2/14/24

Recitation 4B

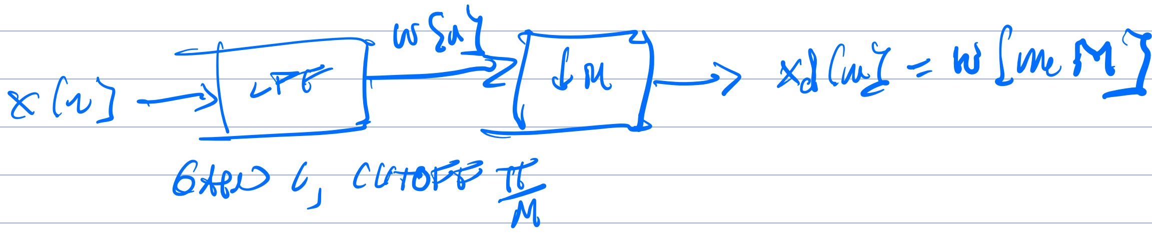
Special LSI Systems, Sampling, Change of Sampling Rate

Down Sampling by M

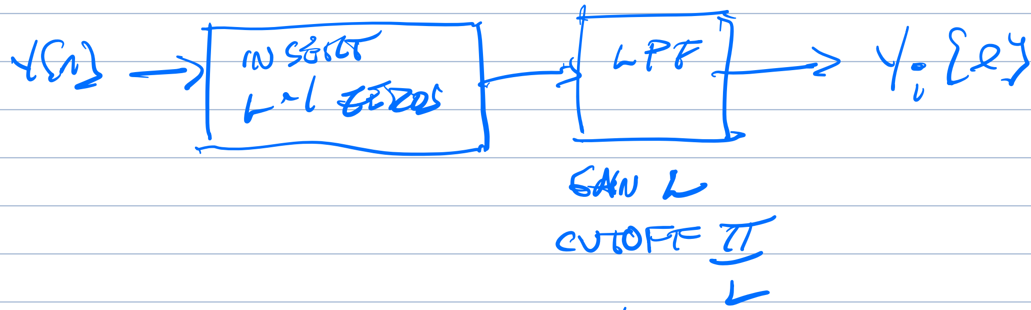
$s[n] = \begin{cases} 1, & n = rM \\ 0, & \text{else} \end{cases}$

To avoid aliasing require $X(e^{j\omega}) = 0$, $\frac{\pi}{M} < |\omega| \le \pi$

