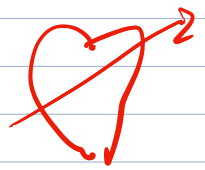
2/14/25

RECITATION 5B



DISCRETE-TIME DECIMATION + INTERPOLATION

DECIMATION BY M

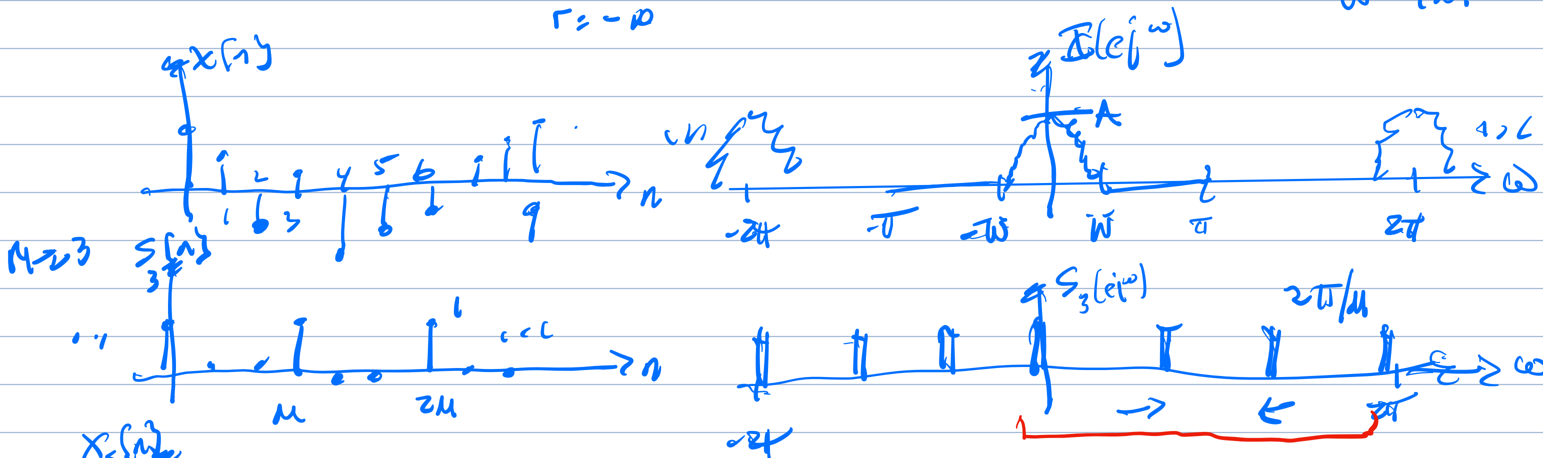
$$x[n] \rightarrow \boxed{\downarrow M} \rightarrow x_d[m] = x[nM]$$

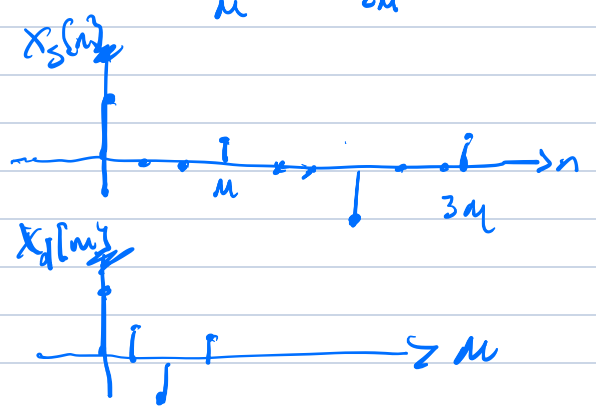
$$x[n] \rightarrow \otimes \xrightarrow{x_s[n]} \boxed{\text{COMPRESS IN TIME}} \rightarrow x_d[m] = x[nM]$$

$$s_M[n] = \sum_{r=-\infty}^{\infty} \delta[n - Mr]$$

$X(e^{j\omega}) = 0,$

$W < |\omega| \leq \pi$





$$S(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{M}\delta\left(\omega - \frac{2\pi k}{M}\right) = \sum_{r=-\infty}^{\infty} \sum_{\ell=0}^{M-1} \cancel{\frac{2\pi}{M}\delta\left(\omega - 2\pi\right.}$$

$$\frac{2\pi}{M}\delta\left(\omega - \frac{2\pi \ell}{M} - 2\pi r\right)$$

$$s[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} S(e^{j\omega}) e^{j\omega n}\, d\omega$$

