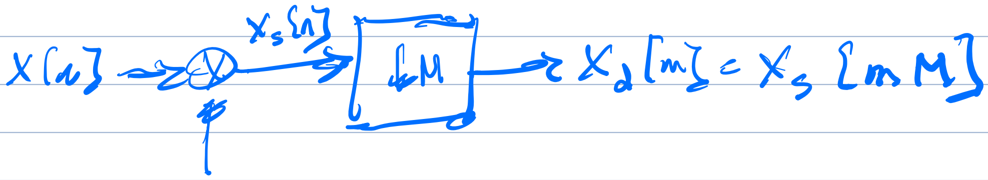
2/16/24

Recitation 4A Special LSI Systems, Sampling, Change of Sampling Rate

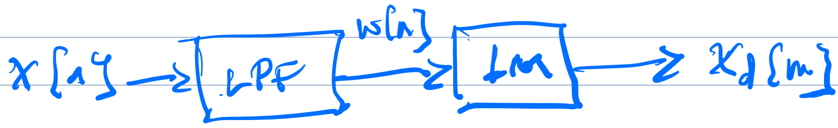
Decimation by M



$$s[n] = \begin{cases} 1, & n = rM \\ 0, & \text{otherwise} \end{cases}$$

To avoid aliasing require $X(e^{j\omega}) = 0,\ \frac{\pi}{M} < |\omega| \le \pi$

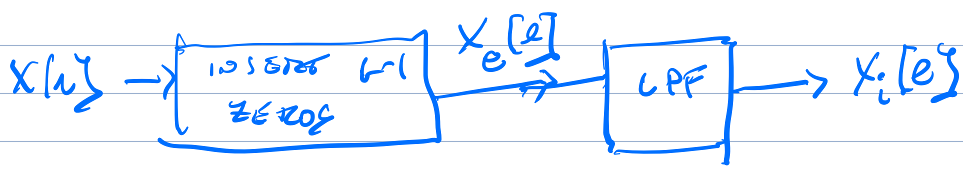
Complete implementation



Gain 1

Cutoff $\frac{\pi}{M}$

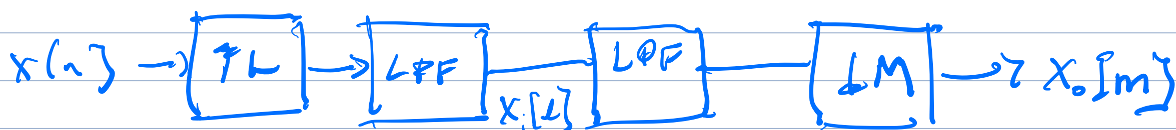
Interpolation by L



Gain L

Cutoff $\frac{\pi}{L}$

Change of sampling rate



Gain L, Cutoff π/L — Gain 1, Cutoff π/M

$$\underbrace{\qquad\qquad\qquad}_{\text{Upsampling by } L} \qquad \underbrace{\qquad\qquad\qquad}_{\text{Downsampling by } M}$$

MORE EFFICIENT IMPLEMENTATION

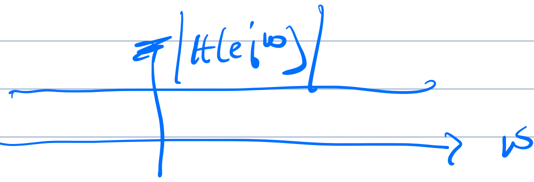
$$x[n] \to \boxed{\uparrow L} \to \boxed{LPF} \to \boxed{\downarrow M} \to x_o[m]$$

GAIN L

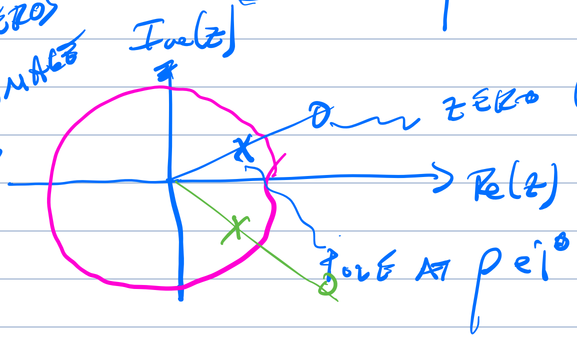
CUTOFF $\min\left(\frac{\pi}{L}, \frac{\pi}{M}\right)$