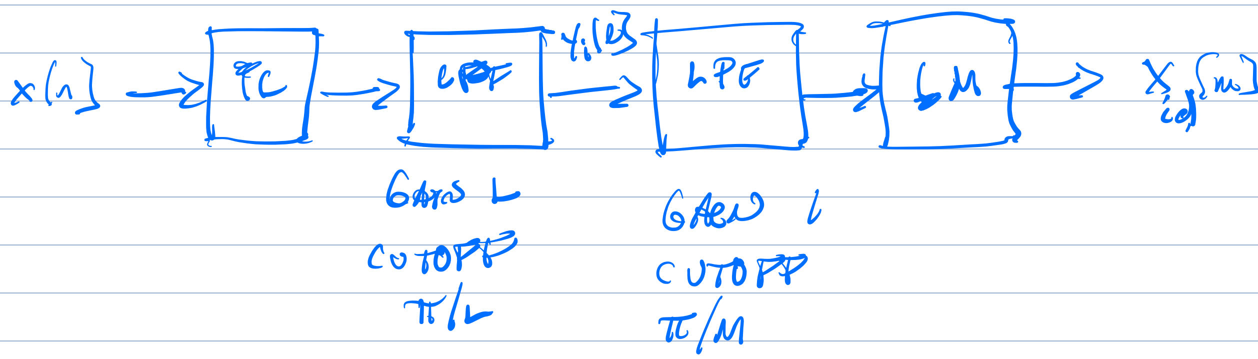
Practical Implementation



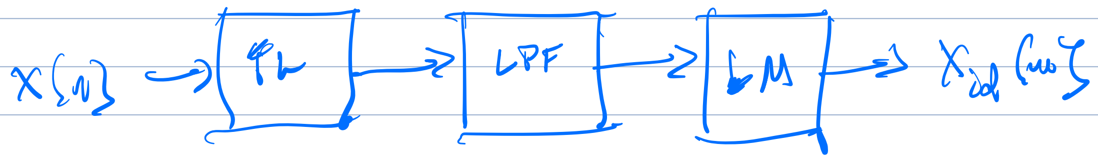
Interpolation by L



Changing Rate by L/M



MORE EFFICIENT IMPLEMENTATION



GAIN L

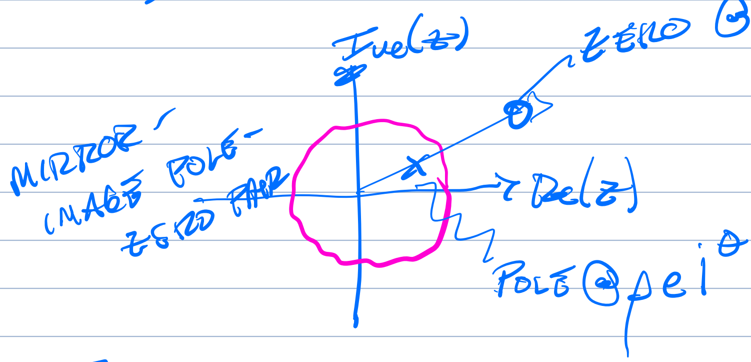
CUTOFF $\min\left(\frac{\pi}{L}, \frac{\pi}{M}\right)$

SPECIAL LTI SYSTEMS

ALL PASS SYSTEM

$|H(e^{j\omega})| = \text{CONST.}$

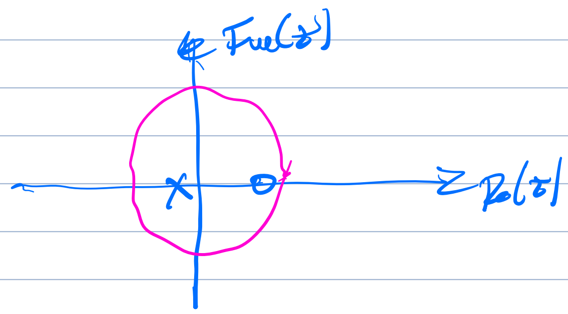
SUFFICIENT CONSTRAINT

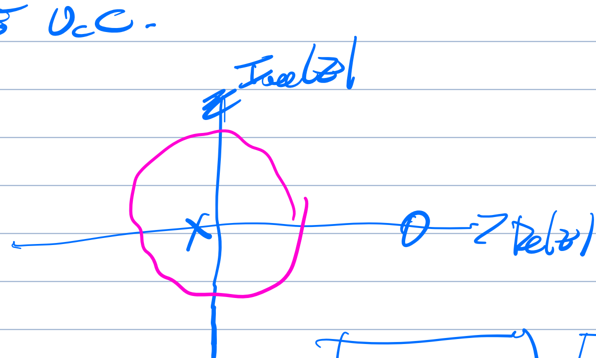


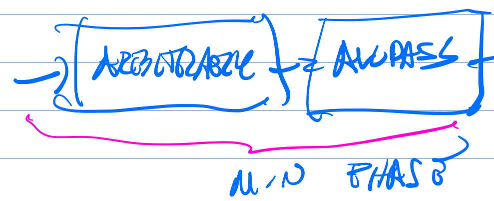
$$H(z) = \frac{z - \frac{1}{\rho} e^{j\theta}}{z - \rho e^{j\theta}}$$

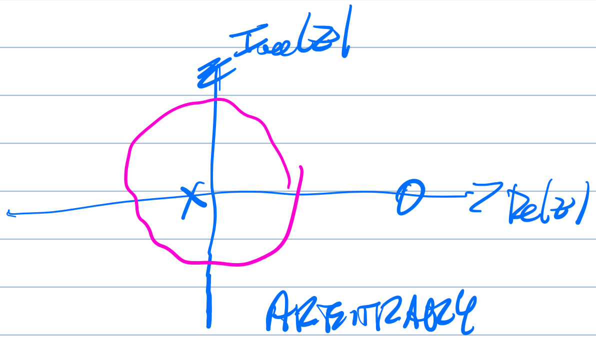
MINIMUM PHASE SYSTEM

ALL POLES, ALL ZEROS INSIDE UC.

