3/31/23 RECITATION Q2A

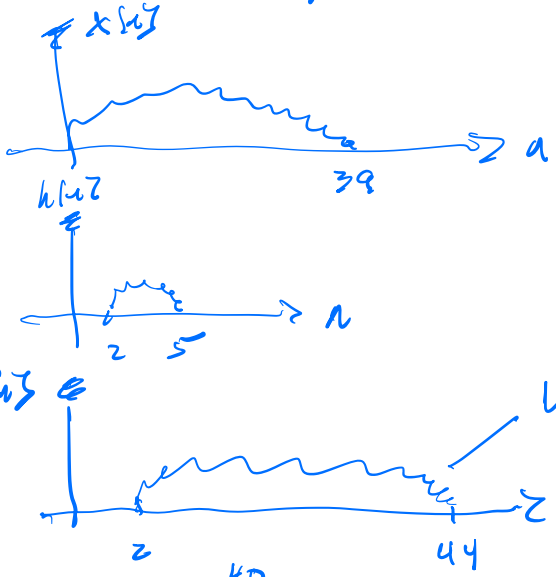
REVIEW OF Q2 FROM 2022

QUESTION 2

$x[n] \neq 0$, $0 \leq n \leq 39$

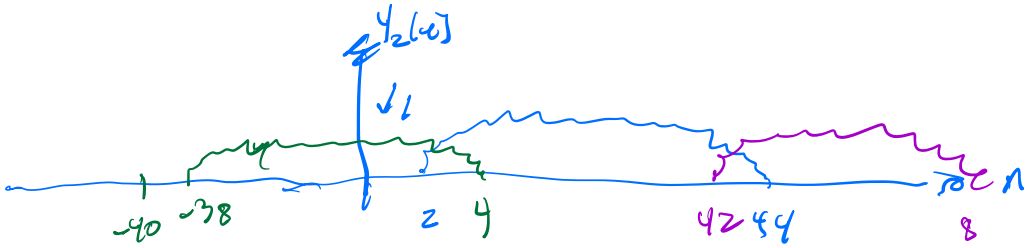
$h[n] \neq 0$, $2 \leq n \leq 5$

(a) POSSIBLE VALUES OF $y[n]$?



(b) $y_2[n] = x[n] \overset{40}{\circledast} h[n]$

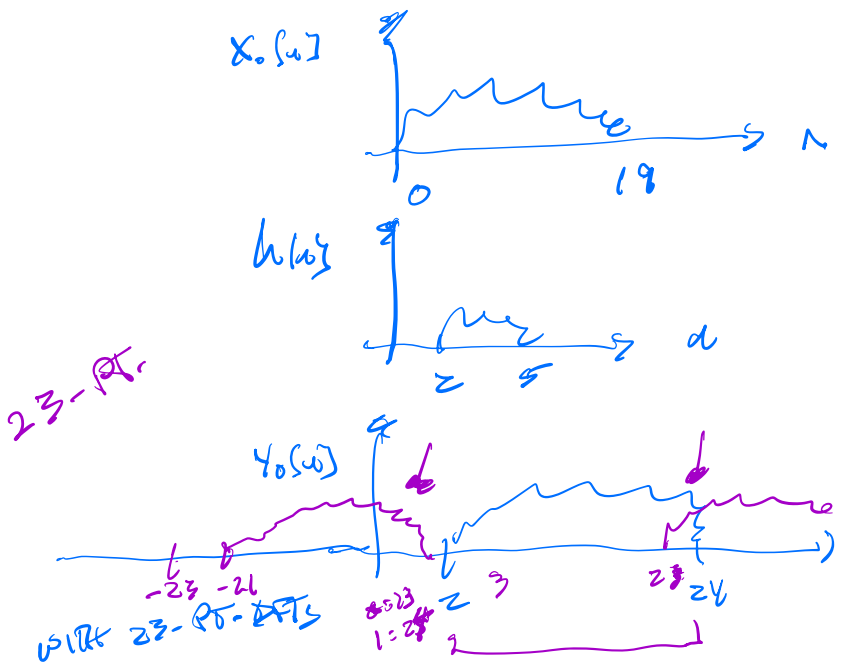
FOR WHICH n DOES $y[n] = y_2[n]$?



