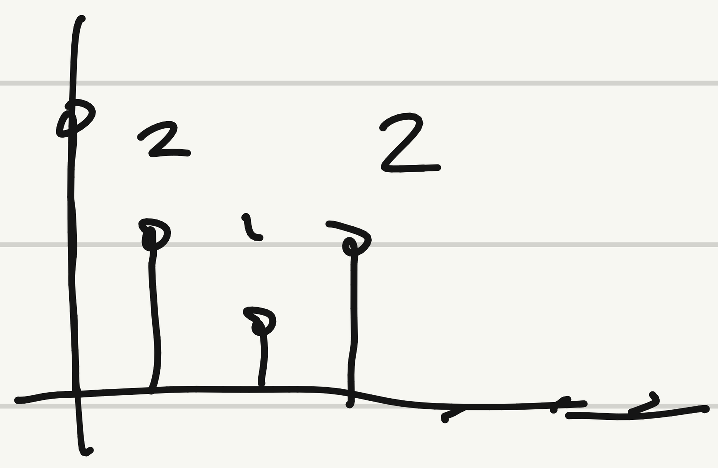
2.)

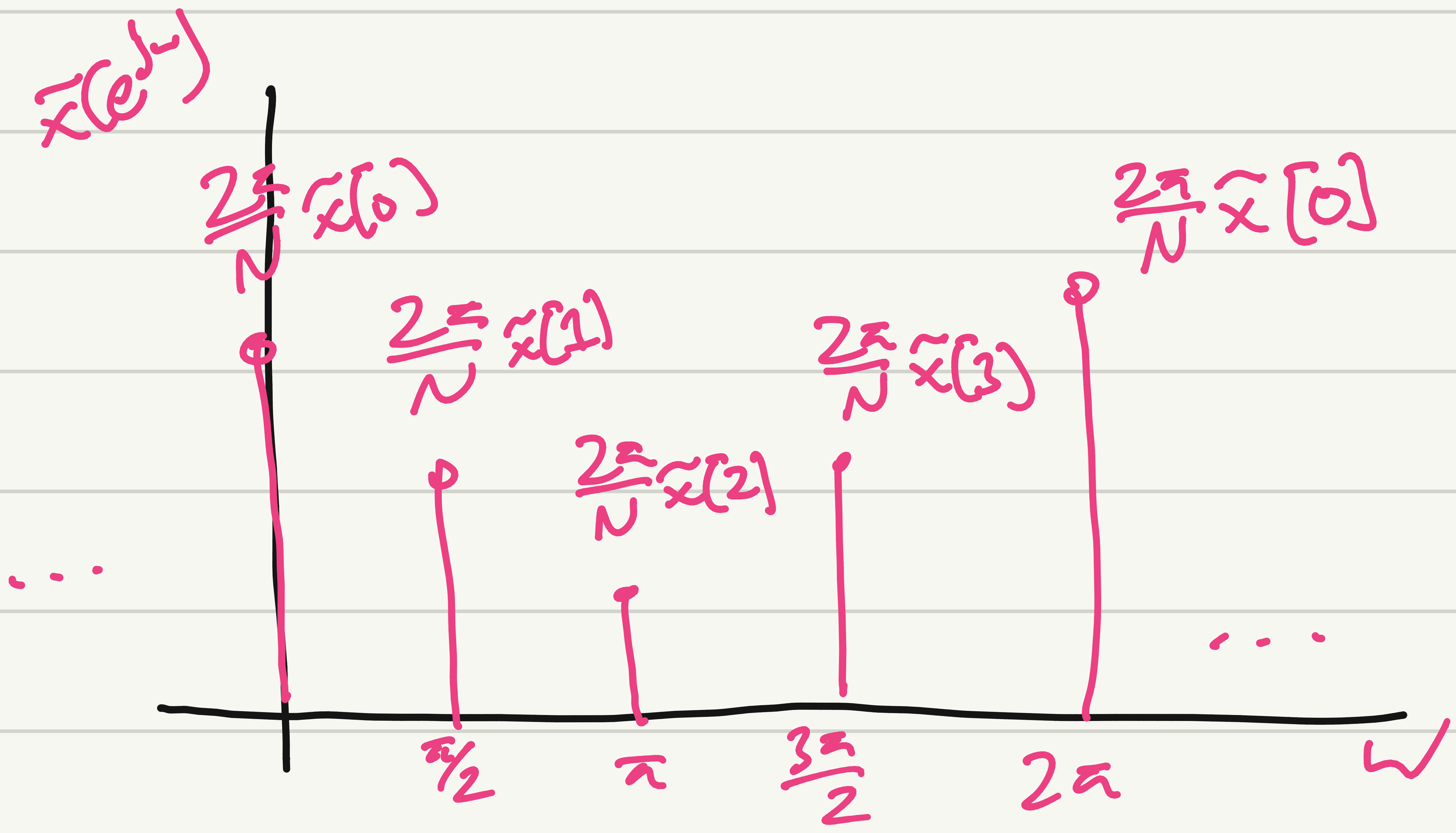
$\tilde{x}[n]$ has $N=4$, where $\tilde{x}[0]=1$,
$\tilde{x}[1]=2$
$\tilde{x}[2]=1$
$\tilde{x}[3]=2$