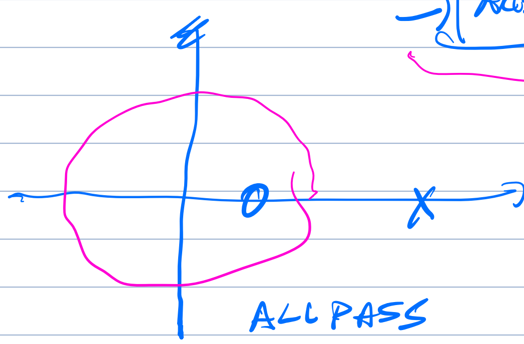




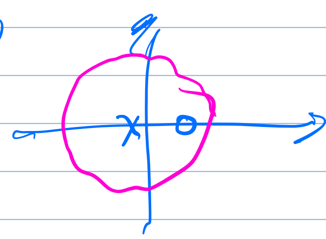




ARBITRARY



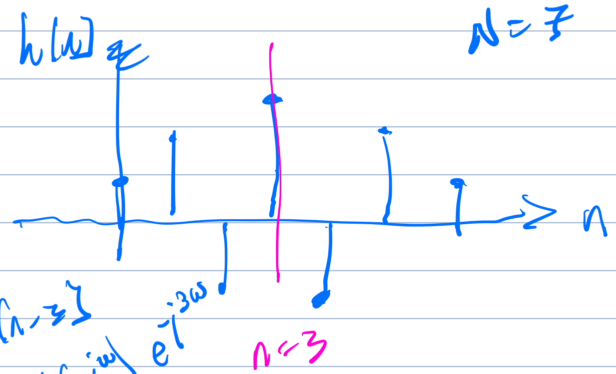
ALLPASS



LINEAR PHASE SYSTEM

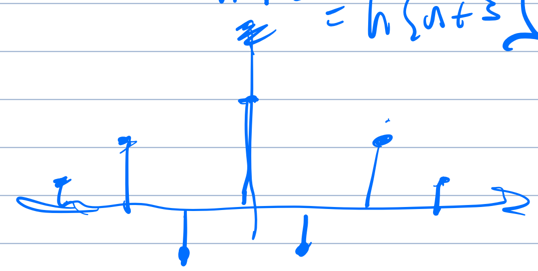
* FIR

$N = 7$



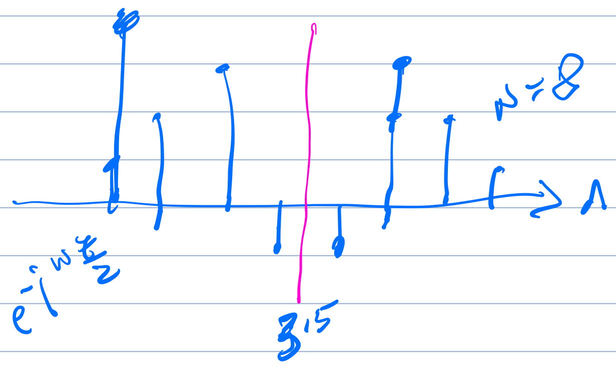
SYMMETRIC ABOUT $\frac{N-1}{2} = 3$

$h'[n] = h[n+3]$



$h[n] = h'[n-3]$

$H(e^{j\omega}) = H'(e^{j\omega}) e^{-j3\omega}$



$H(e^{j\omega}) e^{-j\omega\frac{7}{2}}$

SYMMETRIC ABOUT $\frac{N-1}{2} = \frac{7}{2}$

("GENERALIZED LINEAR PHASE")

$$H(z) = 1 + 2z^{-1} - \frac{3}{2}z^{-2} + \frac{5}{2}z^{-3} - \frac{3}{2}z^{-4} + 2z^{-5} + z^{-6}$$