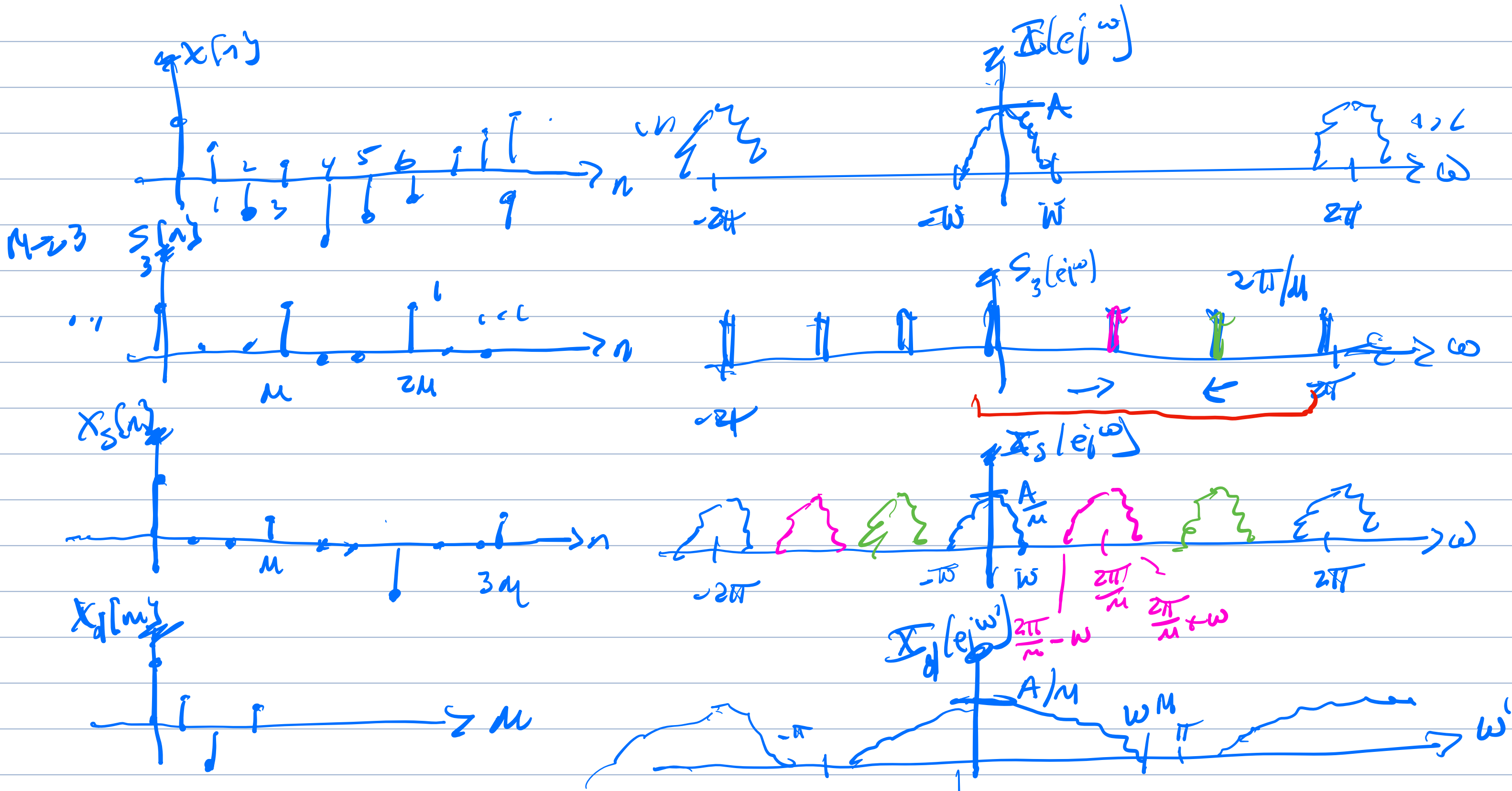
$$= \frac{1}{2\pi}\int_{0-\epsilon}^{2\pi-\epsilon} \frac{2\pi}{M}\sum_{\ell=0}^{M-1} \delta\left(\omega - \frac{2\pi \ell}{M}\right) e^{j\omega n}\, d\omega$$

