2/2/24 RECITATION 2B WORKING WITH DTFTs

FOR $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$ DTFT $x[n] = \frac{1}{2\pi}\int_{\omega=-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

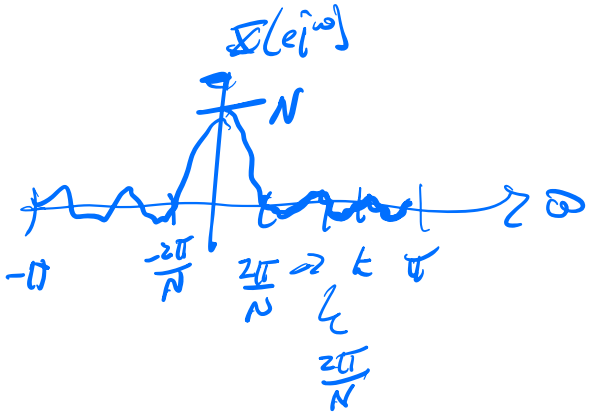
EXAMPLES

1. $\alpha^n u[n] \Leftrightarrow \frac{1}{1-\alpha e^{-j\omega}}$ $|\alpha|<1$

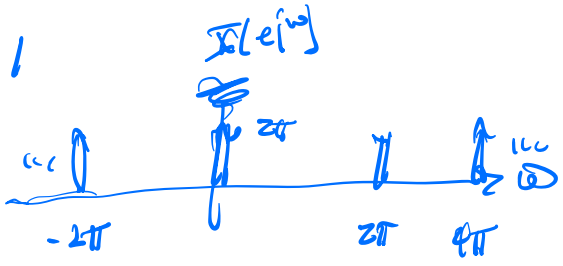
2. 

$\Leftrightarrow \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$

$N = 2L+1$

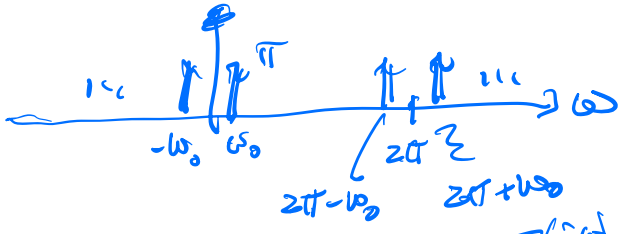


3. $\delta[n] \Leftrightarrow 1$

4. $1 \Leftrightarrow$ 

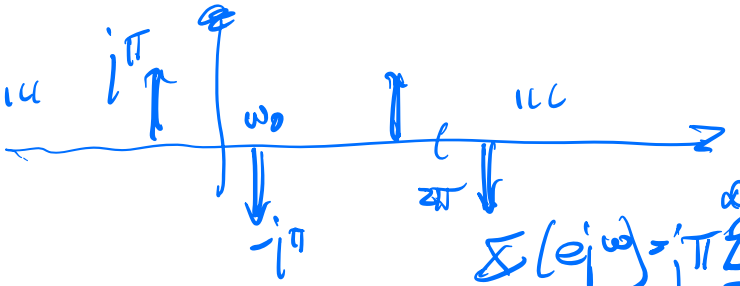
$X(e^{j\omega}) = 2\pi \sum_{r=-\infty}^{\infty} \delta(\omega - 2\pi r)$

5. $\cos(\omega_0 n) \Leftrightarrow$
$= \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}$

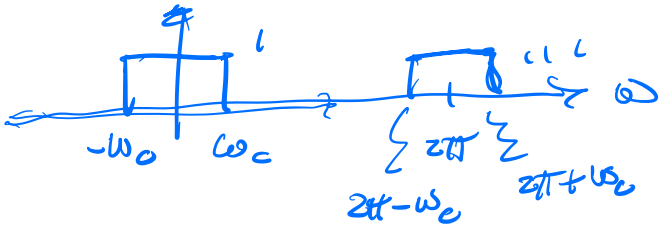


$X(e^{j\omega}) = \pi \sum_{r=-\infty}^{\infty} [\delta(\omega - \omega_0 - 2\pi r) + \delta(\omega + \omega_0 - 2\pi r)]$