ZEROS ⊘ →

$$1 + 2z^{-1} - \frac{3}{2}z^{-2} + \frac{5}{2}z^{-3} - \frac{3}{2}z^{-4} + 2z^{-5} + z^{-6} = 0$$

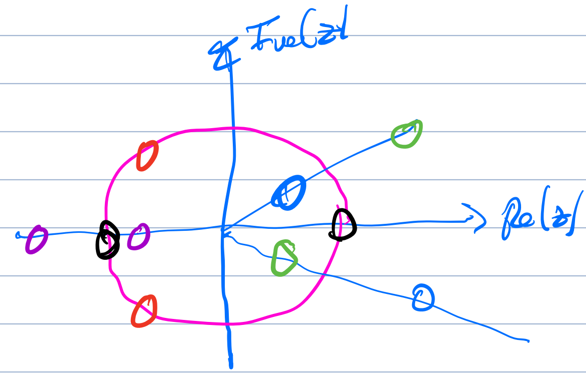
$$z^6 + 2z^5 - \frac{3}{2}z^4 + \frac{5}{2}z^3 - \frac{3}{2}z^2 + 2z + 1 = 0$$

IF z_0 IS A ZERO of $H(z)$

$1/z_0$ WILL BE ZERO AS WELL

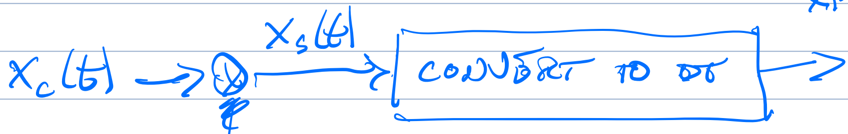
$$z_0 = \rho e^{j\theta}$$

$$\frac{1}{z_0} = \frac{1}{\rho e^{j\theta}} = \frac{1}{\rho} e^{-j\theta}$$



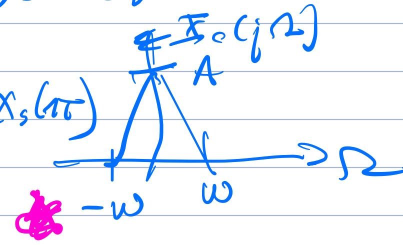
SAMPLING + RECONSTRUCTION OF CT SIGNALS

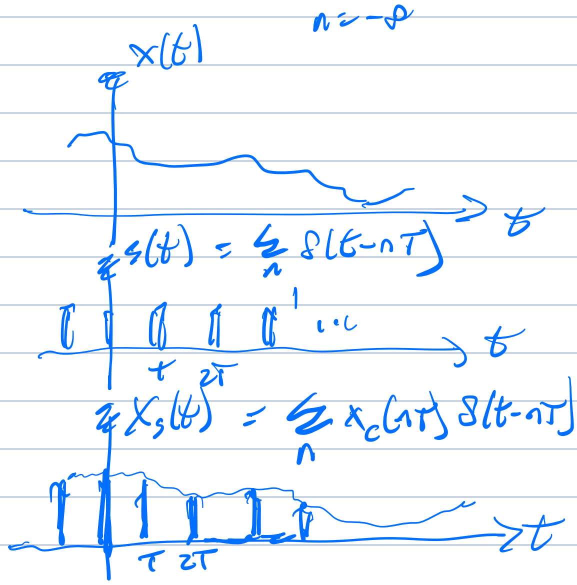
C/D CONVERSION

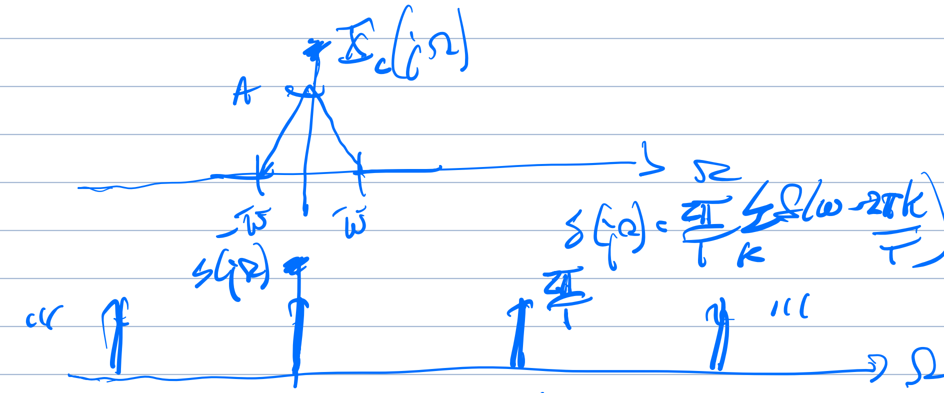


$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$