$y_2[n] = y[n]$ $\quad 5 \leq n \leq 39$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk},$$

(c) OVERLAP ADD



How many sections? 2

COMPLETE SOLUTION

Let $x_0[n] = x[n]$, $0 \le n \le 22$

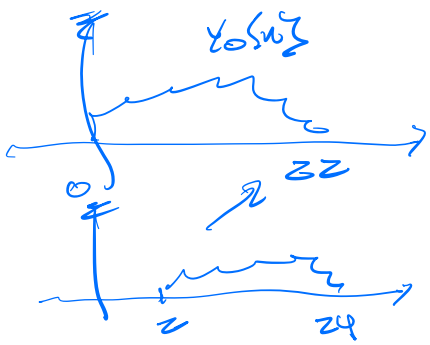
$x_0[n] \overset{23}{\Leftrightarrow} X_0[k]$

$h[n] \overset{23}{\Leftrightarrow} H[k]$

$Y_0[k] = X_0[k]\, H[k]$

$y_0[n] \overset{23}{\Leftrightarrow} Y_0[k]$

~~$y_0[n] = y_0[n-23]$~~



$x_1[n] = x[n+23]$

$x[23] \to x[0]$ $\quad 0 \le n \le 20$

COMPUTE $x_1[n] \circledast h[n]$

$y_1[n]$ CIRC. SHIFT TO THE RIGHT BY 2 SAMPLES

$$y[n] = y_0[n] + y_1[n-2]$$

(a) MARK INPUTS

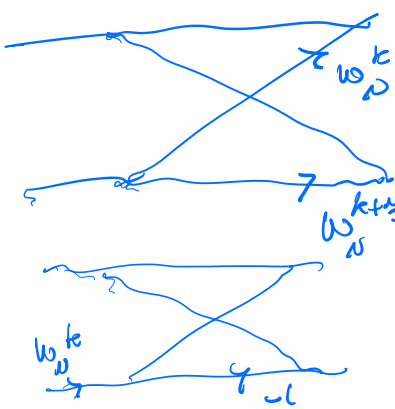
(b) MULTS

(c) MULTS NOT $\pm 1, \pm j$?

$$X[k] = \sum_{\substack{n=0 \\ n \text{ EVEN} \\ n=2r}}^{\frac{N}{2}-1} x[n] W_N^{nk} + \sum_{\substack{n=0 \\ n \text{ ODD} \\ n=2r+1}}^{\frac{N}{2}-1} x[n] W_N^{nk}$$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] \underbrace{W_N^{2rk}}_{W_{\frac{N}{2}}^{rk}} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] \underbrace{W_N^{(2r+1)k}}_{W_{\frac{N}{2}}^{rk} \cdot W_N^k}$$

$$= \underset{\text{of EVENS}}{\text{4-PT. DFT}} + W_N^k \underset{\text{of ODDS}}{\text{4-PT. DFT}}$$



$W_N^0 = 1$

W_N^1

$W_N^2 = -j$

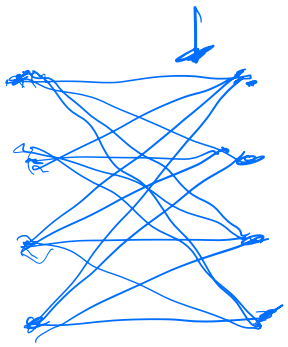
W_N^3

$W_N^4 = -1$

W_N^5

$W_N^6 = +j$

W_N^7



W_N^{nk}

$W_4^0\ W_4^0\ W_4^0\ W_4^0 = 1, 1, 1, 1$

$W_4^0\ W_4^1\ W_4^2\ W_4^3 = 1, -j, -1, j$

$W_4^0\ W_4^2\ W_4^4\ W_4^6 = 1, -1, 1, -1$

$W_4^0\ W_4^3\ W_4^6\ W_4^9 = 1, j, -1, -j$

$q_i = \frac{N}{p_i}$ $\quad X[k] = \sum_{l=0}^{p_i-1} \sum_{n=0}^{q_i-1} x[p_i n + l]\, W_N^{(p_i n + l)k}$