SPECIAL LSI SYSTEMS

ALL PASS



POLES, ZEROS MIRROR IMAGE PAIRS

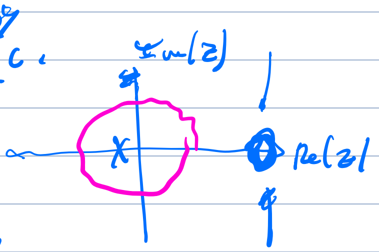


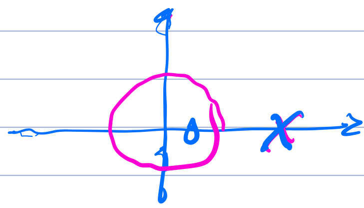
$$H(z) = \frac{z - \frac{1}{\rho} e^{j\theta}}{z - \rho e^{j\theta}}$$

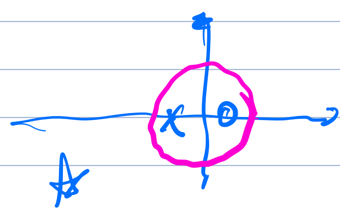
MINIMUM PHASE

$$\overbrace{\to \boxed{\text{ARBITRARY } H(z)} \to \boxed{\text{ALL PASS}}}^{\text{MIN. PHASE}}$$

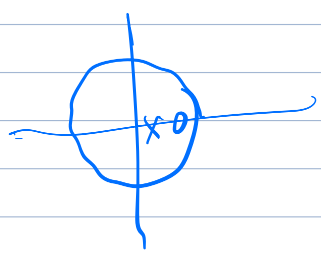
ALL POLES, ALL ZEROS INSIDE U.C.

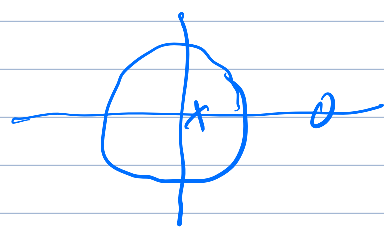






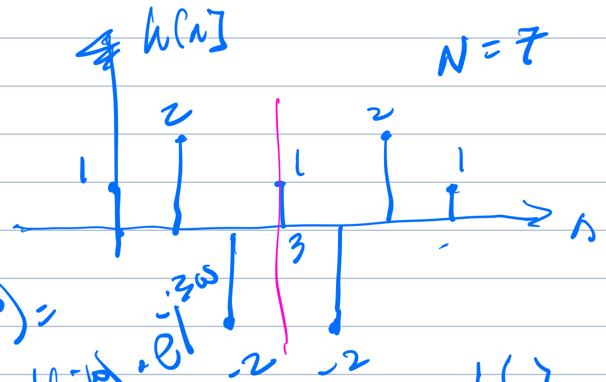
ARBITRARY ALL PASS MINIMUM PHASE





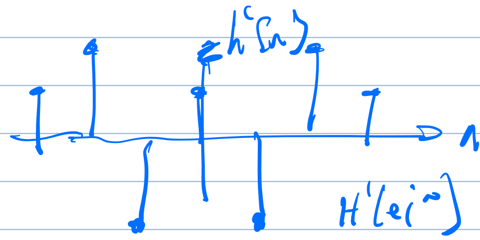
LINEAR PHASE SYSTEM

FIR ★



$N = 7$

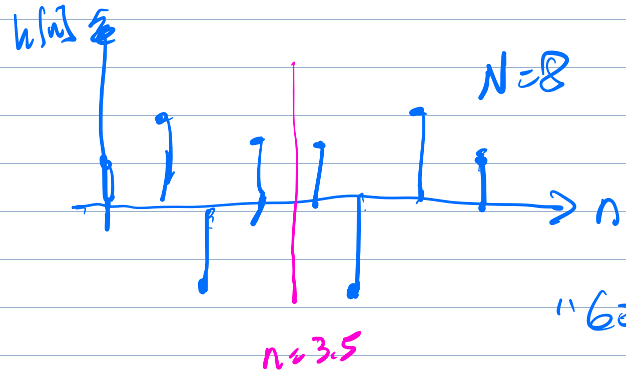
SYMMETRIC ABOUT $n = 3 = \frac{N-1}{2}$



$H(e^{j\omega}) = H'(e^{j\omega}) \cdot e^{-j3\omega}$

$H'(e^{j\omega})$ ZERO PHASE

$h[n] = h[N-n]$



$N = 8$

SYMMETRIC ABOUT $n = \frac{N-1}{2} = \frac{7}{2}$

PHASE $e^{-j\omega\frac{7}{2}}$

"GENERALIZED LINEAR PHASE"