$x[n] = x_s(nT)$

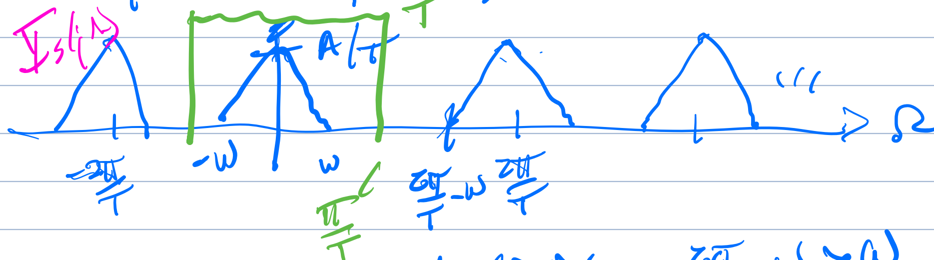






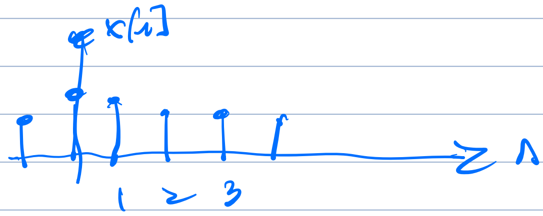
$S(j\Omega) = \frac{2\pi}{T}\sum_k \delta(\omega - \frac{2\pi k}{T})$

$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$



TO AVOID ALIASING $\frac{2\pi}{T} - W > W$

$$W < \frac{\pi}{T}$$

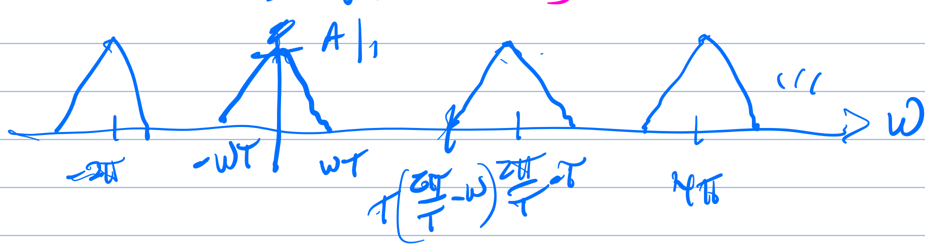


$X(e^{j\omega}) = X_c(j\Omega)\Big|_{\Omega = \frac{\omega}{T}}$

$\omega = \Omega T$

$\Omega = \frac{\omega}{T}$

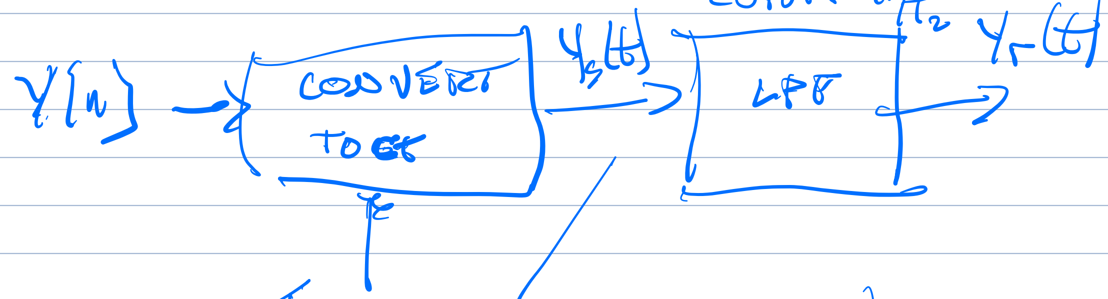
$X(e^{j\omega})$



$$X_s(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\Omega - \frac{2\pi k}{T}\right)\right)$$

