1/26/24 RECITATION 1b

DTFT $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

SYMMETRY PROPERTIES

$x[n] \Leftrightarrow X(e^{j\omega})$

$x[-n] \Leftrightarrow X(e^{-j\omega})$

$\sum_{n=-\infty}^{\infty} x[-n] e^{-j\omega n} = \sum_{\ell=-\infty}^{\infty} x[\ell] e^{-j\omega(-\ell)}$

let $\ell = -n$
$n = -\ell$

$= \sum_{\ell=-\infty}^{\infty} x[\ell] e^{-j(-\omega)\ell}$

$= X(e^{-j\omega})$

$x[n] = x[-n] \Leftrightarrow X(e^{j\omega}) = X(e^{-j\omega})$

EVEN EVEN

$x[n] = -x[-n] \Leftrightarrow X(e^{j\omega}) = -X(e^{-j\omega})$

ODD OD

$x[n]$ REAL $\boxed{x[n] = x^*[n]}$

$x^*[n] \Leftrightarrow X^*(e^{-j\omega})$

HERMITIAN SYMMETRY

$\sum_{n=-\infty}^{\infty} x^*[n] e^{-j\omega n}$

$= \left(\sum_{n=-\infty}^{\infty} x[n] e^{+j\omega n}\right)^*$

$= \left(\underbrace{\sum_{n=-\infty}^{\infty} x[n] e^{-j(-\omega)n}}_{X(e^{-j\omega})}\right)^*$

let $X(e^{j\omega}) = X_R(e^{j\omega}) + j X_I(e^{j\omega})$

$x[n]$ REAL

$X(e^{-j\omega}) = X_R(e^{-j\omega}) + j X_I(e^{-j\omega})$

$X^*(e^{-j\omega}) = X_R(e^{-j\omega}) - j X_I(e^{-j\omega})$

$X_R(e^{j\omega}) = Re\{X(e^{j\omega})\}$

$X_I(e^{j\omega}) = Im\{X(e^{j\omega})\}$

$X_R(e^{j\omega}) = X_R(e^{-j\omega})$ EVEN

$X_I(e^{j\omega}) = -X_I(e^{-j\omega})$ ODD

$x[n] \Leftrightarrow X(e^{j\omega})$

EVEN — EVEN

ODD — ODD

REAL — HERMITIAN SYMM. $X_R(e^{j\omega})$ EVEN

$X_I(e^{j\omega})$ ODD

REAL, EVEN — ~~REAL~~ EVEN

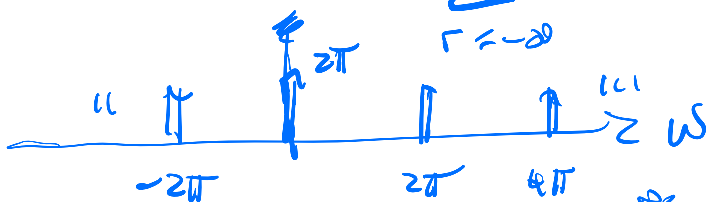
REAL, ODD — IMAG, ODD

DTFT of COSINES ....

DTFT of $e^{j\omega_0 n}$

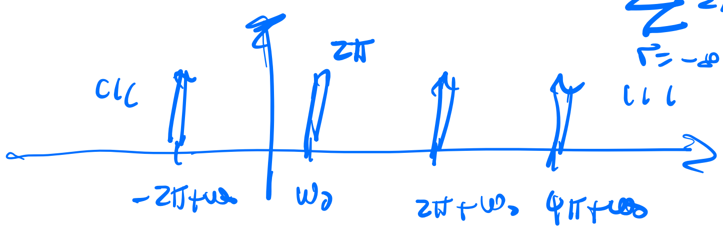
$$\sum_{r=-\infty}^{\infty} 2\pi\,\delta(\omega - 2\pi r)$$