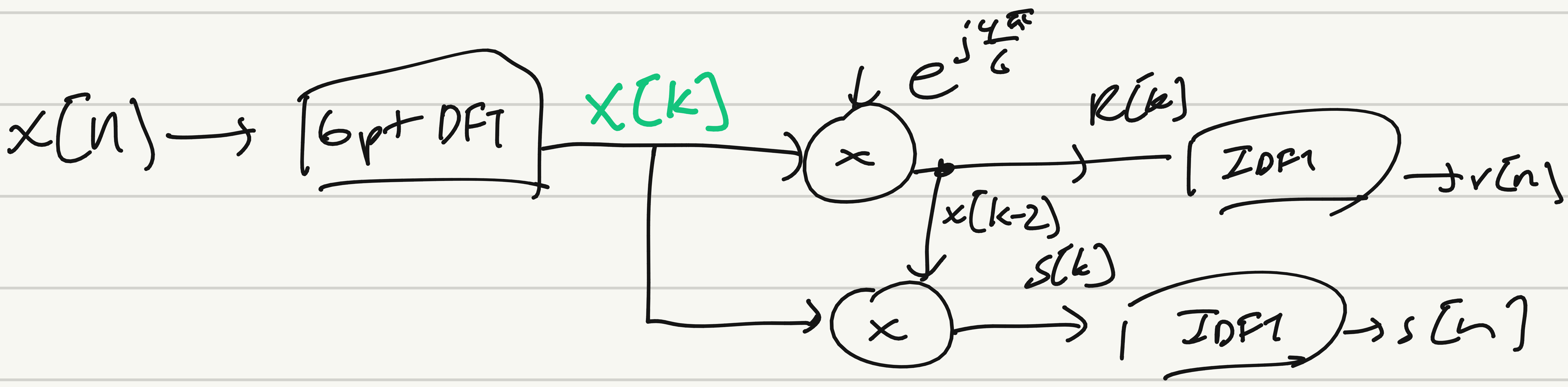


DTFT in terms of DFT:

$$\tilde{X}(e^{j\omega}) = \sum_{r=-\infty}^{\infty} \sum_{k=0}^{N-1} \frac{2\pi}{N} \tilde{X}[k]\, \delta\left(\omega - \frac{2\pi k}{N} - 2\pi r\right)$$

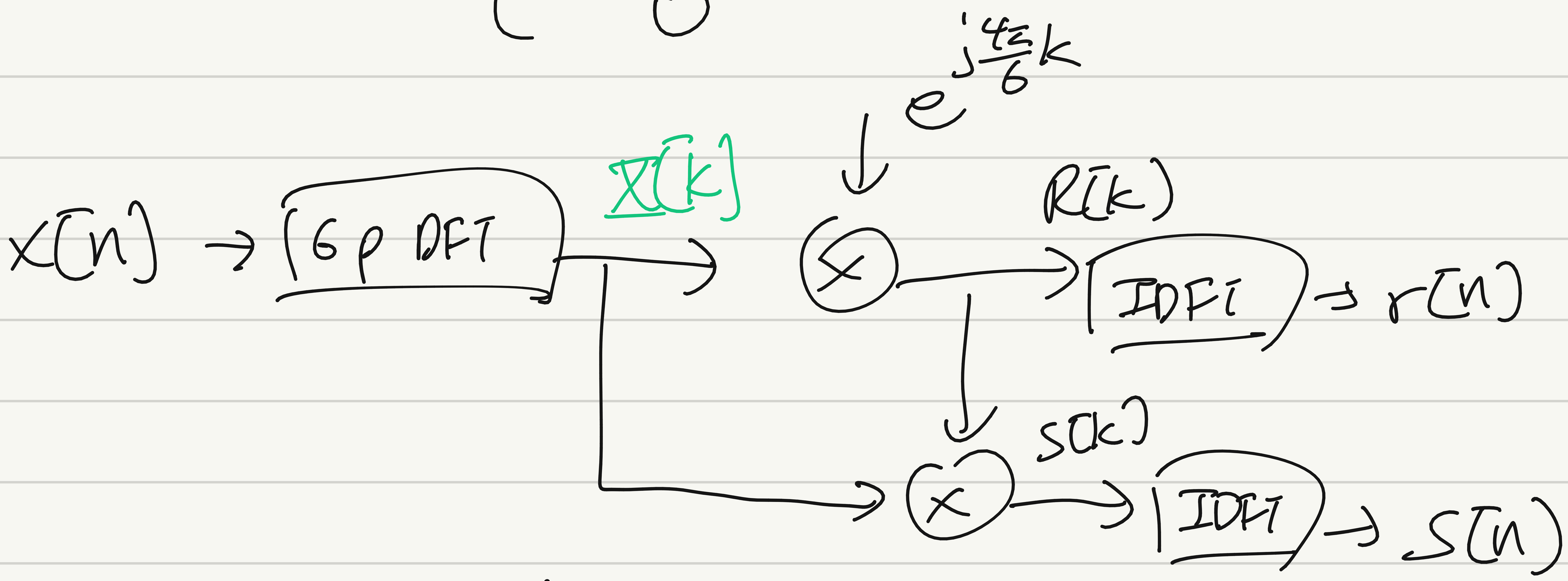
a.) in this case, $N=4$





2.)