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\Omega T - \frac{2\pi k}{T}T\right)\right)$$

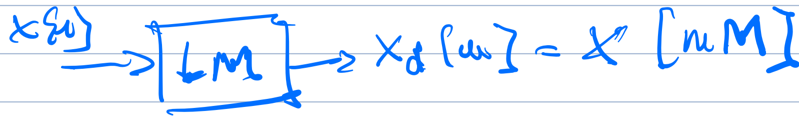
D/C CONVERSION



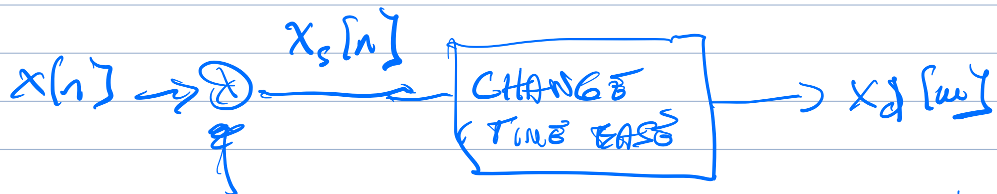
$$y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t - nT_2)$$

DECIMATION & INTERPOLATION

DECIMATION BY M



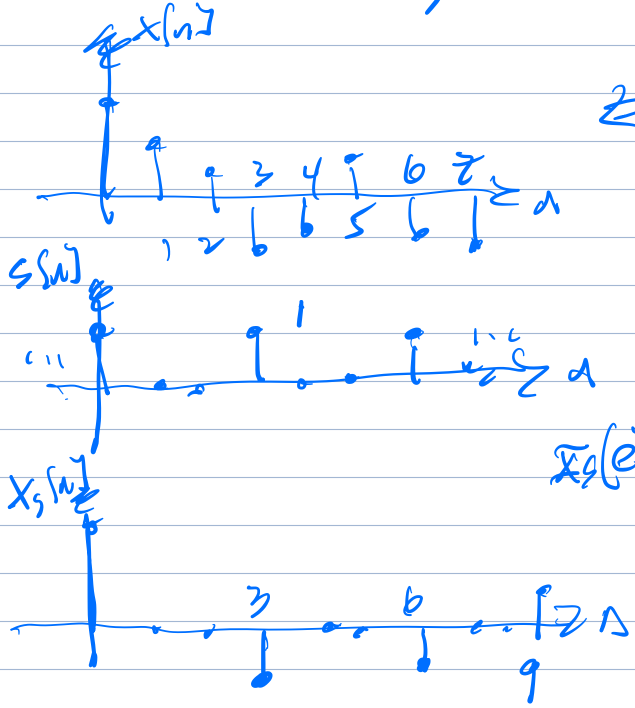
$x_d[n] = x[nM]$

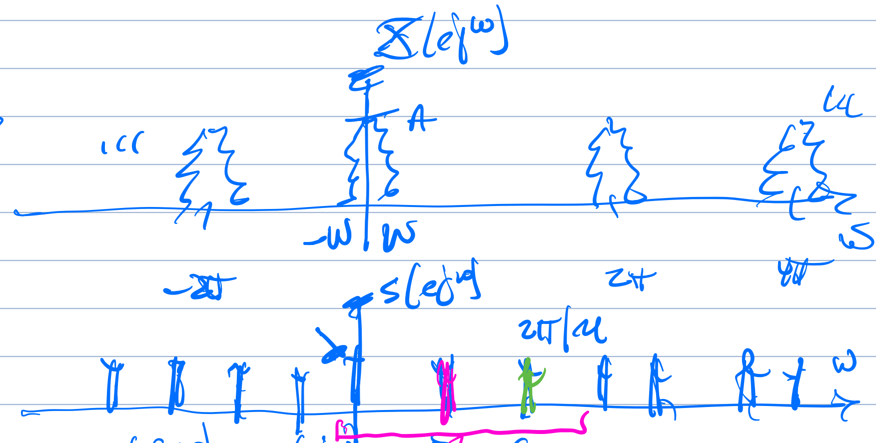


$M = 3$

