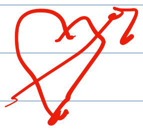
2/14/25

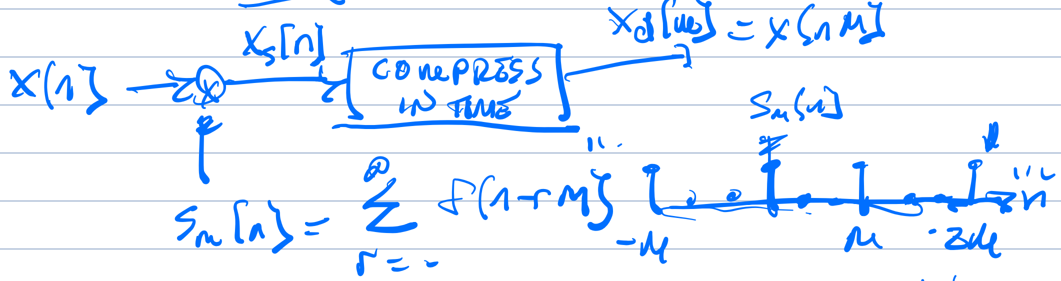


RECITATION 5A

CHANGE OF SAMPLING RATE IN DT

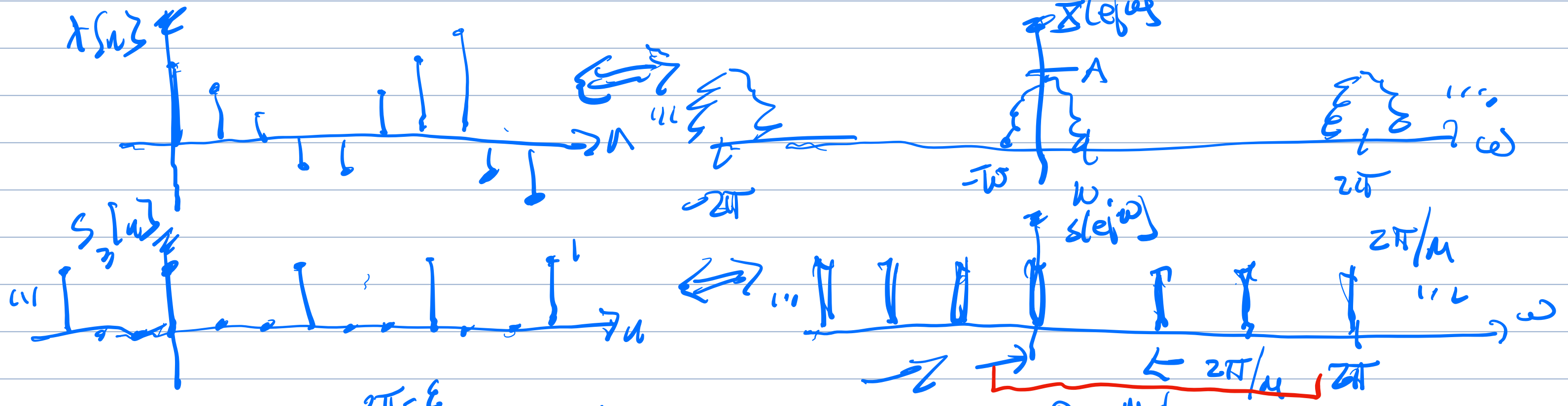
DT DECIMATION DOWNSAMPLING

$$x[n] \rightarrow \boxed{\downarrow M} \rightarrow x_d[m] = x[nM]$$



let $M=3$

$$s_M[n] = \sum_{r=-\infty}^{\infty} \delta[n-rM]$$



$$s_3[n] = \frac{1}{2\pi}\int_{\cancel{-\pi}\;0-\epsilon}^{2\pi-\epsilon} S_3(e^{j\omega}) e^{j\omega n} d\omega$$

$$S(e^{j\omega}) = \sum_{r=-\infty}^{\infty}\sum_{l=0}^{M-1} \frac{2\pi}{M}\delta\left(\omega - \frac{2\pi l - 2\pi r}{M}\right)$$