$= \sum_{l=0}^{p_i-1} W_N^{lk} \sum_{n=0}^{q_i-1} x[p_i n + l]\, W_{q_i}^{nk}$

$W_N^{p_i n k} \cdot W_N^{lk} = W_N^{nk} W_N^{lk}$

q_i-PT. DFT

Q 5 $\quad y[n] = -3y[n-1] - 4y[n-2] - 2y[n-3]$

$+ x[n] - 3x[n-1] + 2x[n-2]$

ZEROS @ $z = 0, 1, 2$

POLES @ $z = -1, -1+j, -1-j$

(a) 1 DFII

2 DFII TRANSPOSED

3 CASCADE FORM

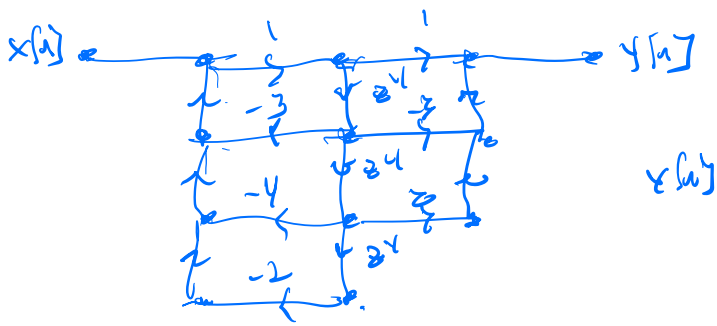
$$y[n] = -3y[n-1] - 4y[n-2] - 2y[n-3] + x[n] + 3x[n-1] + 2x[n-2]$$

$$y[n] + 3y[n-1] + 4y[n-2] + 2y[n-3] = x[n] + 3x[n-1] + 2x[n-2]$$

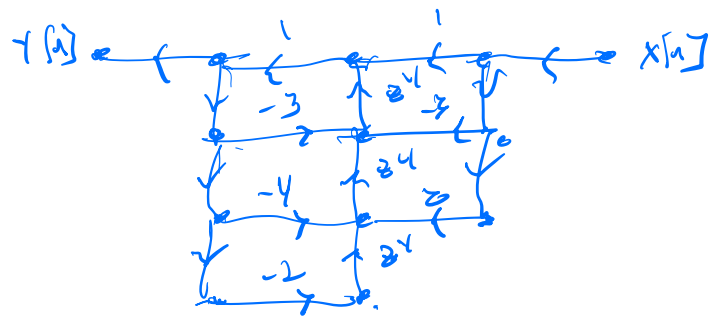
$$Y(z)(1 + 3z^{-1} + 4z^{-2} + 2z^{-3}) = X(z)(1 + 3z^{-1} + 2z^{-2})$$

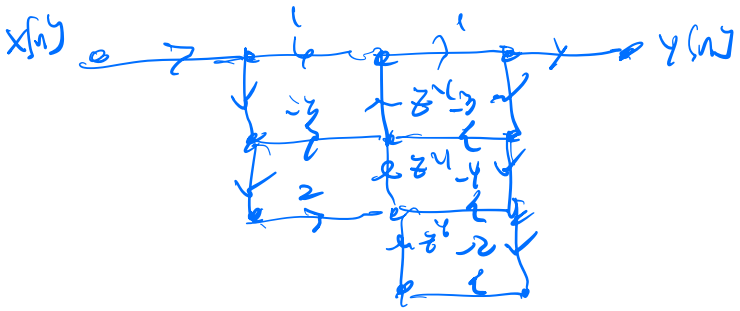
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + 3z^{-1} + 4z^{-2} + 2z^{-3}} = \frac{\sum_{l=0}^{M} b_l z^{-l}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{l=0}^{M} b_l x[n-l]$$



DF II – Transposed

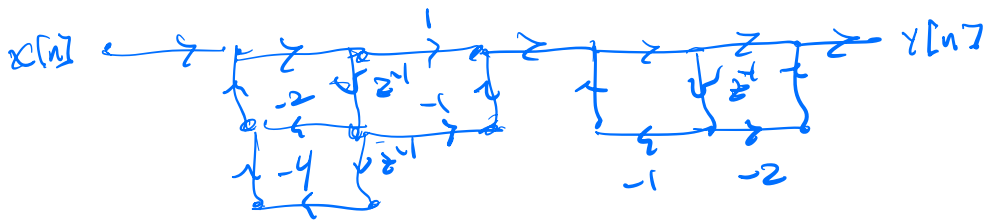




ZEROS @ $z = 0, 2$

POLES @ $z = -1, -1+j, -1-j$

CASCADE FORM



$$H_1(z) = \frac{(1-c_1 z^{-1})(1-c_2 z^{-1})}{(1-d_1 z^{-1})(1-c_1 z^{-1})} = \frac{1-z^{-1}}{1+2z^{-1}+4z^{-2}} = \frac{z(z-1)}{z^2+2z+4}$$