$$s_M[n] = \begin{cases} 1, & n = rM \\ 0, & \text{else} \end{cases}$$

$|X(e^{j\omega})| = 0, \; W < |\omega| \le \pi$

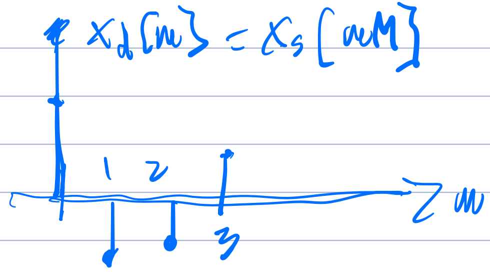


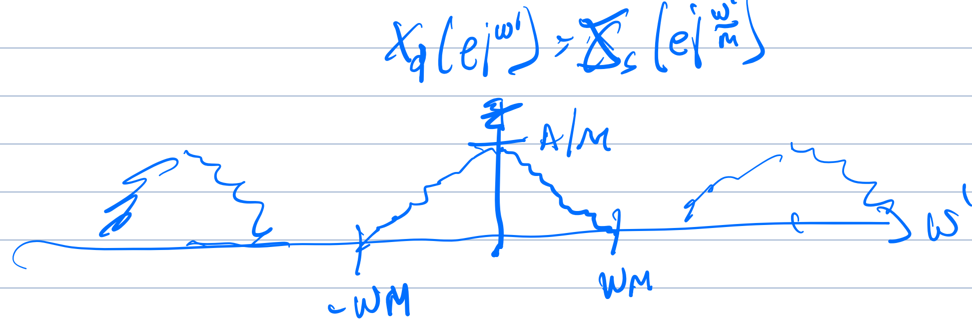


$$X_s(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \circledast S(e^{j\omega})$$

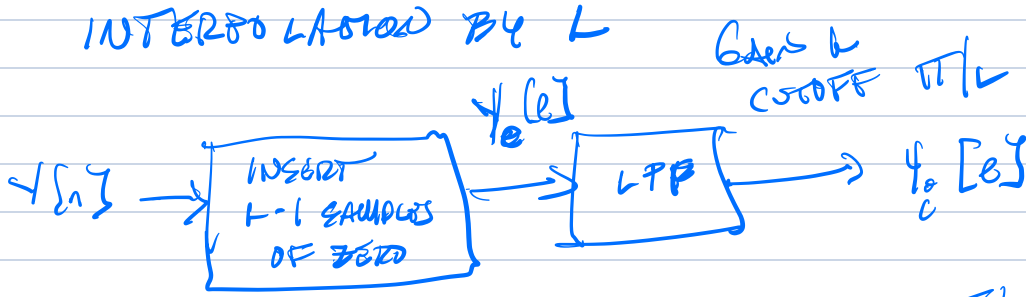
REQUIRE $\frac{2\pi}{M} - W > W$ $\quad \frac{2\pi}{M} - W$

$W < \pi/M$ (NYQUIST)