$$= \frac{1}{\cancel{2\pi}} \sum_{\ell=0}^{M-1} \frac{\cancel{2\pi}}{M} \int_{0-\epsilon}^{2\pi-\epsilon} \delta\left(\omega - \frac{2\pi \ell}{M}\right) e^{j\omega n}\, d\omega$$

$$s[n] = \sum_{\ell=0}^{M-1} \frac{1}{M} e^{j\frac{2\pi \ell n}{M}} = 1, \quad n = 0, M, 2M$$

For $n \neq rM$, $s[n] = \sum_{\ell=0}^{M-1} \frac{1}{M}\left(e^{j\frac{2\pi n}{M}}\right)^{\ell}$

$$= \frac{1}{M}\, \frac{1 - e^{j\frac{2\pi n}{M}M}}{1 - e^{j2\pi n/M}} = \begin{cases} 1, & n = rM \\ 0, & \text{ELSE} \end{cases}$$



$x_s[n] = x[n]\, s[n] \Leftrightarrow \frac{1}{2\pi} X(e^{j\omega}) \circledast S(e^{j\omega})$

$X_s(e^{j\omega}) = \frac{1}{M}\sum_{l=0}^{N-1} X\left(e^{j\left(\omega - \frac{2\pi l}{M}\right)}\right)$

TO PREVENT ALIASING REQUIRE

$W < \frac{2\pi}{M} - W$; $2W < \frac{2\pi}{M}$

$\boxed{W < \frac{\pi}{M}}$

$X_d(e^{j\omega'}) = \sum_{m=-\infty}^{\infty} x_d[m]\, e^{-j\omega' m} = \sum_{m=-\infty}^{\infty} x_s[mM]\, e^{-j\omega' m}$

$m = rM$

LET $s = mM$, $m = \frac{s}{M}$

$= \sum_{s=-\infty}^{\infty} x_s[s]\, e^{-j\omega' \frac{s}{M}} = \sum_{s=-\infty}^{\infty} x_s[s]\, e^{-j\frac{\omega'}{M} s} = X_s\left(e^{j\frac{\omega'}{M}}\right)$

$X_d(e^{j\omega'}) = \sum_{l=0}^{M-1} \frac{1}{M} X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi l}{M}\right)}\right)$