$1 \Leftrightarrow$

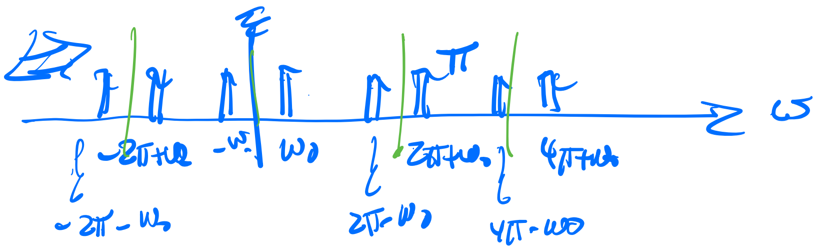


$$\sum_{r=-\infty}^{\infty} 2\pi\,\delta(\omega - \omega_0 - 2\pi r)$$

$e^{j\omega_0 n} = 1 \cdot e^{j\omega_0 n} \Leftrightarrow$

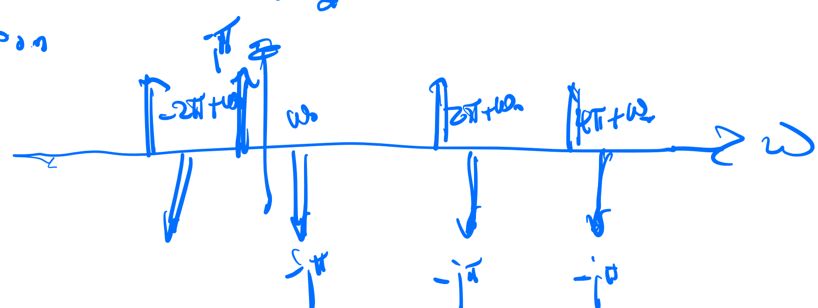


$\cos(\omega_0 n) = \dfrac{e^{j\omega_0 n}}{2} + \dfrac{e^{-j\omega_0 n}}{2} \Leftrightarrow$



$$\sum_{r=-\infty}^{\infty} \pi\left(\delta(\omega - \omega_0 - 2\pi r) + \delta(\omega + \omega_0 - 2\pi r)\right)$$

$\sin(\omega_0) = \dfrac{e^{j\omega_0 n}}{2j} - \dfrac{e^{-j\omega_0 n}}{2j}$



$$= j\pi \sum_{r=-\infty}^{\infty} \left(-\delta(\omega - \omega_0 - 2\pi r) + \delta(\omega + \omega_0 - 2\pi r)\right)$$

SOME MORE PROPERTIES .....

PARSEVAL'S THM.

$$\underbrace{\sum_{n=-\infty}^{\infty} |x[n]|^2}_{\text{ENERGY}} \Longleftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

ENERGY SPECTRAL DENSITY FUNCTION

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

IDTFT FOR $n=0$

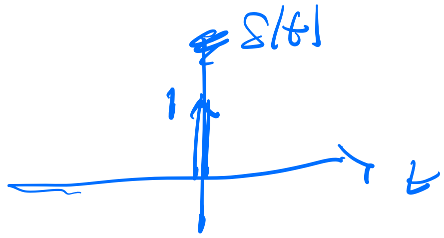
$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) \cancel{e^{j\omega 0}} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$$

DTFT FOR $\omega=0$

$$X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n]$$

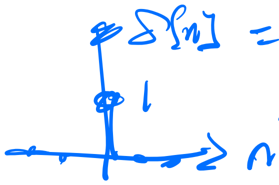
WORKING WITH DELTA FUNCTIONS



AREA 1

INFINITE @ $t=0$

$=0$, $t \neq 0$



$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$

$$\begin{cases} \delta(t) = 0, \quad t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)\, dt = 1 \end{cases}$$

IMPLICIT DEF. of $\delta(t)$

Let $\phi(t)$ CONTINUOUS $\forall t$

$$\int_{-\infty}^{\infty} \delta(t-a)\,\phi(t)\, dt = \phi(a)$$

ANSWERS

1. VARIABLE INTEGRATED? t
2. VALUE of VARIABLE THAT CAUSES $(\;) = 0$? $t=a$
3. VALUE of INTEGRAND THERE? $\phi(a)$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = \int_{-\infty}^{\infty} \delta(t)\cdot 1\, dt = 1$$

$$\int_{-\infty}^{\infty} \delta(2t)\,dt \;? \;= \int_{-\infty}^{\infty} \delta(t')\,\frac{dt'}{2}$$

Let $t' = 2t$

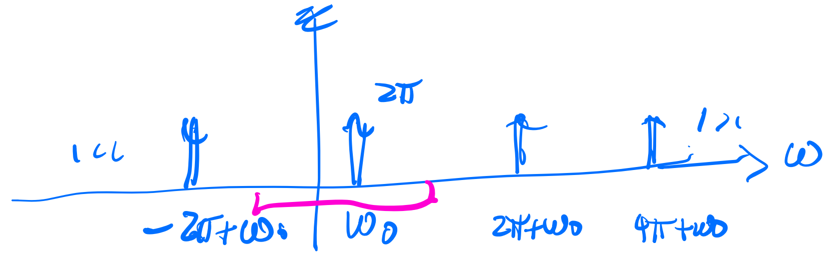
$t = \frac{t'}{2}$

$dt = \frac{dt'}{2}$

$$= \frac{1}{2}\underbrace{\int_{-\infty}^{\infty} \delta(t')\,dt'}_{1} = \frac{1}{2}$$