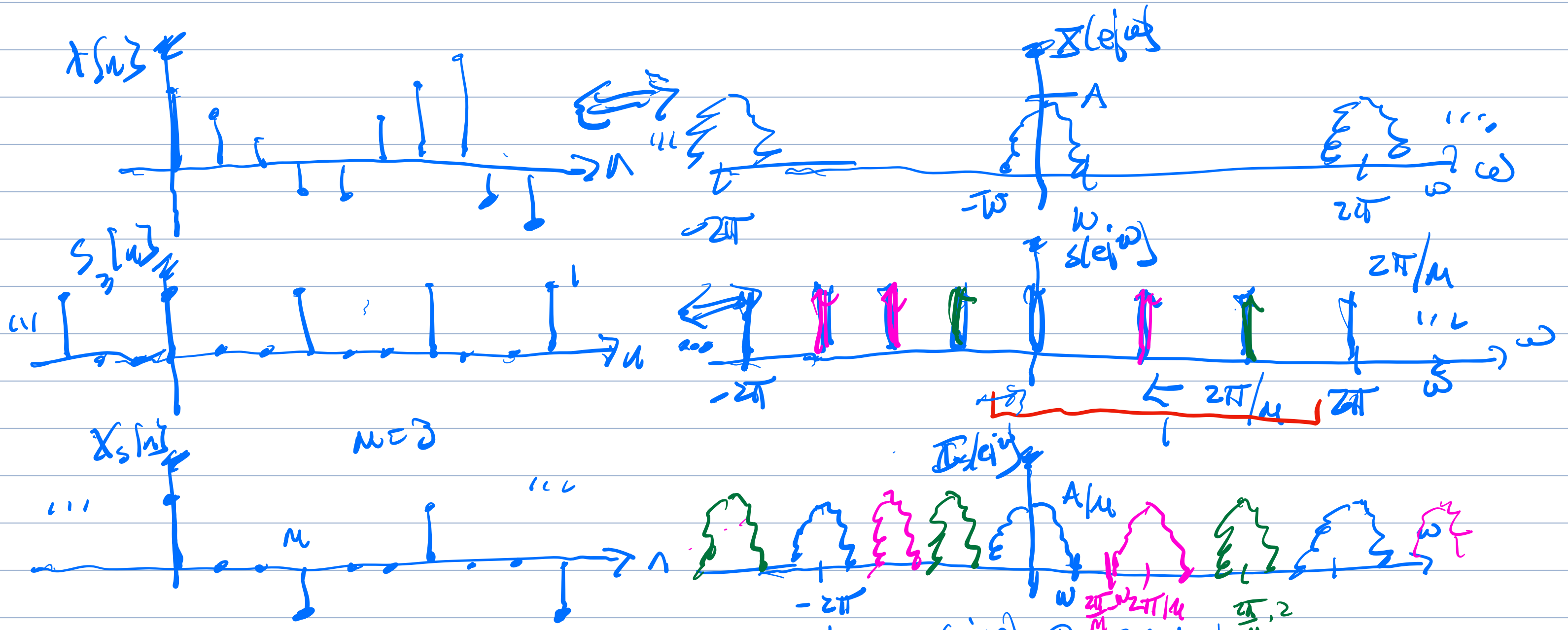
$$= \sum_{k=-\infty}^{\infty} \frac{2\pi}{M}\delta\left(\omega - \frac{2\pi k}{M}\right)$$

$$s_3[n] = \frac{1}{2\pi}\int_{0-\epsilon}^{2\pi-\epsilon}\sum_{l=0}^{M-1}\frac{2\pi}{M}\delta\left(\omega-\frac{2\pi l}{M}\right) e^{j\omega n} d\omega$$

$$= \frac{1}{\cancel{2\pi}}\sum_{l=0}^{M-1}\frac{\cancel{2\pi}}{M}\int_{0-\epsilon}^{2\pi-\epsilon}\delta\left(\omega-\frac{2\pi l}{M}\right)e^{j\omega n}d\omega$$

$$s_3[n] = \frac{1}{M}\sum_{l=0}^{M-1} e^{j\frac{2\pi l}{M}n} = \frac{1}{M}\sum_{l=0}^{M-1}\left(e^{j\frac{2\pi n}{M}}\right)^l$$

$$= \frac{1}{M}\,\frac{1-e^{j\frac{2\pi n}{M}\cdot M}}{1-e^{j\frac{2\pi n}{M}}} = 0,\; n \neq rM$$