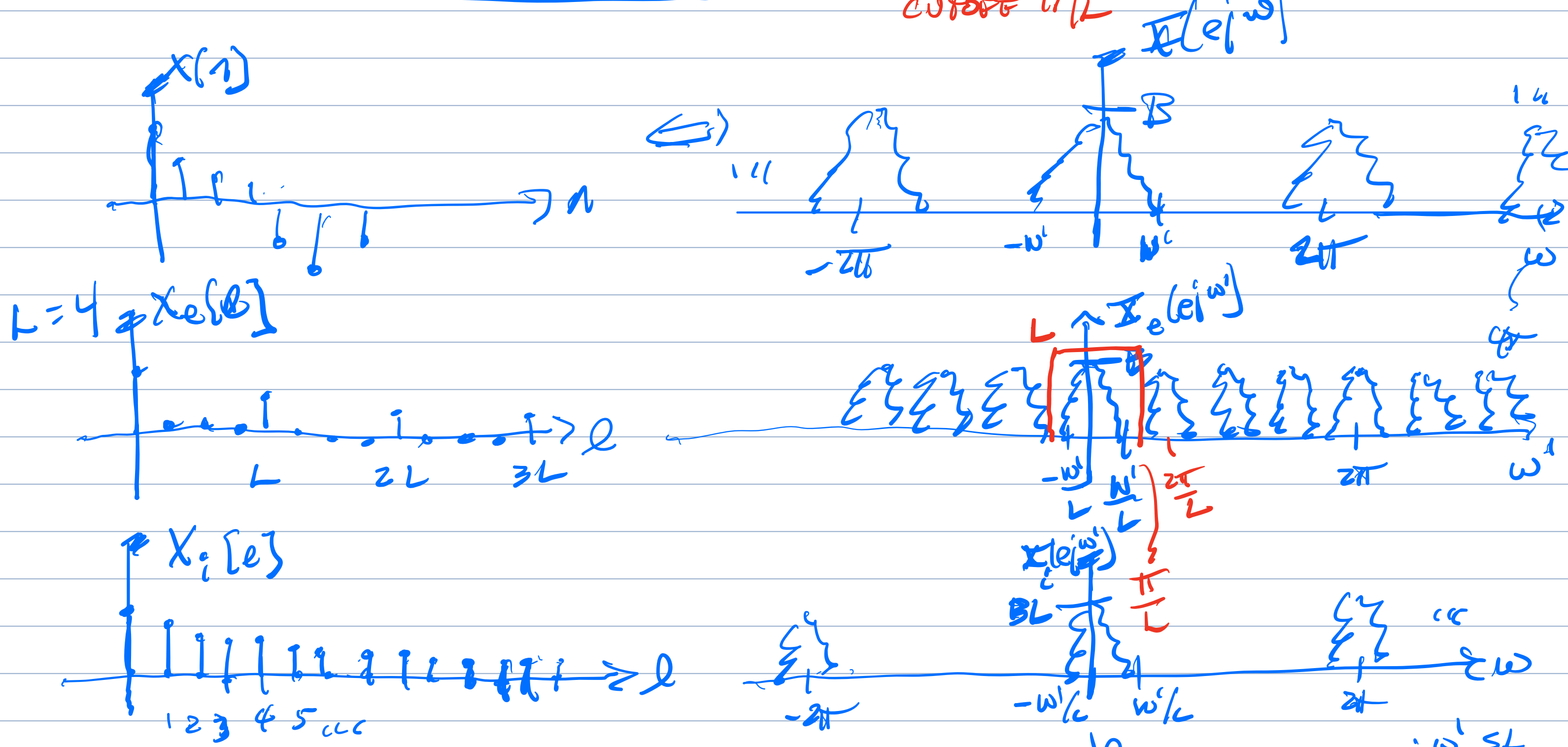
DT INTERPOLATION

$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow x_i[\ell] = x\left[\frac{\ell}{L}\right]$, $\frac{\ell}{L}$ INTEGER

SOMETHING ELSE OTHERWISE

$x[n] \longrightarrow$ $\boxed{\text{INSERT } L-1 \text{ ZERO SAMPLES BET. INPUT SAMPLES}}$ $\xrightarrow{x_e[\ell]}$ $\boxed{\text{LPF}}$ $\longrightarrow x_i[\ell]$

GAIN L
CUTOFF π/L



$$X_e(e^{j\omega'}) = \sum_{n=-\infty}^{\infty} x_e[\ell] e^{-j\omega'\ell} = \sum_{\substack{n=-\infty \\ n=rL}}^{\infty} x\left[\frac{\ell}{L}\right] e^{-j\omega'\ell} = \sum_{s=-\infty}^{\infty} x[s] e^{-j\omega' sL}$$