6. $\sin(\omega_0 n)$
$= \frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}$



$X(e^{j\omega}) = j\pi \sum_{r=-\infty}^{\infty} [-\delta(\omega - \omega_0 - 2\pi r) + \delta(\omega + \omega_0 - 2\pi r)]$

2a. $\frac{\sin(\omega_c n)}{\pi n} \Leftrightarrow$ 

DTFT PROPERTIES

LINEARITY $\quad a x_1[n] + b x_2[n] \Leftrightarrow a X_1(e^{j\omega}) + b X_2(e^{j\omega})$

TIME SHIFT $\quad x[n-n_0] \Leftrightarrow e^{-j\omega n_0} X(e^{j\omega})$

MULT BY COMP. EXP. $\quad e^{j\omega_0 n} x[n] \Leftrightarrow X(e^{j(\omega-\omega_0)})$

CONVOLUTION IN TIME $\quad x[n] * h[n] \Leftrightarrow X(e^{j\omega}) \cdot H(e^{j\omega})$

MULT IN TIME $\quad x_1[n] \cdot x_2[n] \Leftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta}) H(e^{j(\omega-\theta)})\, d\theta$

$= \frac{1}{2\pi} X(e^{j\omega}) \circledast X_2(e^{j\omega})$

SYMMETRY PROPERTIES

$x[n] \Leftrightarrow X(e^{j\omega})$

$x[-n] \Leftrightarrow X(e^{-j\omega})$

$x^*[n] \Leftrightarrow X^*(e^{-j\omega})$

$x[n] = x[-n] \Leftrightarrow X(e^{j\omega}) = X(e^{-j\omega})$

EVEN $\qquad$ EVEN

$x[n] = -x[-n] \Leftrightarrow X(e^{j\omega}) = -X(e^{-j\omega})$

ODD $\qquad$ ODD

$x[n] = x^*[n] \Leftrightarrow X(e^{j\omega}) = X^*(e^{-j\omega})$

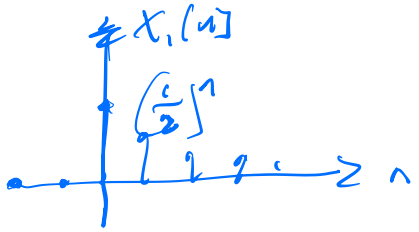
REAL $\qquad$ HERMITIAN

REAL, EVEN $\Leftrightarrow$ REAL, EVEN

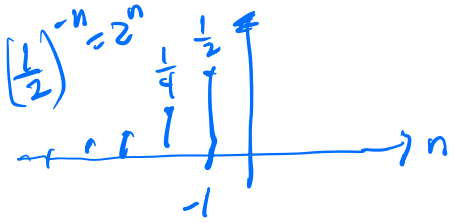
REAL, ODD $\Leftrightarrow$ IMAG, ODD

DTFT EXAMPLES

1. $x_1[n] = \left(\frac{1}{2}\right)^n u[n] \Leftrightarrow \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$



2. $x_2[n] = z^n u[-n-1]$



$$\Leftrightarrow \sum_{n=-\infty}^{-1} z^n e^{j\omega n} = \sum_{\ell=1}^{\infty} z^{-\ell} e^{+j\omega \ell}$$

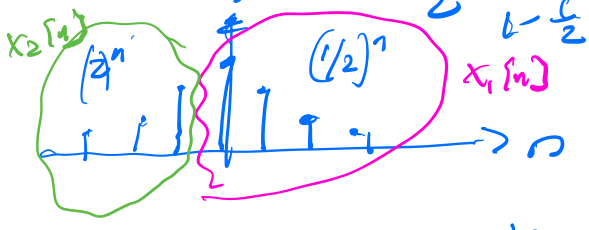
$\ell = -n$, $n = -\ell$

$$= \sum_{r=0}^{\infty} z^{-(r+1)} e^{j\omega(r+1)} = \frac{1}{2} e^{j\omega} \sum_{r=0}^{\infty} \left[\frac{1}{2} e^{j\omega}\right]^r$$