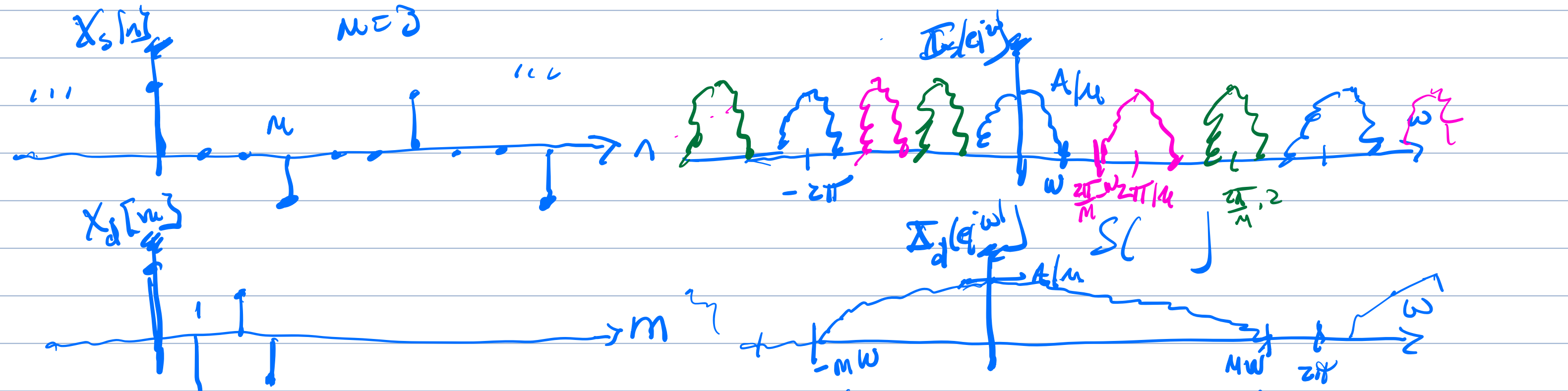
$$x_s[n] = x[n]\cdot s[n] \Leftrightarrow \frac{1}{2\pi} X(e^{j\omega}) \circledast S(e^{j\omega})$$

$$X_s(e^{j\omega}) = \sum_{\ell=0}^{M-1} \frac{1}{M} X\left(e^{j(\omega - \frac{2\pi \ell}{M})}\right)$$

TO AVOID ALIASING NEED

$$W < \frac{2\pi}{M} - W ; \quad 2W < \frac{2\pi}{M}$$

$$\boxed{W < \frac{\pi}{M}}$$



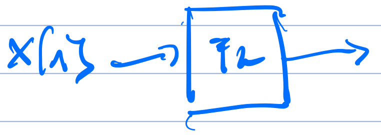
$$X_d(e^{j\omega'}) = \sum_{m=-\infty}^{\infty} x_d[m]\, e^{-j\omega' m} = \sum_{m=-\infty}^{\infty} x[mM]\, e^{-j\omega' m}$$

let $\ell = mM$, $m = \frac{\ell}{M}$ $$= \sum_{\ell=-\infty}^{\infty} x[\ell]\, e^{-j\omega' \frac{\ell}{M}} = \sum_{\ell=-\infty}^{\infty} x[\ell]\, e^{-j\frac{\omega'}{M}\ell}$$

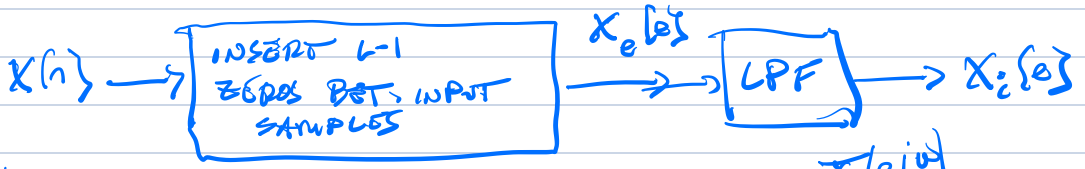
$$X_d(e^{j\omega'}) = X\left(e^{j\frac{\omega'}{M}}\right)$$