b.) $x[n] = \begin{cases} \tilde{x}[n] & 0 \le n \le 3 \\ 0 & \end{cases}$



$R[k] = X[k]\, e^{j\frac{4\pi}{6}k}$

$\Updownarrow$

$\underline{R[k]} = X[k]\, W_6^{2k}$

$= X[k+2]$

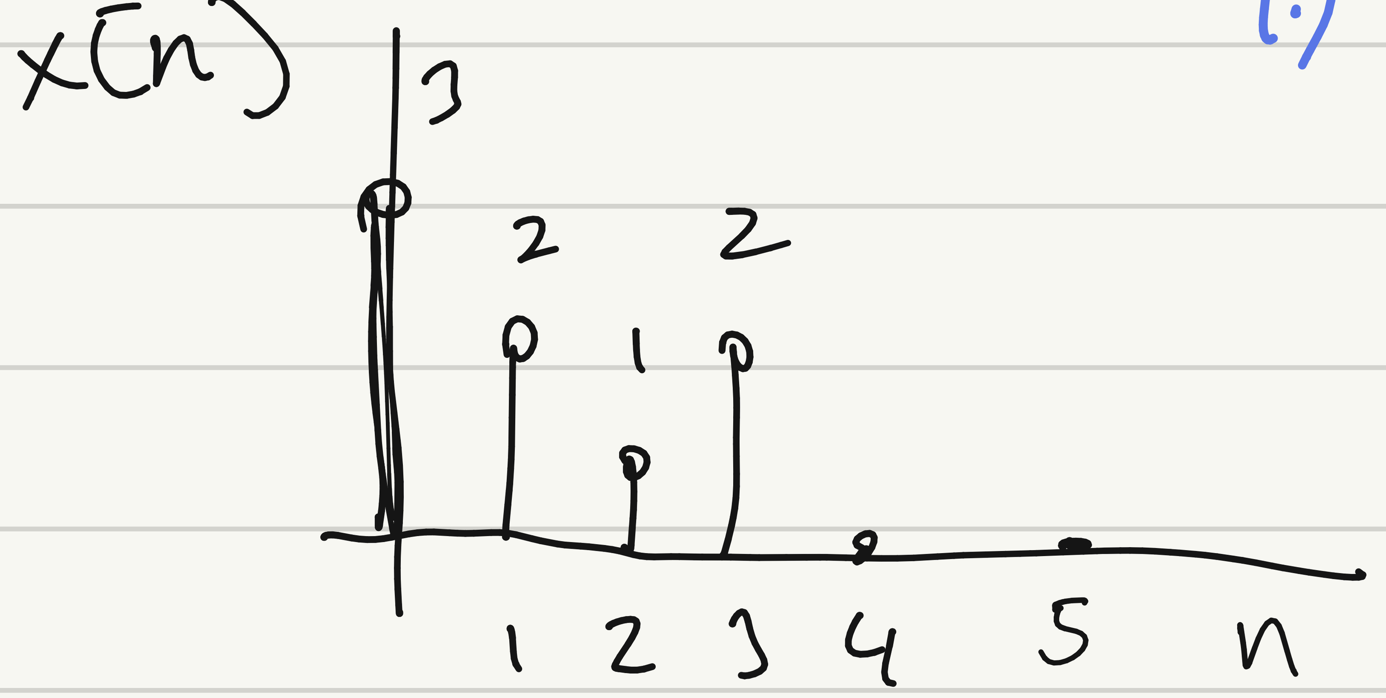
$\tilde{x}[n-m] \Leftrightarrow W_N^{km}\, \tilde{X}[k]$

in this case, "m" = -2

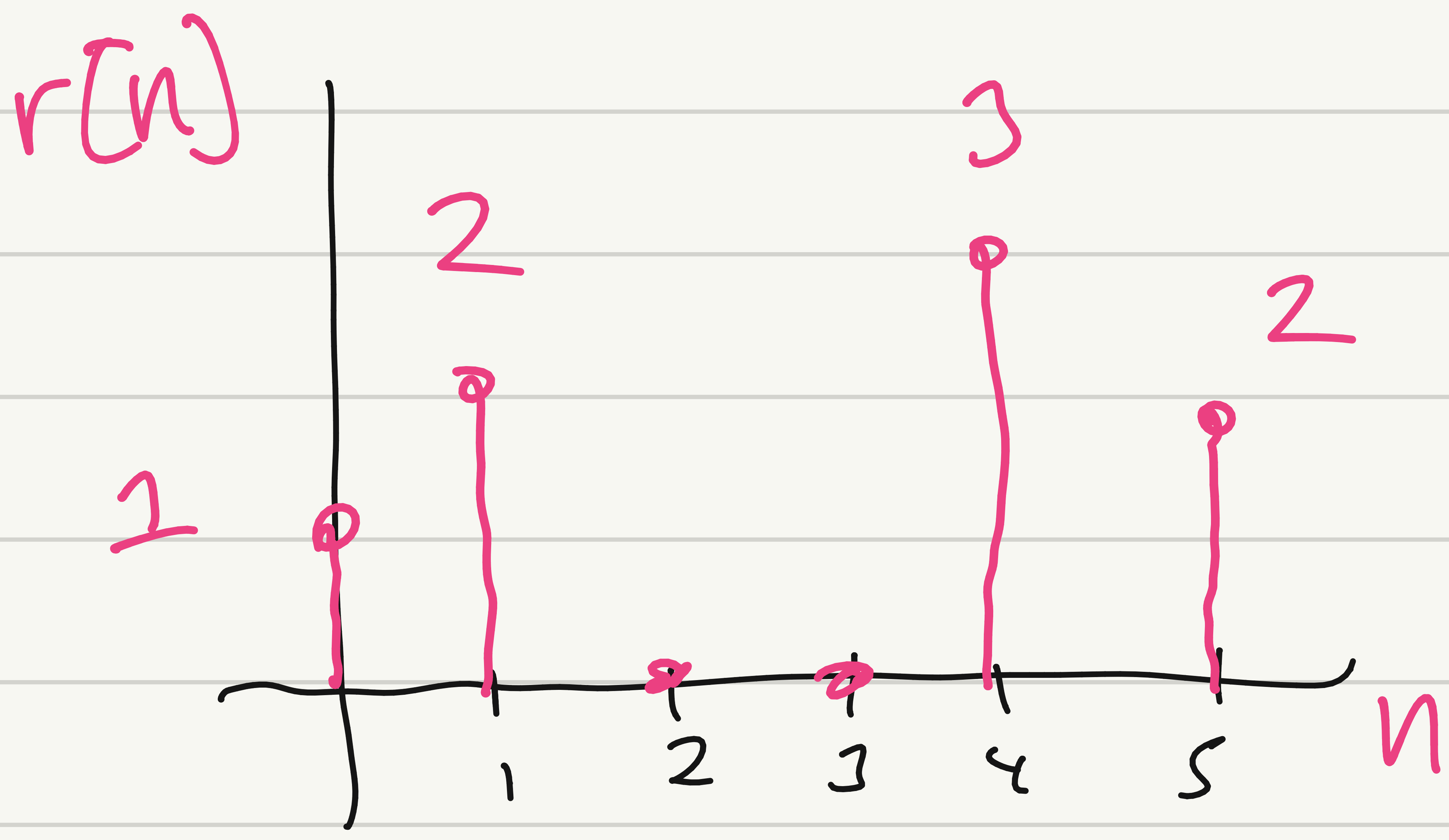
$\tilde{x}[n-m] \Leftrightarrow e^{-j\frac{2\pi}{N}km}\, \tilde{X}[k]$