$N=7 \quad H(z) = 1 + 2z^{-1} - 2z^{-2} + z^{-3} - 2z^{-4} + 2z^{-5} + z^{-6}$

ZEROS ARE ROOTS OF

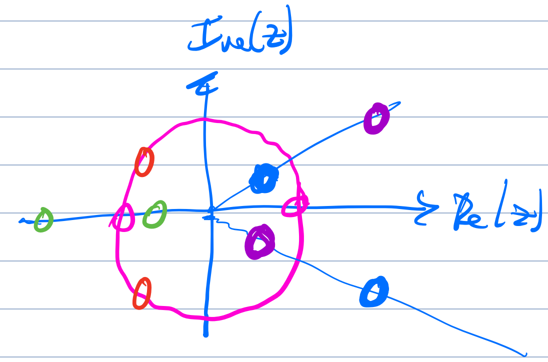
$$1 + 2z^{-1} - 2z^{-2} + z^{-3} - 2z^{-4} + 2z^{-5} + z^{-6} = 0$$

$$z^6 + 2z^5 - 2z^4 + z^3 - 2z^2 + 2z + 1 = 0$$

IF z_0 IS A ZERO of $H(z)$, $\frac{1}{z_0}$ WILL BE A ZERO AS WELL

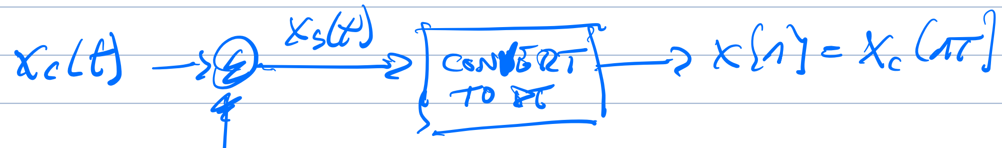
$$z_0 = \rho e^{j\theta}$$

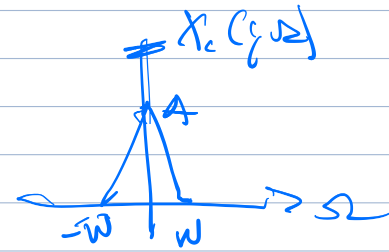
$$\frac{1}{z_0} = \frac{1}{\rho e^{j\theta}} = \frac{1}{\rho} e^{-j\theta}$$



