]$$

$$Y[n] = \sum_{i=0}^{\infty} Y_i[n]$$

OLS $h(n)$ LENGTH P

$x(n)$ LENGTH L

DFT SIZE LENGTH L



OLS DFT SIZE 512, $P=106$

P	INPUT		OUTPUT	
	BEG	END	BEG	END
0	0	511	99	511
1	413	924	512	924
2	826	1337	925	1337

COOLESY-TURKEY DECIIMATION IN TIME

$$X[k] = \sum_{n=0}^{N-1} x(n) w_N^{nk}$$

$$w_N = e^{-j2\pi/N}$$

$$X[k] = \sum_{\substack{n=0 \\ \text{EVEN}}}^{N-1} x(n) w_N^{nk} + \sum_{\substack{n=0 \\ \text{ODD}}}^{N-1} x(n) w_N^{nk}$$

$$n = 2r$$

$$n = 2r+1$$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] \omega_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] \omega_N^{(2r+1)k}$$

$$\omega_N^{2rk} = e^{j \frac{2\pi}{N} 2rk} = e^{-j \frac{2\pi}{N/2} rk} = \omega_{N/2}^{rk}$$

$$\omega_N^{(2r+1)k} = \omega_N^{2rk} \cdot \omega_N^k = \omega_{N/2}^{rk} \cdot \omega_N^k$$

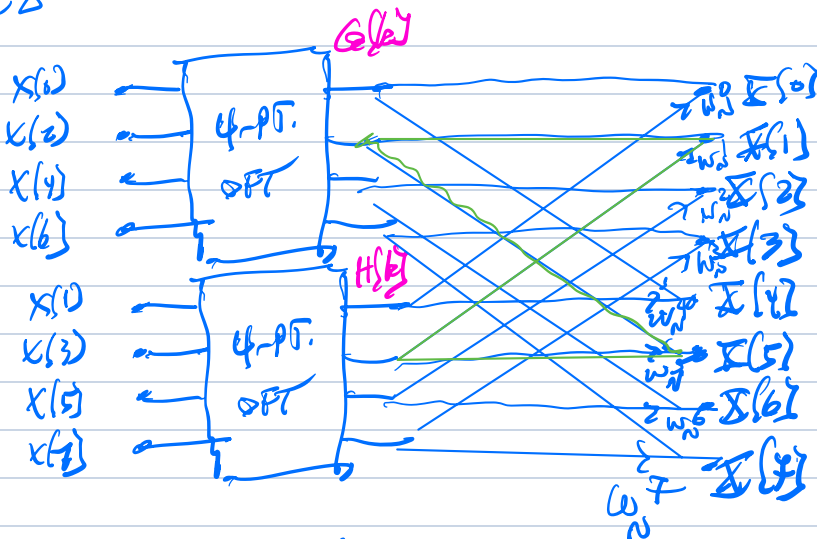
$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] \omega_{N/2}^{rk} + \omega_N^k \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] \omega_{N/2}^{rk}$$

$\frac{N}{2}$ -PT. DFT of EVEN INDICES of $x[n]$

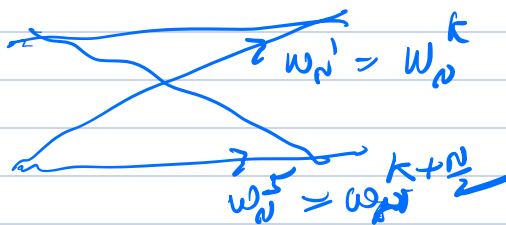
$\frac{N}{2}$ -PT. DFT of ODD INDICES of $x[n]$

$$X[k] = G[k] + \omega_N^k H[k]$$

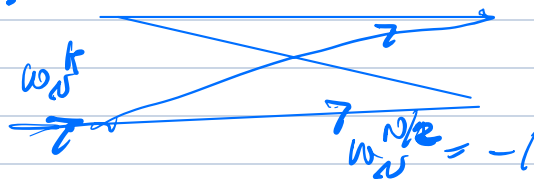
$N=8$



$N \log_2 N$ MULTS



$\frac{N}{2} \log_2 N$ MULTS.



$x_1(n), x_2(n)$
LENGTH N

SIMPLE CONVOLUTION $\approx N^2$ MULTS

DFT COMPUTATION = N^2 MULTS

CONVOLUTION USING DFTS = $3N^2 + N$

↑
DFT/IDFT

CONVOLUTION USING FFTS

$$= 3 \left(\frac{N}{2} \log_2(N) \right) + N$$

OZALOUS WITH B SECTIONS

$$2B \left(\frac{N}{2} \log_2(N) \right) + \left(\frac{N}{2} \log_2(N) \right)$$

$H(k)$

TRN

NO.3 RADIX-2 DFTS ...

$$N = p_1 p_2 p_3 \dots p_m$$

$$q_1 = \frac{N}{p_1}, q_2 = \frac{N}{p_1 p_2} \text{ ETC.}$$

$$X[k] = \sum_{r=0}^{q_1-1} x[p_1 r] \omega_N^{p_1 r k} + \sum_{r=0}^{q_1-1} x[p_1 r + 1] \omega_N^{(p_1 r + 1) k} + \dots + \sum_{r=0}^{q_1-1} x[p_1 r + (q_1 - 1)] \omega_N^{(p_1 r + (q_1 - 1)) k}$$

$$X[k] = \sum_{r=0}^{p_1-1} \sum_{s=0}^{q_1-1} x[p_1 r + s] \omega_N^{p_1 r k + s k}$$

$$\omega_N^{(p_1 r + s) k} = \omega_N^{p_1 r k} \omega_N^{s k}$$

$$X[k] = \sum_{r=0}^{p_1-1} \omega_N^{r k} \sum_{s=0}^{q_1-1} x[p_1 r + s] \omega_N^{s k} = \omega_N^{r k} \omega_N^{s k} = \omega_N^{r k} \omega_N^{s k}$$

q_1 -PT. DFT of $x[p_1 r + s]$