DFS, DFT notes on website

R[k] is x[k] shifted left by 2 samples

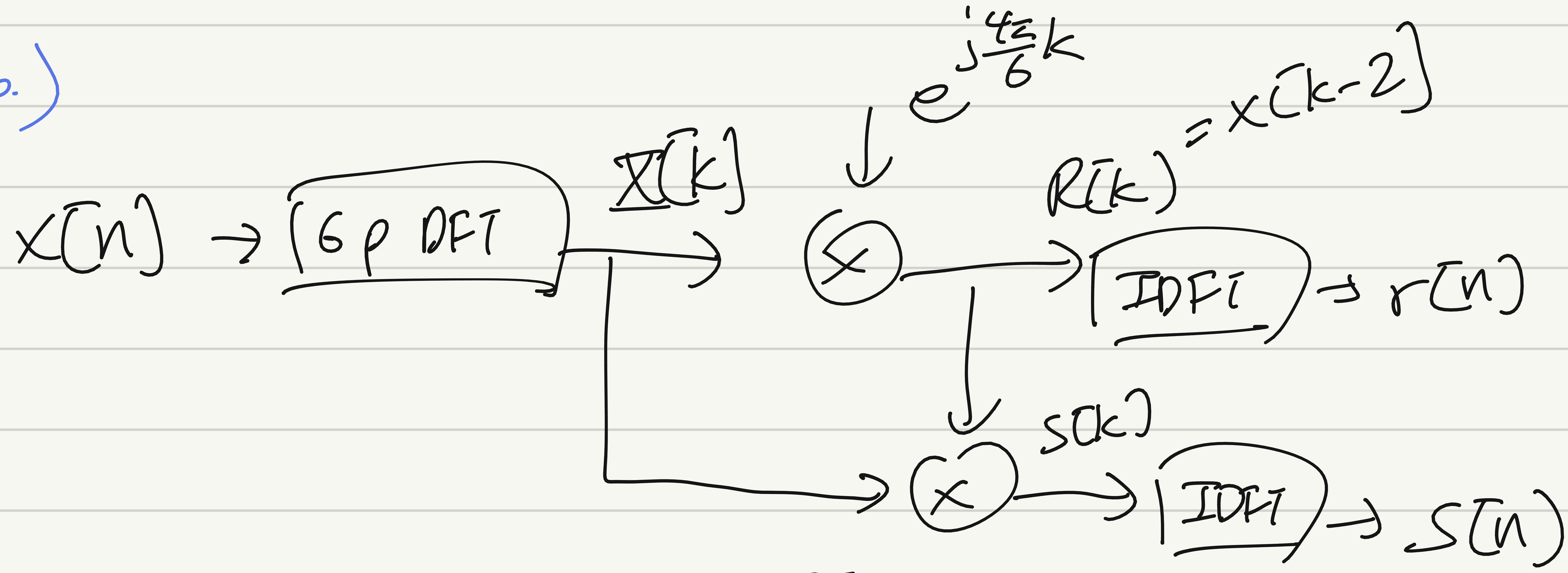


1.)



2.)

b.)