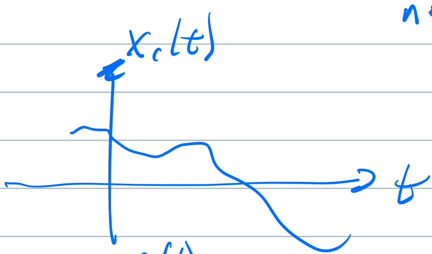
CONJUGATE RECIPROCAL LOCATIONS

SAMPLING CT SIGNALS

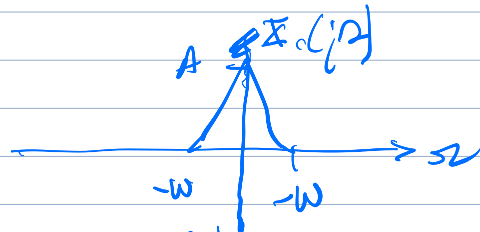


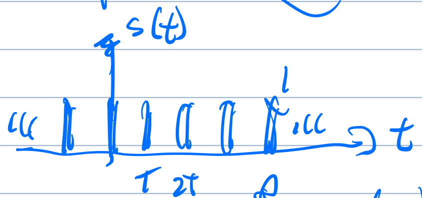


$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

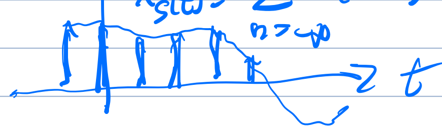


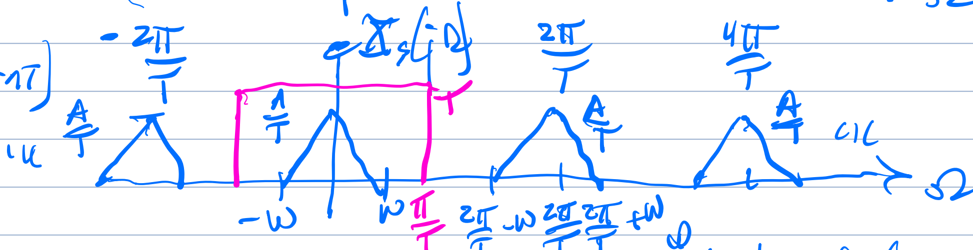
$\Longleftrightarrow$





$$x_s(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t - nT)$$





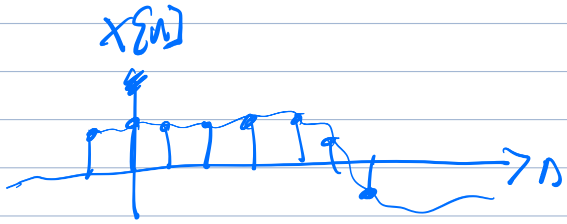
$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T} X_c\left(j\left(\Omega - \frac{2\pi k}{T}\right)\right)$$

TO PREVENT OVERLAP

REQUIRE $\frac{2\pi}{T} - W > W$

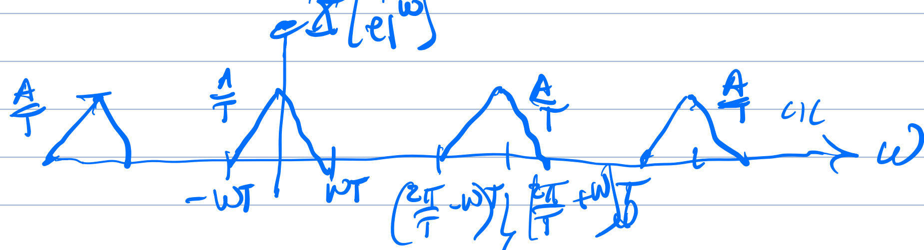
$2W < \frac{2\pi}{T} \Rightarrow \boxed{W < \frac{\pi}{T}}$

NYQUIST LIMIT



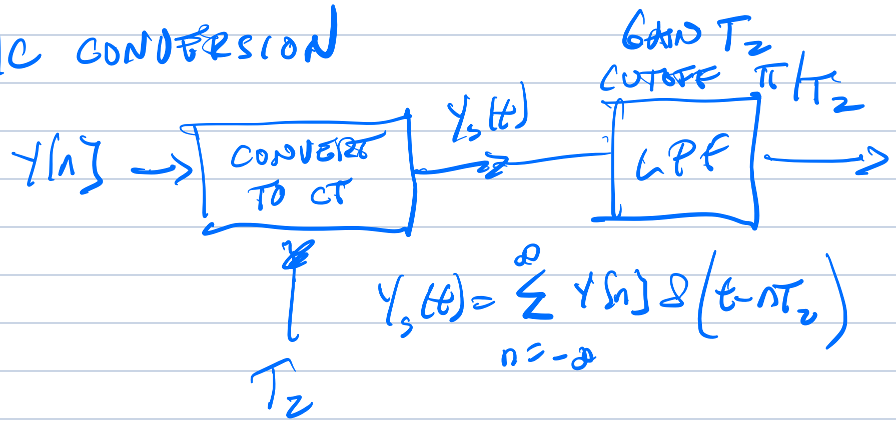
$$\Longleftrightarrow X(e^{j\omega}) = X_s(j\Omega)\Big|_{\Omega = \frac{\omega}{T}}$$

$\omega = \Omega T$



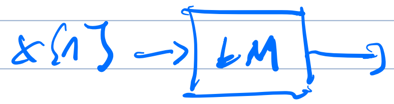
$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} X_c(j(\Omega T - 2\pi k)) \qquad \frac{2\pi}{T}\cdot T$$