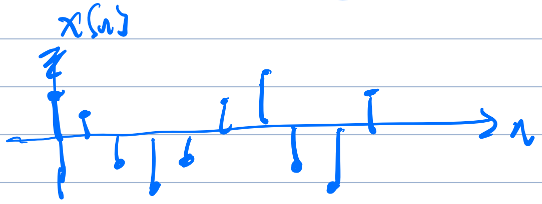
$$X_d(e^{j\omega'}) = \sum_{\ell=0}^{M-1} \frac{1}{M} X\left(e^{j\left(\frac{\omega'}{M} - \frac{2\pi}{\ell}\right)}\right)$$

DISCRETE-TIME INTERPOLATION

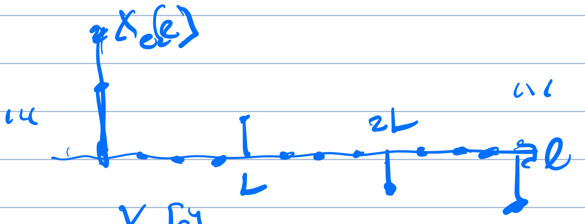


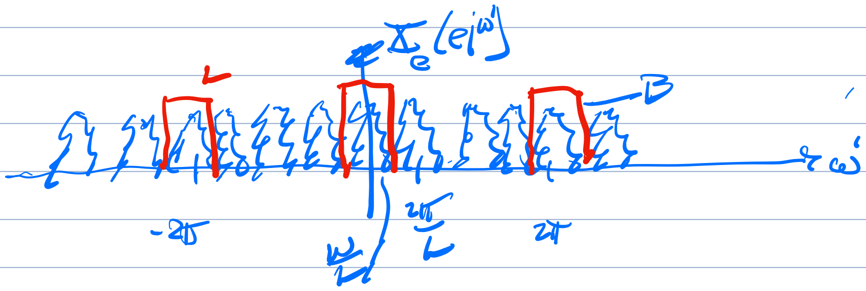
$x_i[\ell] = x\left[\frac{\ell}{L}\right], \quad \frac{\ell}{L}$ INTEGER

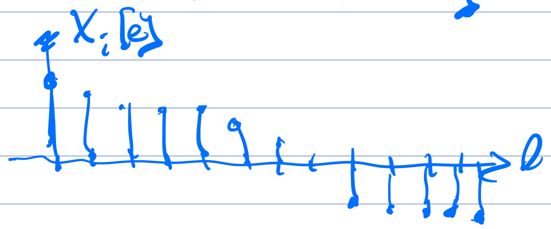




L=4







$$\text{LPF} \quad H(e^{j\omega'}) = \begin{cases} L, & |\omega'| < \frac{\pi}{M} \\ 0, & \frac{\pi}{L} < |\omega'| \le \pi \end{cases}$$

$$x_e[\ell] = x[\ell/L], \quad \ell = rL$$
$$0, \text{ OTHERWISE}$$

$$X_e(e^{j\omega'}) = \sum_{\ell=-\infty}^{\infty} x_e[\ell] e^{-j\omega'\ell} = \sum_{\substack{\ell=-\infty \\ \ell=rL}}^{\infty} x\left[\frac{\ell}{L}\right] e^{-j\omega'\ell} = \sum_{s=-\infty}^{\infty} x[s] e^{-j\omega' sL}$$

Let $s = \frac{\ell}{L}$, $\ell = sL$

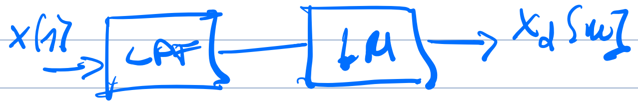
$$= \sum_{s=-\infty}^{\infty} x[s] e^{-j(\omega' L)s} = X(e^{jL\omega})$$