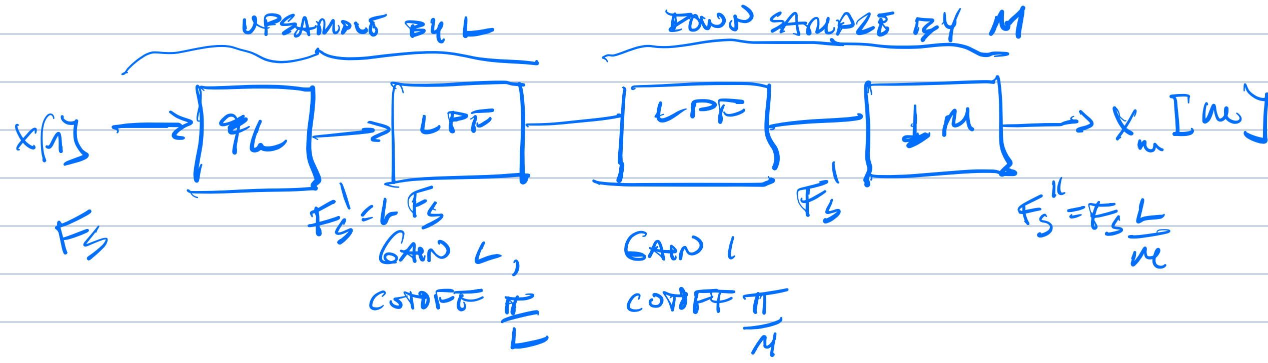
let $s = \frac{\ell}{L}$

$\ell = sL$

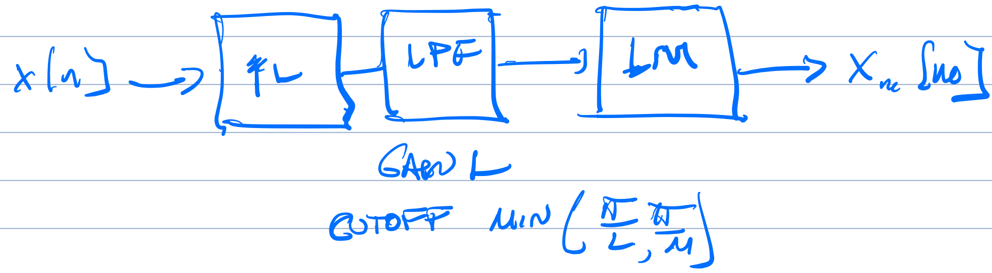
$$X_e(e^{j\omega'}) = \sum_{s=-\infty}^{\infty} x[s] e^{-j\omega' L s} = X(e^{j\omega' L})$$

$$X_i(e^{j\omega'}) = \begin{cases} X(e^{j\omega' L}), & |\omega'| \le \frac{\pi}{L} \\ 0, & \frac{\pi}{L} < |\omega'| \le \pi \end{cases}$$