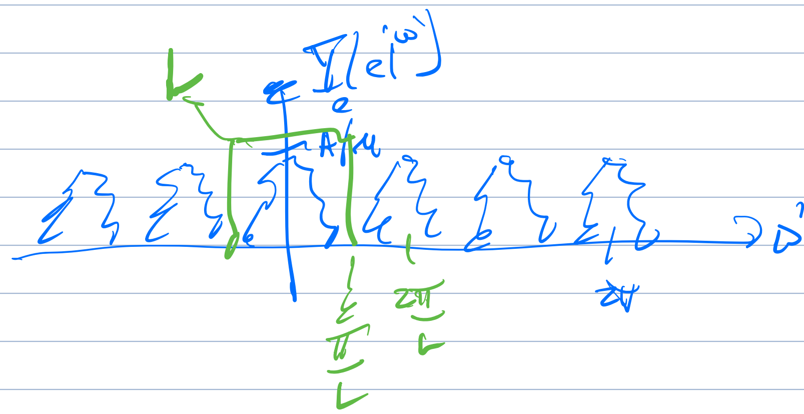




INTERPOLATION BY L

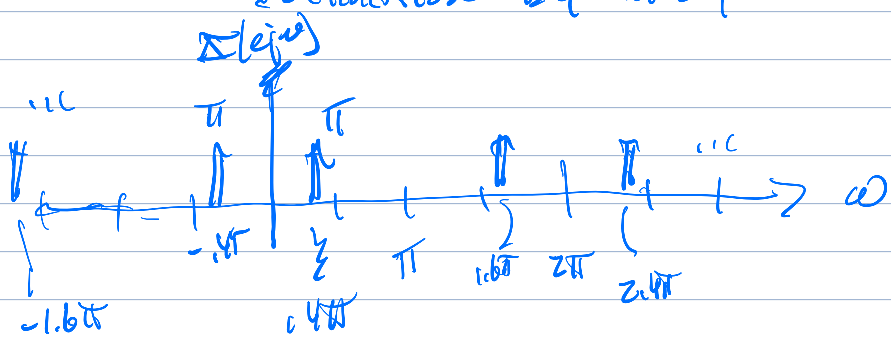


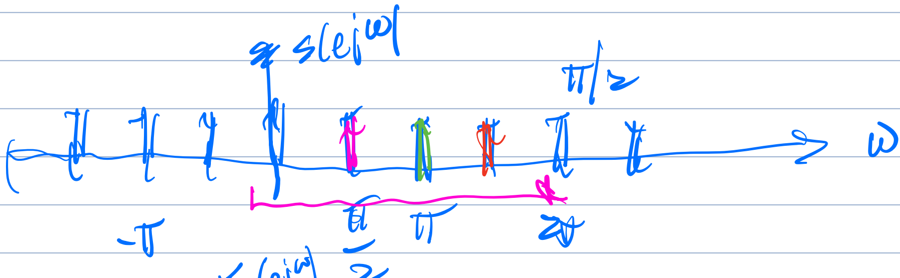
$$Y_e(e^{j\omega'}) = Y(e^{j\omega' L})$$

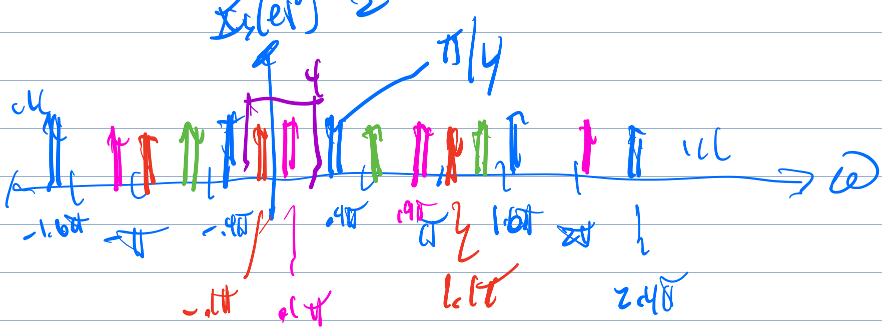


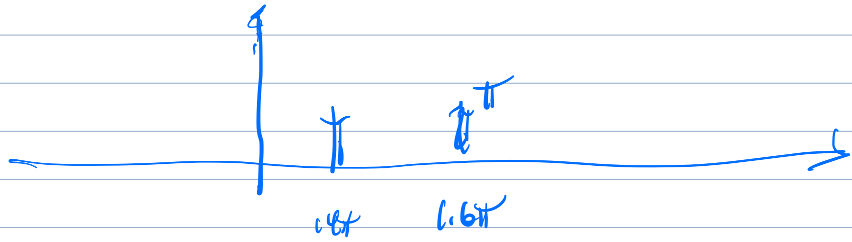
CONSIDER $x(n) = \cos(.4\pi n)$

DECIMATION BY $M = 4$








.4π
1.6π