$$X_i(e^{j\omega'}) = L\, X(e^{j\omega' L}), \quad |\omega'| \le \frac{\pi}{L}$$

CHANGE of SAMPLING RATE

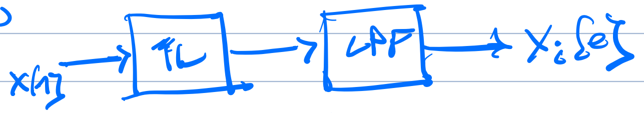
IN PRACTICE

DECIMATION BY M

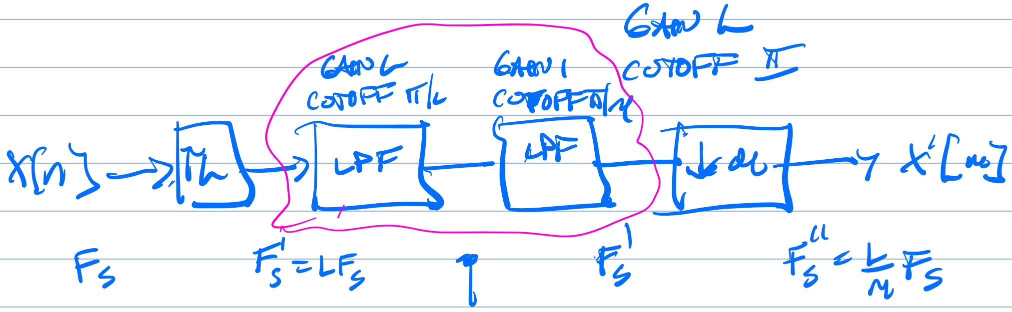


GAIN 1
CUTOFF $\frac{\pi}{M}$

INTERPOLATION BY L

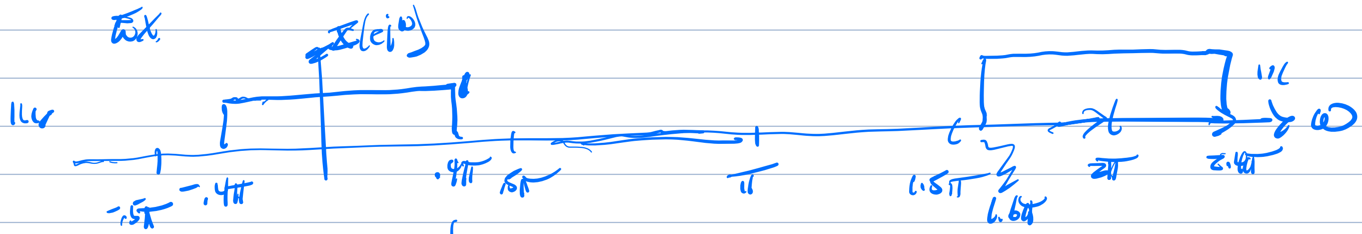


GAIN L
CUTOFF π

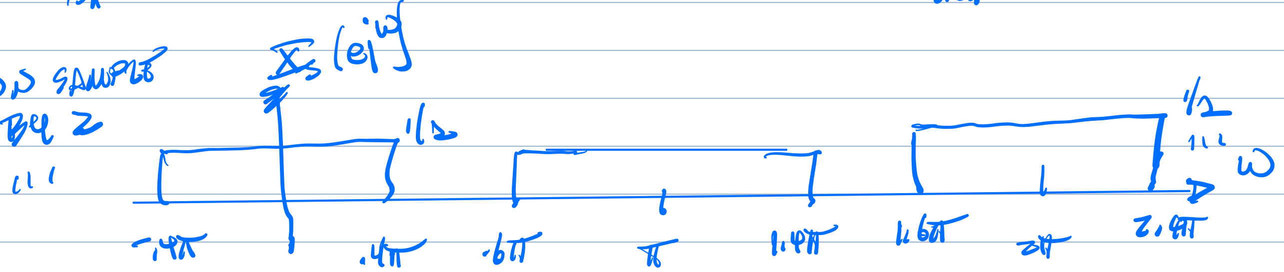


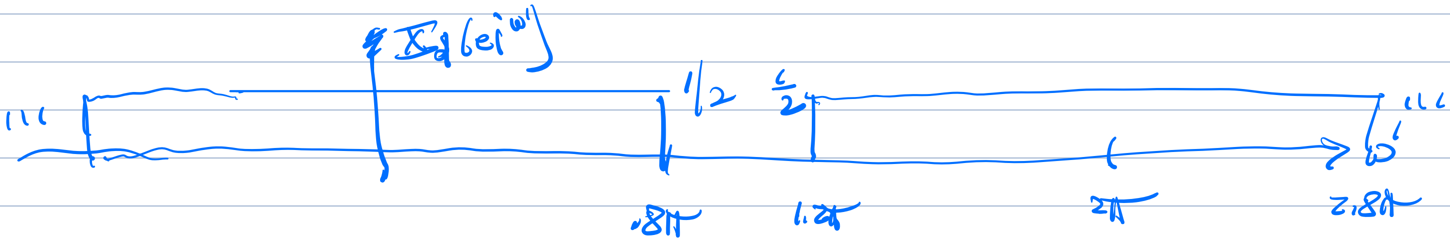
GAIN L
CUTOFF MIN $\left(\frac{\pi}{M}, \frac{\pi}{L}\right)$

EX.