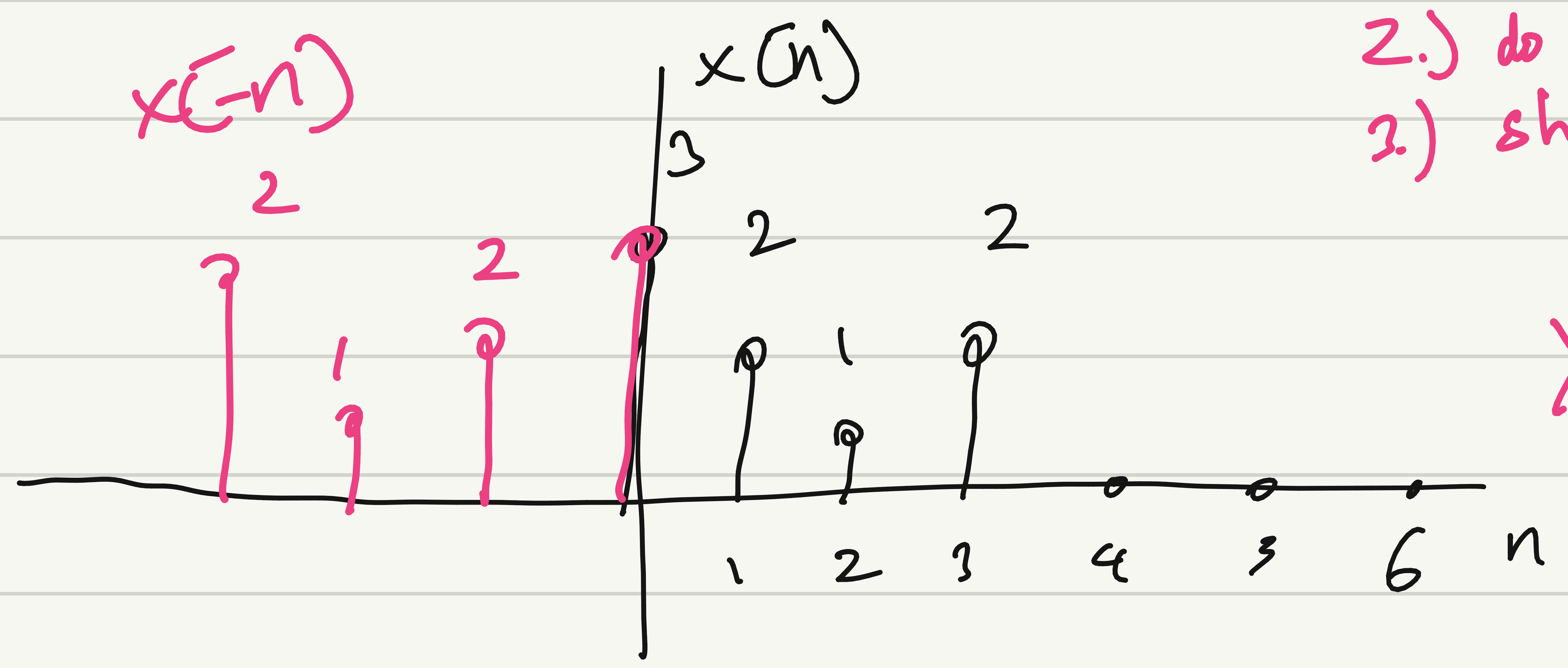
$$S[k] = X[k] \cdot X[k+2] \overset{IDFT}{\Longrightarrow} s[n]$$

$$\Updownarrow$$

$$s[n] = x[n] \circledast x[n+2]$$
$$= (x[n] \circledast x[n]) \circledast \delta[n+2]$$

* flip & drag (convolution)

1.) do linear conv
2.) do circular conv (check aliasing)
3.) shift by 2 samples!



$$y[n] = \sum_{k=-\infty}^{\infty} x[n]x[k-n]$$

n	$x[n] * x[n]$	$x[n] \circledast_6 x[n]$
0	9	13
1	3(2) + 2(3) = 12	12
2	3(1) + 2(2) + 3(1) = 3+4+3 = 10	10
3	3(2) + 1(2) + 2(1) + 3(2) = 16	16
4	2(2) + (1)(1) + 2(2) = 9	9
5	1(2) + 2(1) = 4	4
6	2(2) = 4	