CHANGE IN SAMPLING RATE BY L/M



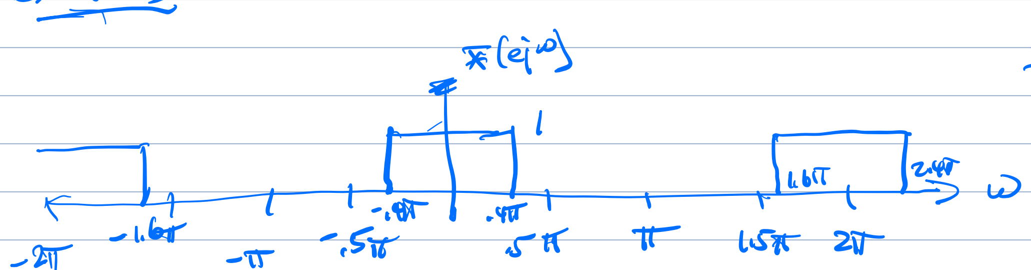
SIMPLER REPRESENTATION



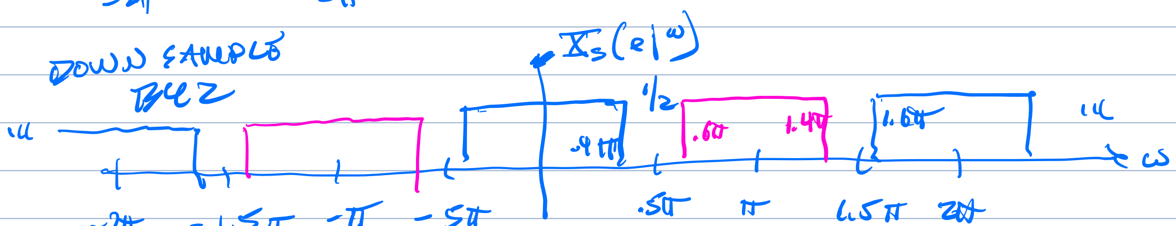
EXAMPLES

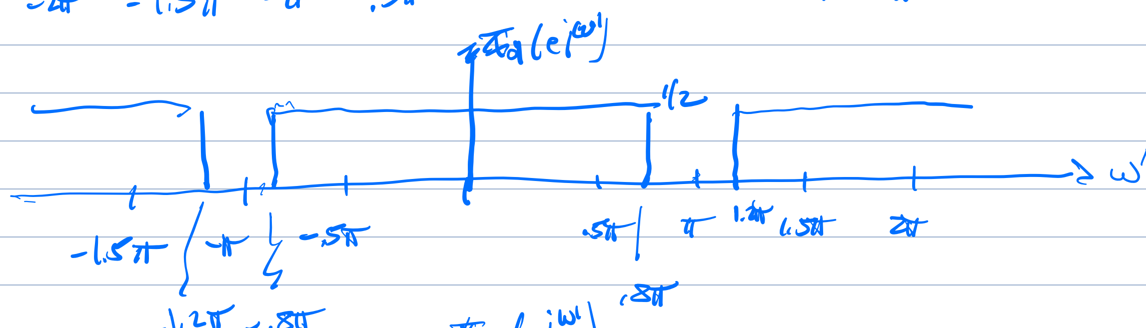
$X(e^{j\omega}) = 1, \; |\omega| \le .4\pi$