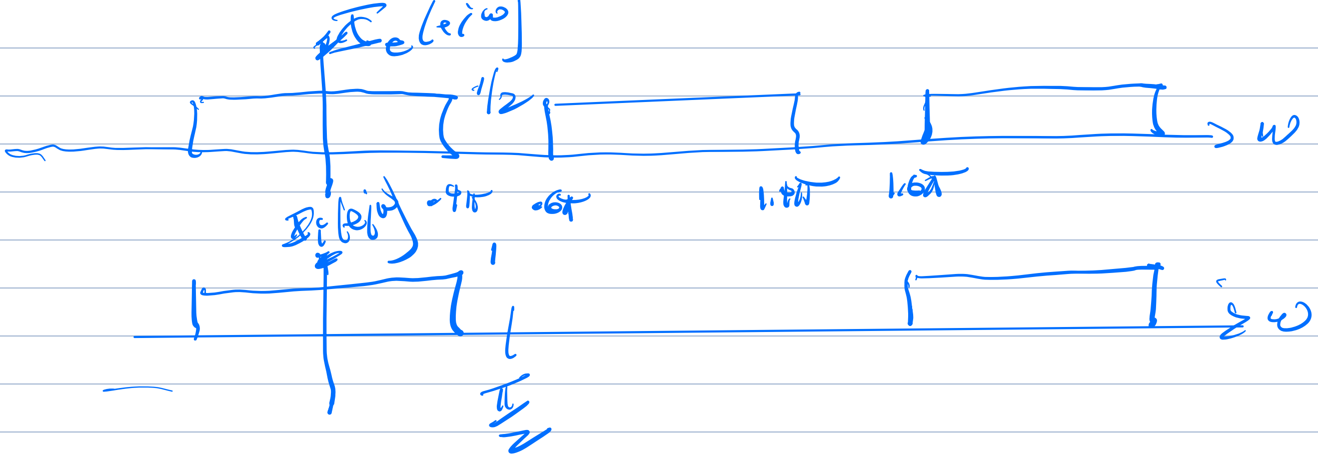


DOWN SAMPLE BY 2



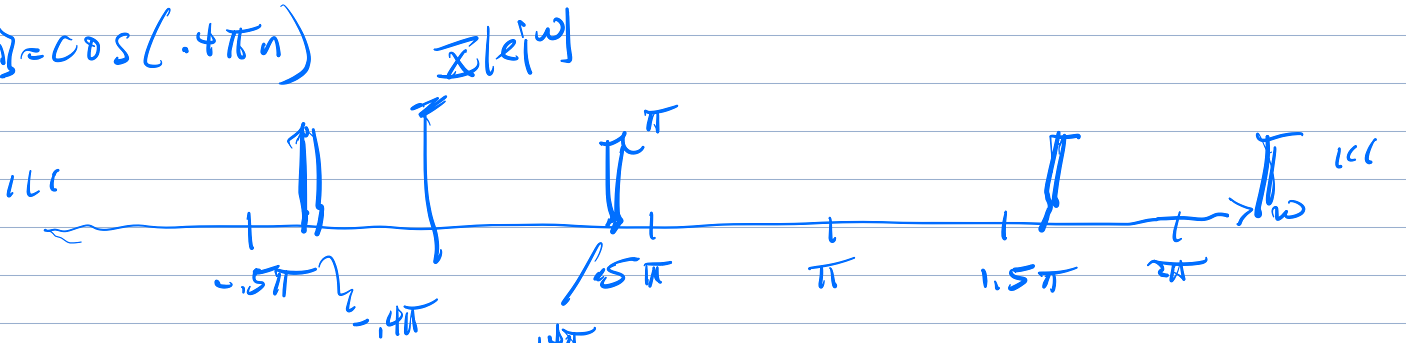


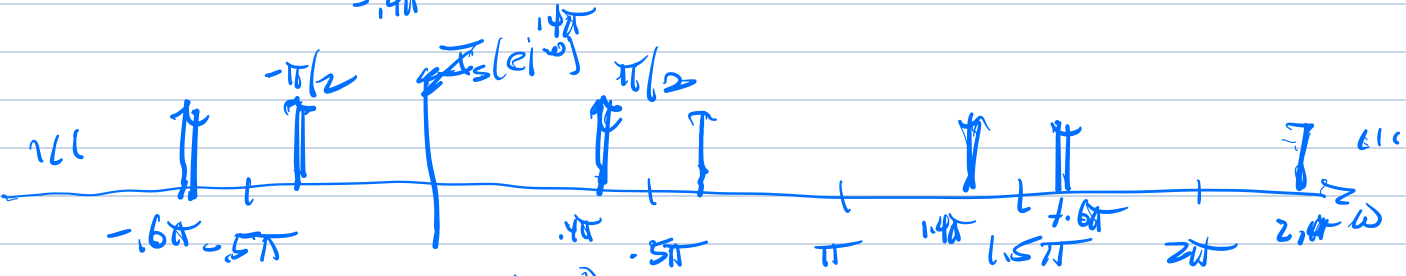