$r = \ell - 1$, $\ell = r+1$

$$X_3[n] = \frac{1}{2} \frac{e^{j\omega}}{1 - \frac{1}{2} e^{j\omega}}$$

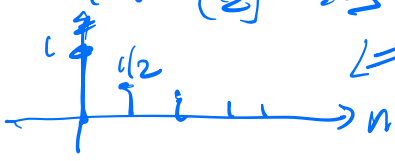
3. $x_3[n] = \left(\frac{1}{2}\right)^{|n|}$, for n



$$x_3[n] = x_1[n] + x_2[n] \Leftrightarrow X_3(e^{j\omega}) = \frac{1}{1 - \frac{1}{2} e^{j\omega}} + \frac{\frac{1}{2} e^{j\omega}}{1 - \frac{1}{2} e^{j\omega}}$$

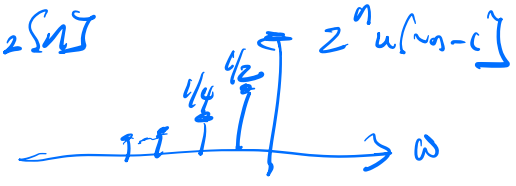
$$= \frac{1 - \frac{1}{2} e^{j\omega} + \frac{1}{2} e^{j\omega} - \frac{1}{4}}{1 - \frac{1}{2} e^{j\omega} - \frac{1}{2} e^{j\omega} + \frac{1}{4}} = \frac{3/4}{\frac{5}{4} - \cos(\omega)}$$

$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$



$$\Leftrightarrow \frac{1}{1 - \frac{1}{2} e^{j\omega}}$$

$x_2[n]$



$$x_2[n] = \frac{1}{2} x_1[-(n+1)]$$

TIME REVERSAL $\quad \frac{1}{1 - \frac{1}{2} e^{j\omega}}$

MULT BY $\frac{1}{2}$ $\quad \frac{1/2}{1 - \frac{1}{2} e^{j\omega}}$

SHIFT TO LEFT BY 1 $\quad \frac{\frac{1}{2} e^{j\omega}}{1 - \frac{1}{2} e^{j\omega}}$ *

WORKING WITH DELTA FUNCTIONS

$$\int_{-\infty}^{\infty} \delta(t-a)\,\phi(t)\,dt = \phi(a)$$

1. WHAT IS INTEGRATED?
2. VALUE THAT CAUSES $(\cdot) = 0$?
3. VALUE of ϕ @ ARGUMENT VALUE

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\int_{-\infty}^{\infty} \delta(2t)\,dt$$

let $z = 2t$

$dz = 2dt$

$dt = \frac{1}{2}dz$

$$= \int_{-\infty}^{\infty} \delta(z)\,\frac{1}{2}\,dz$$

$$= \frac{1}{2}$$

$$\int_{-\infty}^{\infty} e^{-2t}\,\delta\left(\frac{3t-5}{7}\right)dt = \int_{-\infty}^{\infty} e^{-2t}\,\delta\left(\frac{3t}{7} - \frac{5}{7}\right)dt$$

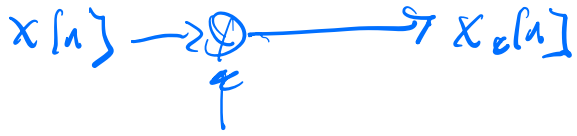
let $t' = \frac{3t}{7}$, $t = \frac{7t'}{3}$