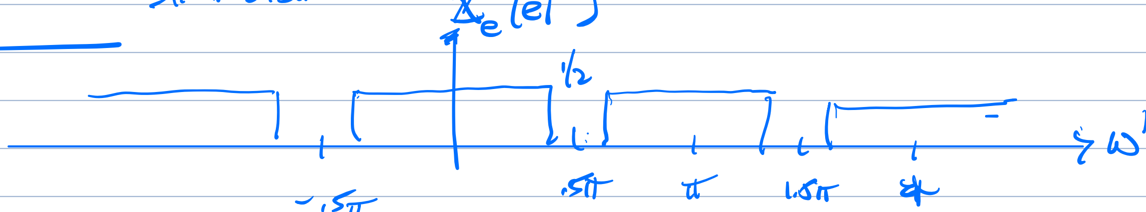
$0, \; .4\pi \le |\omega| \le \pi$

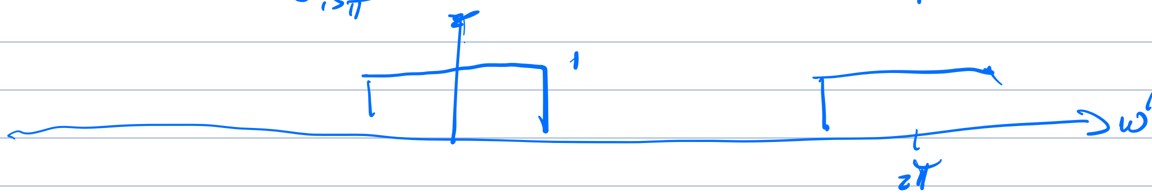


DOWN SAMPLE BY 2

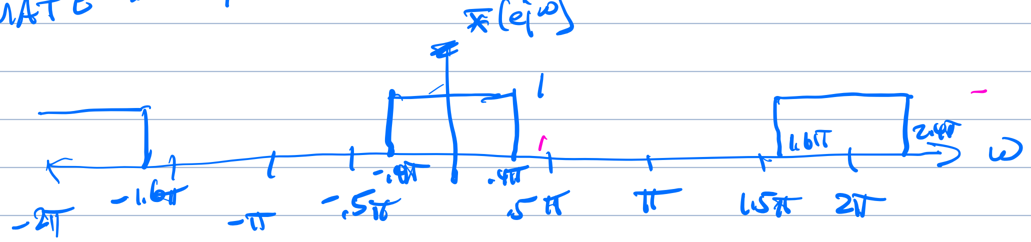


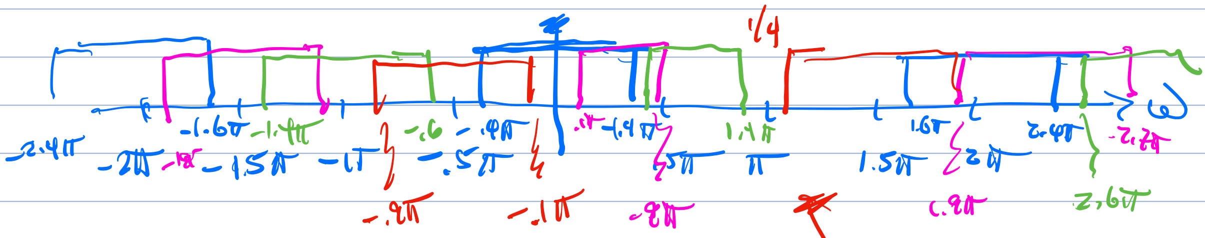






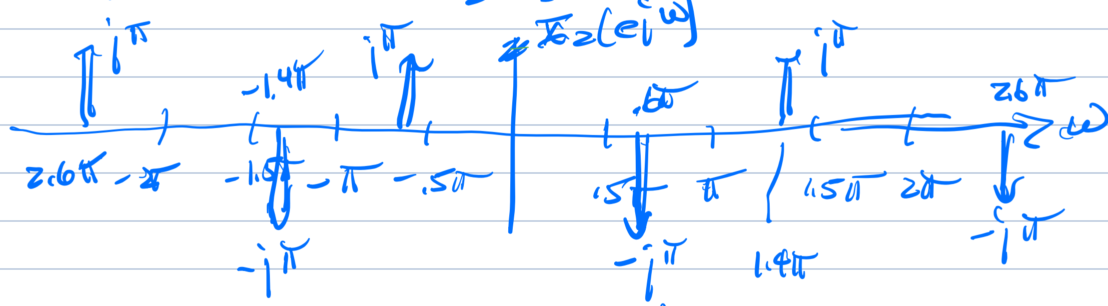
DECIMATE BY 4

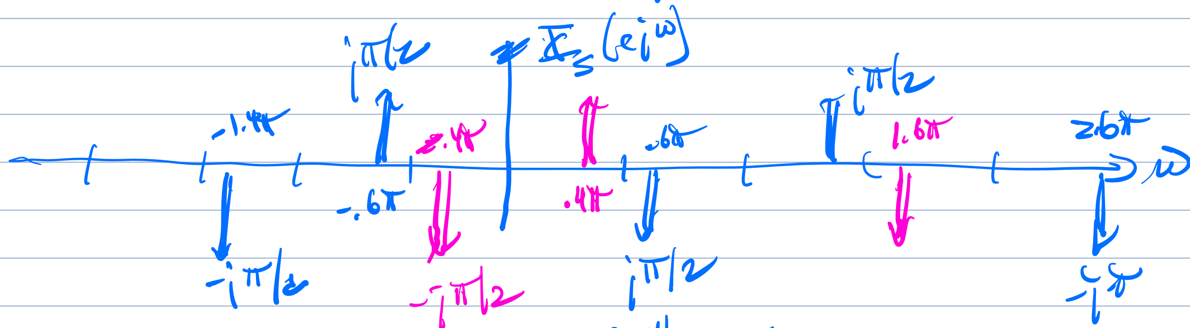


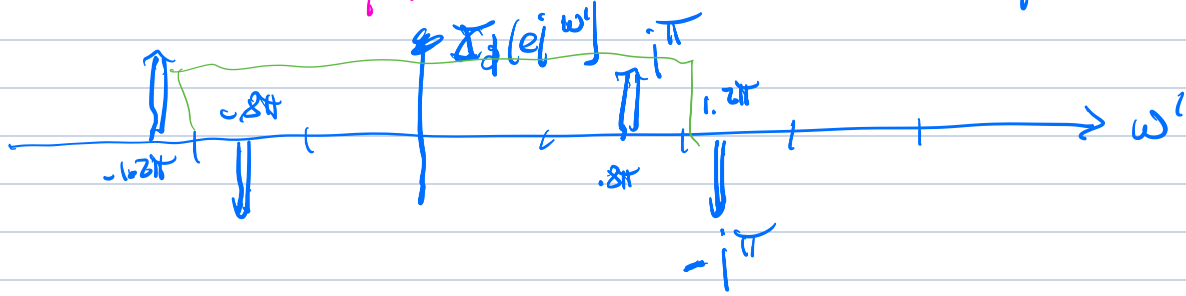


(PROBABLY WRONG)

$x_2[n] = \sin(.6\pi n)$, $M = 2$







$x_d[m] = -\sin(.8\pi m)$