$$H_2(z) = \frac{1-2z^{-1}}{1+z^{-1}}$$

$$H(z) = \frac{(z)(z-1)}{(z+1)(z-2)}$$

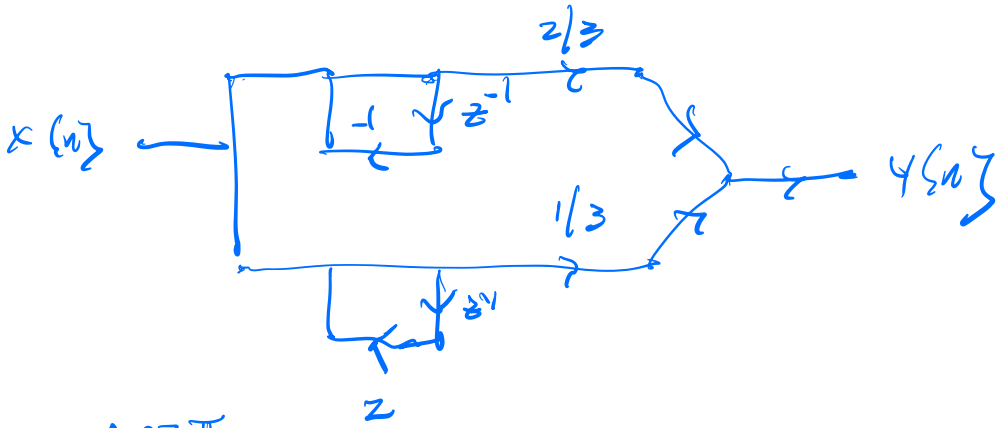
FIRST-ORDER SECTIONS

$$= \frac{1-z^{-1}}{(1+z^{-1})(1-2z^{-1})} = \frac{A_1}{1+z^{-1}} + \frac{A_2}{1-2z^{-1}}$$

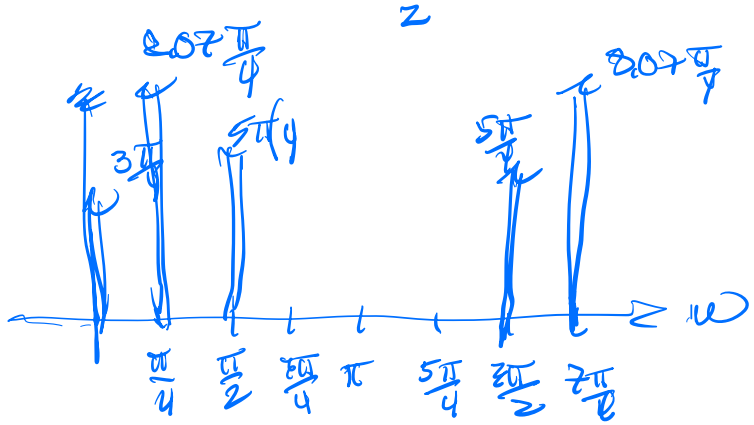
$$A_1 = \left.\frac{1-z^{-1}}{1-2z^{-1}}\right|_{z=-1} = \frac{1+1}{1+2} = \frac{2}{3}$$

$$A_2 = \left.\frac{1-z^{-1}}{1+z^{-1}}\right|_{z=2} = \frac{\frac{2}{2}-1}{\frac{2}{2}+1} = \frac{1/2}{3/2} = \frac{1}{3}$$

$$H(z) = \frac{2/3}{1+z^{-1}} + \frac{1/3}{1-2z^{-1}}$$


x[n]
-1
z^{-1}
2/3
1/3
y[n]
z^{-1}
z

Q.1


2.07
2.07
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$\frac{3\pi}{2}$
$\frac{7\pi}{8}$