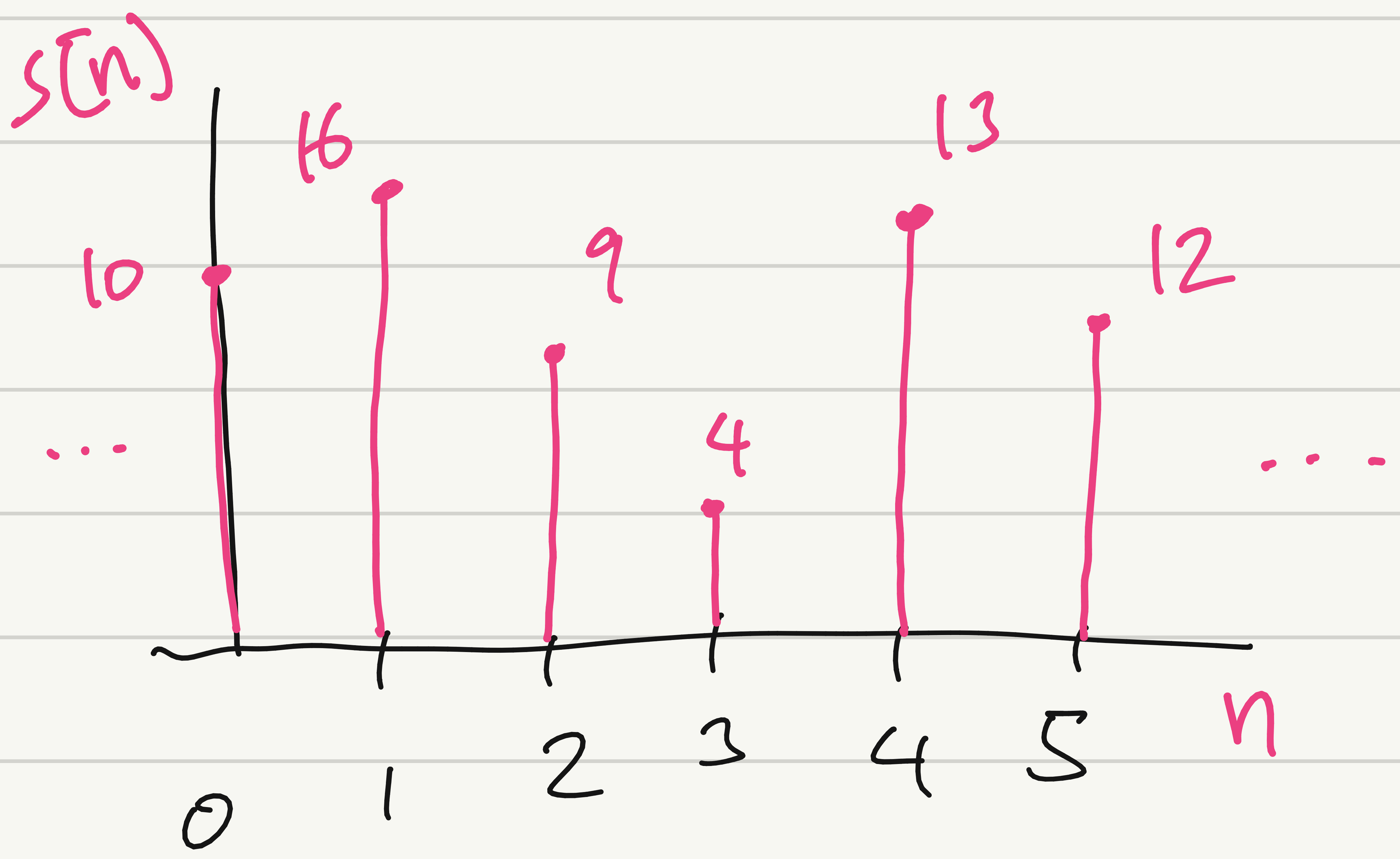
(see next page)

SHIFT BY 2

n	$x[n] * x[n]$	$x[n] \circledast_6 x[n]$	$x[n] \circledast_6 x[n] \circledast \delta[n+2]$
0	9	13	10
1	12	12	16
2	10	10	9
3	16	16	4
4	9	9	13
5	4	4	12
6	4		

aliasing!

$$s[n] = \left(x[n] \circledast_6 x[n]\right) \circledast \delta[n-2]$$



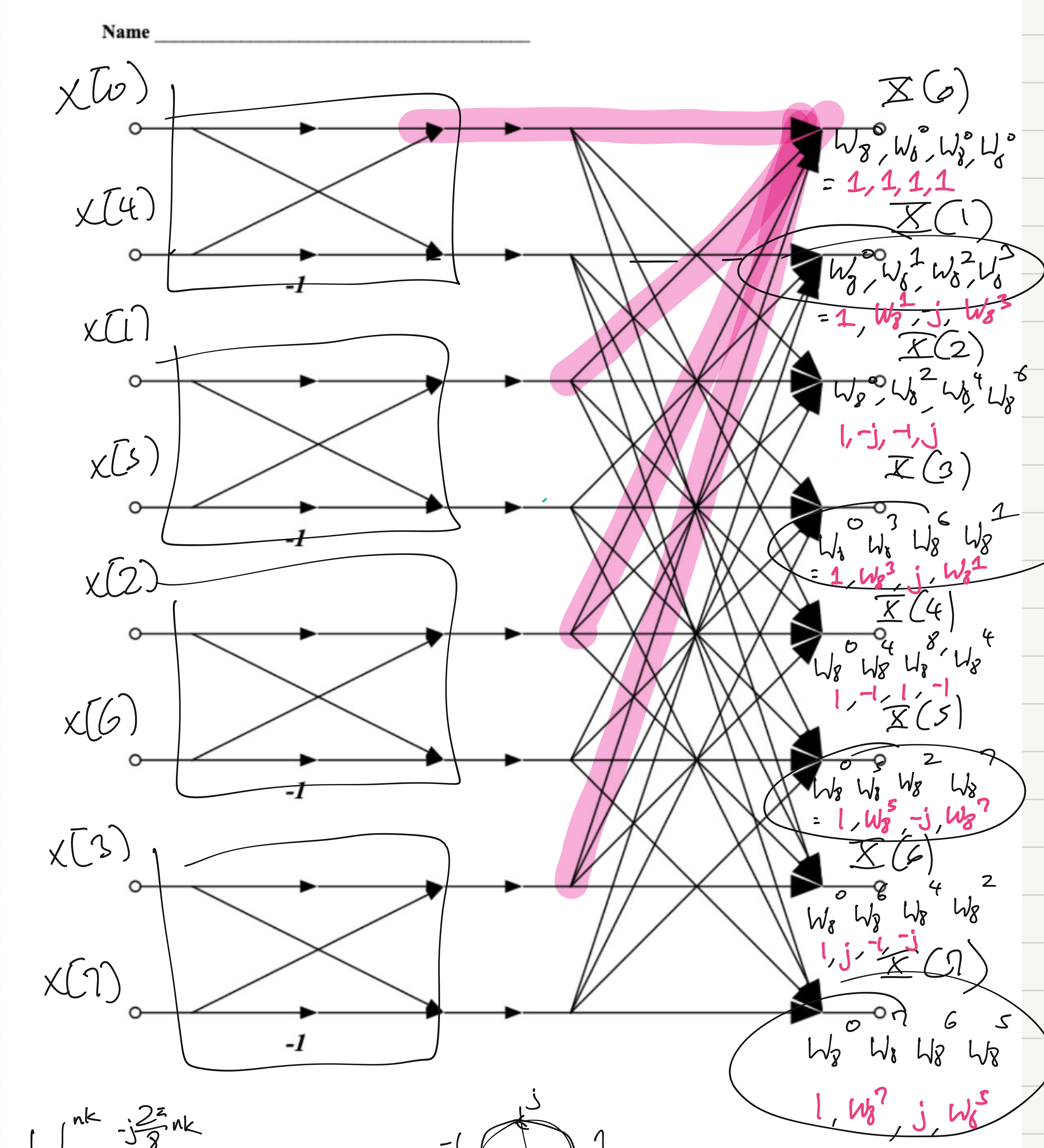
$$X[k] = \sum_{l=0}^{p-1} W_N^{lk} \sum_{r=0}^{q-1} x[pr+l]\, W_q^{rk}$$