D/C CONVERSION

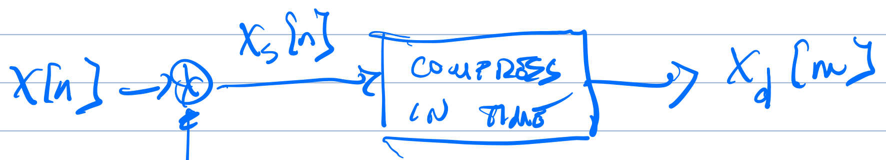


$$y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t - nT_2)$$

DECIMATION

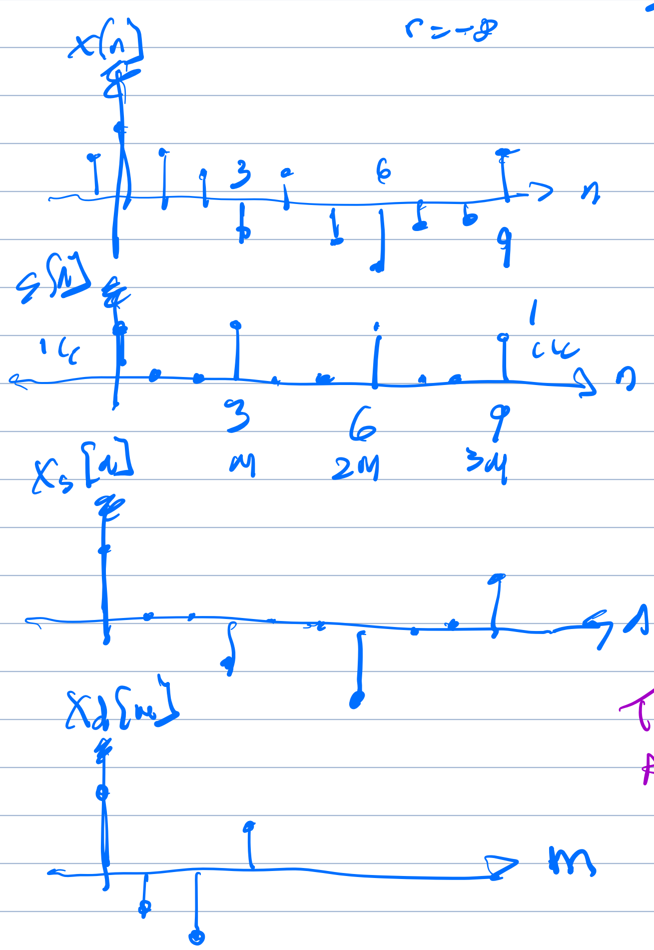


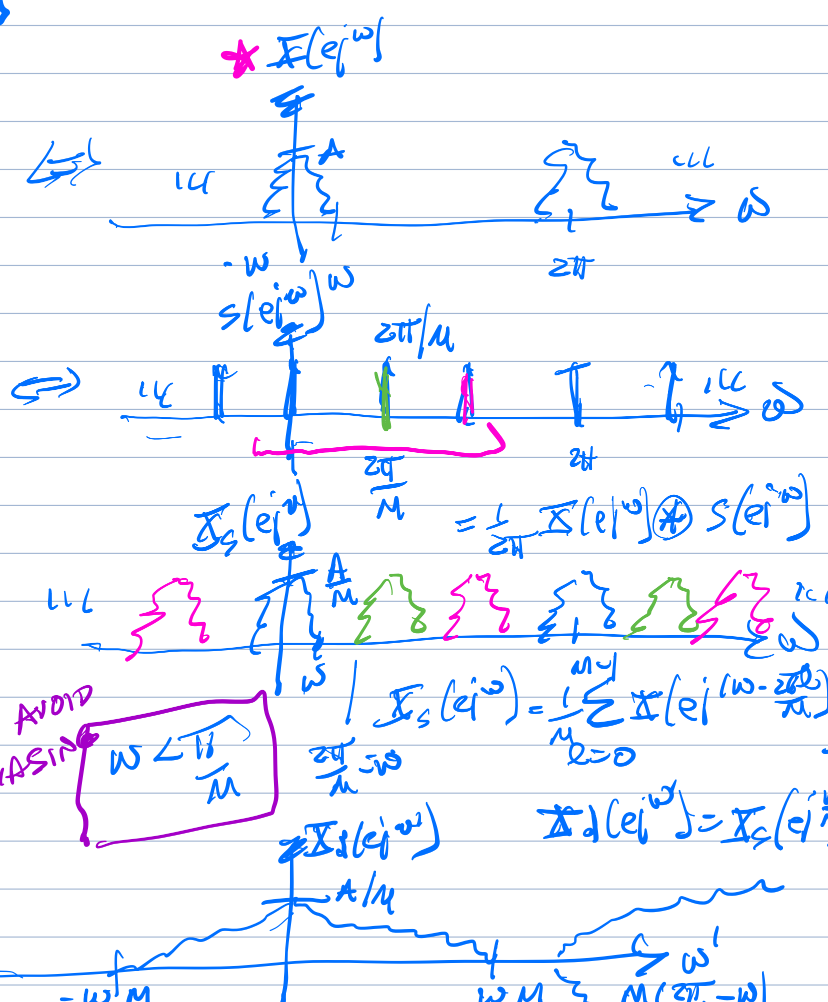
$x_d[m] = x[mM]$



$$s_M[n] = \begin{cases} 1, & n = rM \\ 0, & \text{ELSE} \end{cases}$$

$M = 3$ $\qquad s_M[n] = \sum_{r=-\infty}^{\infty} \delta[n - rM]$





$= \frac{1}{2\pi} X(e^{j\omega}) \circledast S(e^{j\omega})$

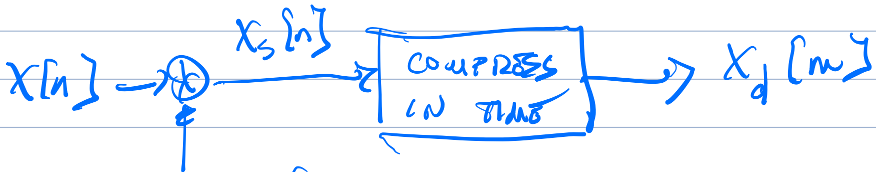
$$X_s(e^{j\omega}) = \frac{1}{M}\sum_{\ell=0}^{M-1} X\left(e^{j(\omega - \frac{2\pi \ell}{M})}\right)$$