DTFT of

$e^{j\omega_0 n}$



$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\,\delta(\omega-\omega_0)\,e^{j\omega n}\,d\omega$$

$$= e^{j\omega_0 n}$$

SYSTEM PROPERTIES

LINEARITY

SHIFT INVARIANCE

CAUSALITY $\longrightarrow$ IF SYSTEM LSI: $h[n] = 0,\ n < 0$

MEMORYLESSNESS $\longrightarrow$ $h[n] = 0,\ n < 0,\ n > 0$

STABILITY $\longrightarrow$ $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

EX. 1. $y[n] = x[n]\tan(\omega_0 n)$

LINEAR

NOT SI

CAUSAL

MEMORYLESS

NOT STABLE

SI $\downarrow$

$\sum_{k=-\infty}^{n} x[k]$

2. $y[n] = \sum_{k=0}^{n} x[k]$

$\sum_{k=n-n_0}^{n+n_0} x[k]$

LINEAR

NOT SI

CAUSAL

NOT MEMORYLESS

NOT STABLE

PERIODICITY

$\cos\left(\frac{2\pi}{N} \cdot n\right)$

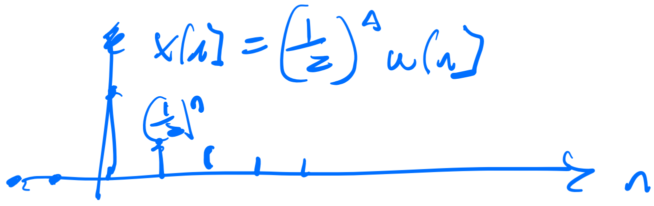
N integer $\rightarrow$ periodic, period $= N$

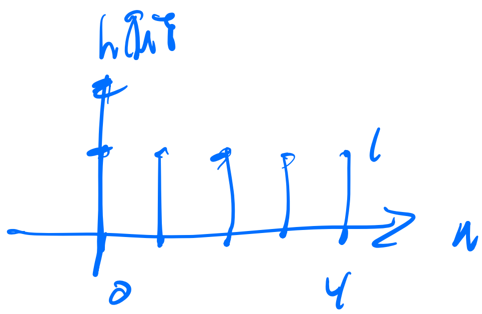
N rational fraction $\rightarrow$ periodic, period $\neq N$

N irrational $\rightarrow$ not periodic

CONVOLUTION EXAMPLES

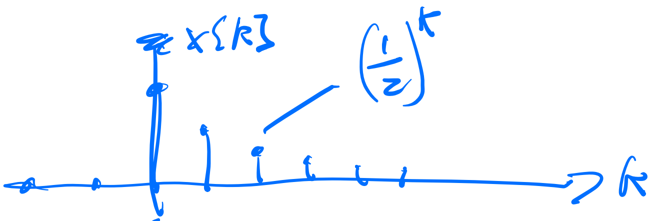
$x[n] = \left(\frac{1}{2}\right)^n u[n]$

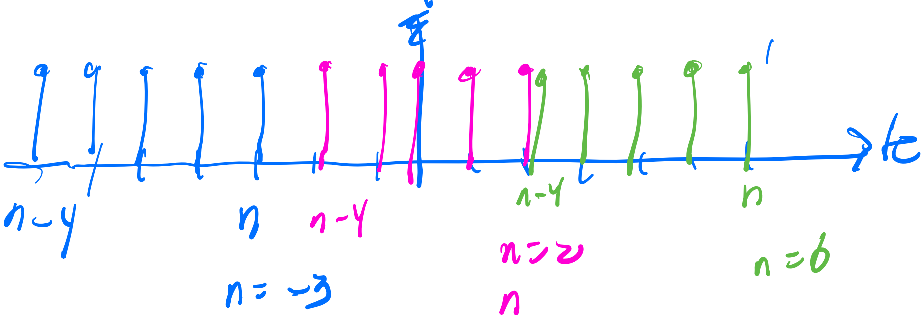




$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$





For $n < 0$, $y[n] = 0$

For $0 \le n \le 4$ $\quad y[n] = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} = 2 - \left(\frac{1}{2}\right)^n$

For $n \ge 4$, $y[n] = \sum_{k=n-4}^{n} \left(\frac{1}{2}\right)^k$

Let $l = k-(n-4)$

$k = l + (n-4)$

$$y[n] = \sum_{l=0}^{4} \left(\frac{1}{2}\right)^{l+(n-4)} = \left(\frac{1}{2}\right)^{n-4} \sum_{l=0}^{4} \left(\frac{1}{2}\right)^{l}$$

$$= \left(\frac{1}{2}\right)^{n-4} \cdot \frac{1-\left(\frac{1}{2}\right)^5}{1-\frac{1}{2}} = \left(\frac{1}{2}\right)^{n-4}\left(2-\left(\frac{1}{2}\right)^4\right)$$

Ex 2 $\quad x[n] = \cos(\omega_0 n), \quad h[n] = u[n]$

$$y[n] = \sum_{k=0}^{n} \cos(\omega_0 k)$$