$N = pq = 8$

$p = 4$

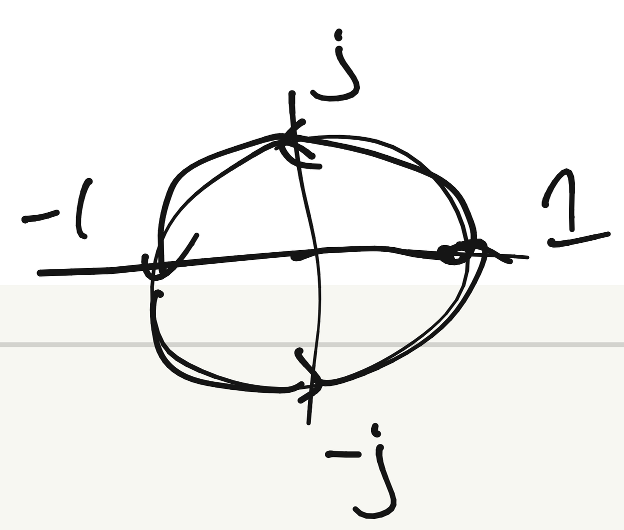
$q = 2$ point DFT blocks

$$X(k) = \sum_{l=0}^{3} W_8^{lk} \sum_{r=0}^{1} x[4r+l]\, W_2^{rk}$$



$W_8^{nk} = e^{-j\frac{2\pi}{8}nk}$

$e^{j\frac{2\pi}{N}k}$

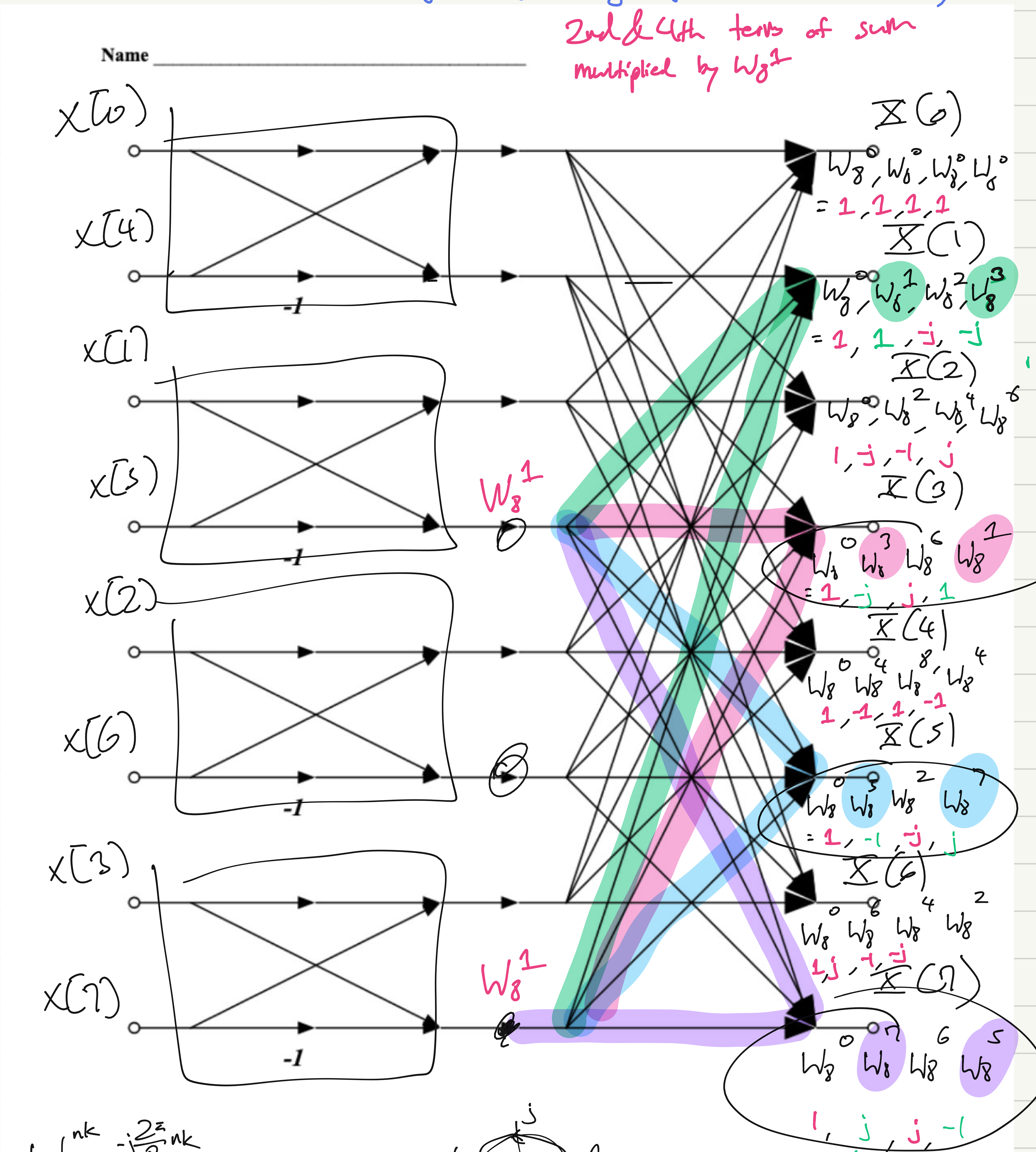


$$X[k] = \sum_{l=0}^{p-1} W_N^{lk} \sum_{r=0}^{q-1} x[pr+l] W_q^{rk}$$

note that: $W_8^3 = W_8^2 \cdot W_8^1 = -j \cdot W_8^1$

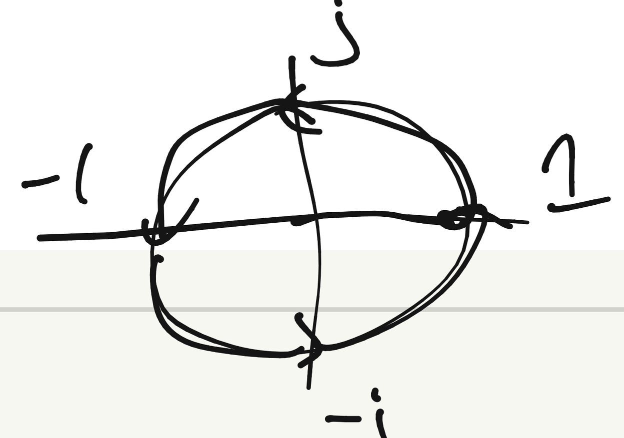
$W_8^5 = W_8^4 \cdot W_8^1 = -1 \cdot W_8^1$ (simplify further!)