$dt = \frac{7}{3}dt'$

$$= \int_{-\infty}^{\infty} e^{-\frac{14}{3}t'}\,\delta\left(t' - \frac{5}{7}\right)\frac{7}{3}\,dt'$$

$$= e^{-\frac{14}{3}\cdot\frac{5}{7}}\,\frac{7}{3} = \frac{7}{3}\,e^{-10/3}$$

DOUBLE SIDEBAND SUPPRESSED CARRIER (DSB/SC) MODULATION

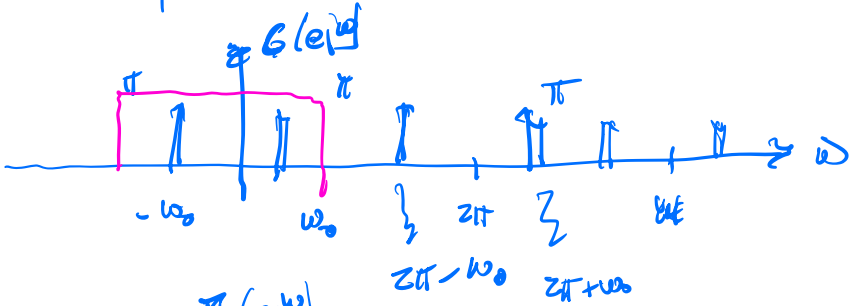


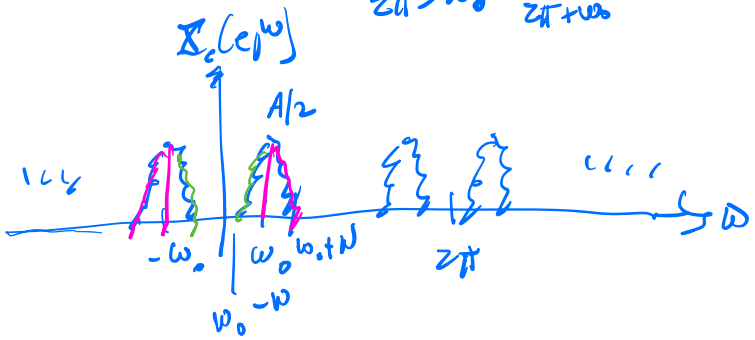
$$X_c(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \circledast C(e^{j\omega})$$

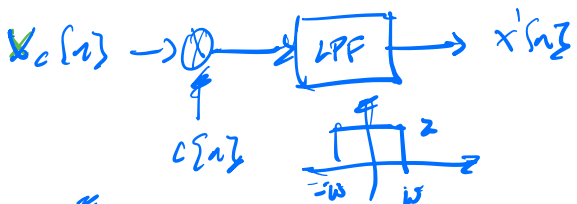
$c[n] = \cos(\omega_0 n)$

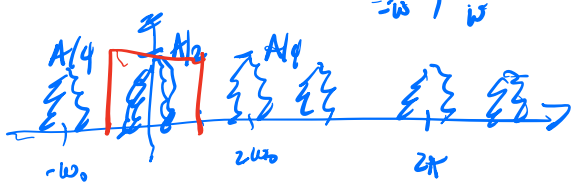
$\Xi X(e^{j\omega})$

$|X(e^{j\omega})| = 0, \quad W < \omega < 2\pi - W$

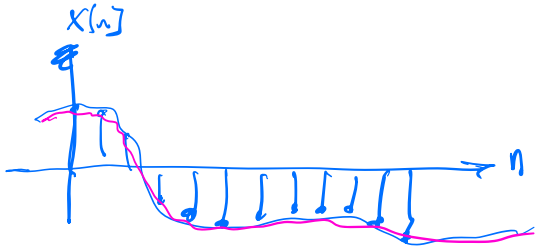








AM



$$x_c[n] = (A + x[n])\cos(\omega_0 n)$$

$$X_c(e^{j\omega}) = A\cos(\omega_0 n) + X[n]\cos(\omega_0$$