INTERPOLATE BY 2...

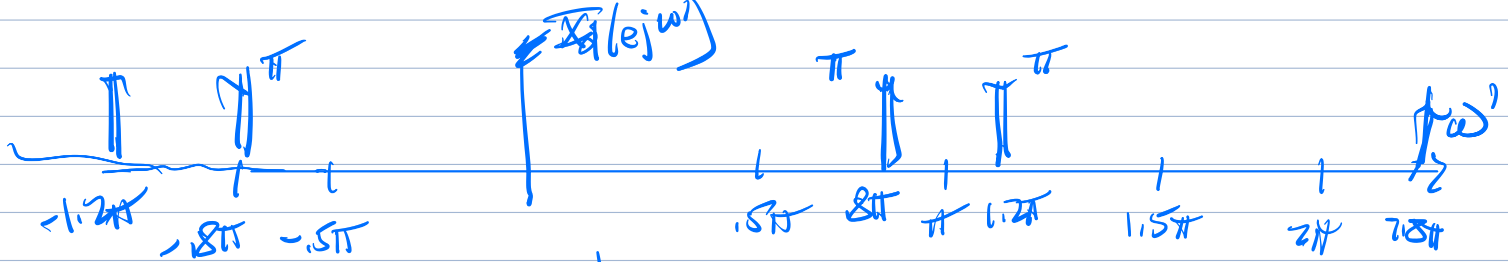


$x_2[n]=\cos(.4\pi n)$

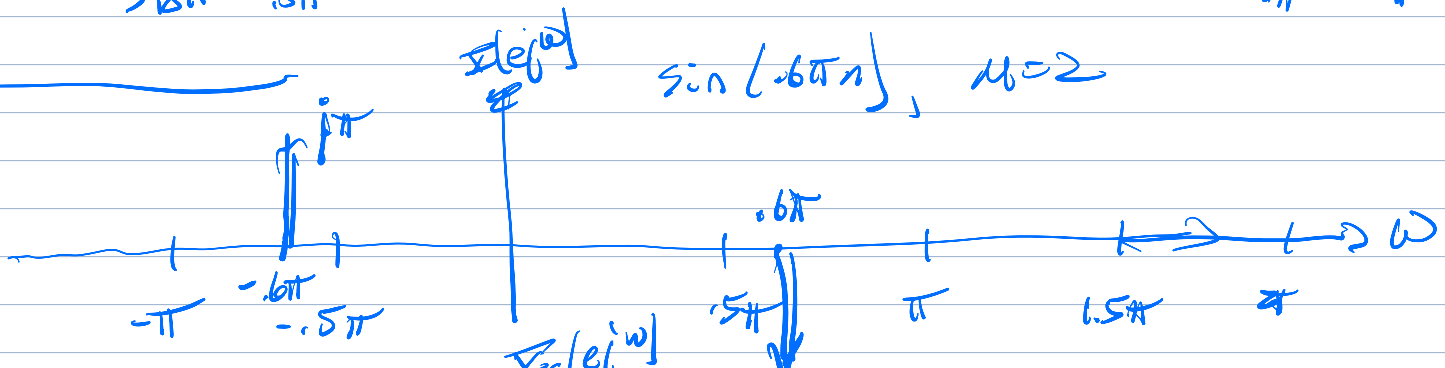
M=2







$\sin(.6\pi n)$, M=2



M=2