TO AVOID ALIASING $\boxed{\omega < \frac{\pi}{M}}$

$$X_d(e^{j\omega}) = X_s(e^{j\frac{\omega}{M}})$$

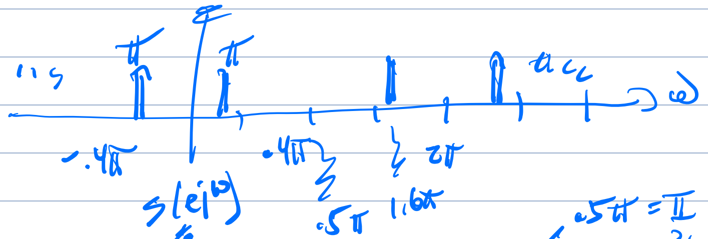
$\omega M < M\left(\frac{2\pi}{M} - \omega\right)$

EX $x[n] = A\cos(.4\pi n)$, $M = 4$

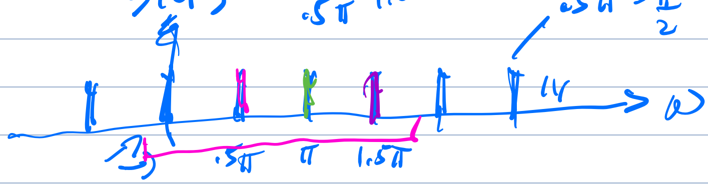


$$s_M[n] = \begin{cases} 1, & n = rM \\ 0, & \text{ELSE} \end{cases}$$

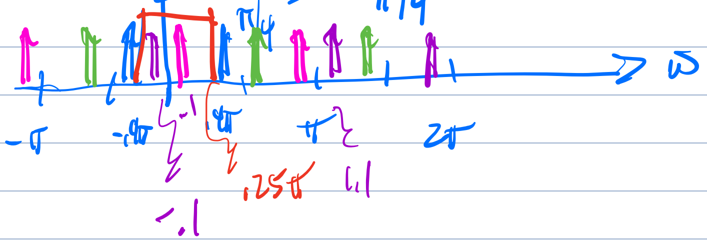
$X(e^{j\omega})$



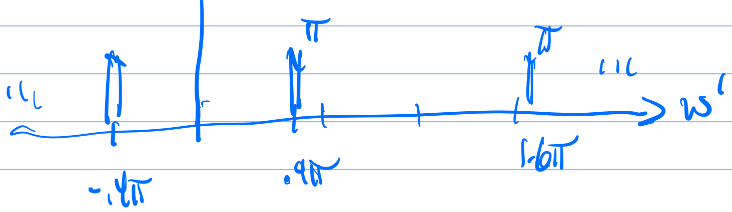
$S(e^{j\omega})$



$X_s(e^{j\omega})$



$X_d(e^{j\omega})$



AFTER UPSAMPLING

$\cos(.1\pi n)$