$W_8^7 = W_8^6 \cdot W_8^1 = j \cdot W_8^1$



$W_8^{nk} = e^{-j\frac{2\pi}{8}nk}$

$e^{j\frac{2\pi}{N}k}$



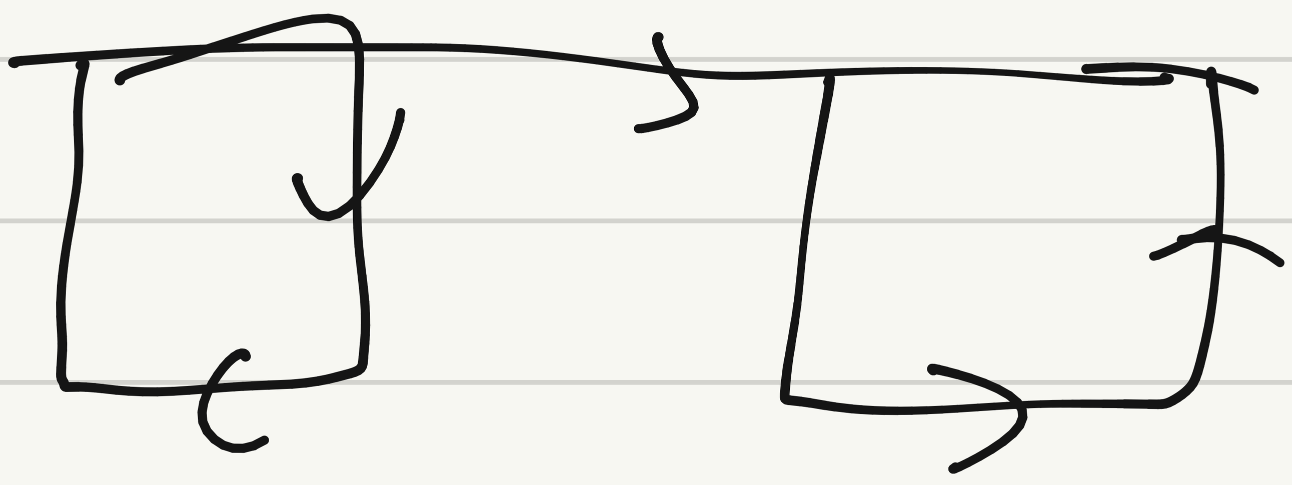
$W_8^6 \cdot W_8^1 = j$

$W_8^4 \cdot W_8^1 = -1$

4.)

$$H(z) = \frac{1 + 1.42z^{-1} - 0.125z^{-2} - 0.792z^{-3} - 0.25z^{-4}}{1 - \underline{1.25}z^{-1} + \underline{0.5}z^{-2} - \underline{0.0625}z^{-3}}$$

$$= \frac{80.3}{1-\frac{1}{2}z^{-1}} + \frac{-0.17}{(1-\frac{1}{2}z^{-1})^2} + \frac{-110}{1-\frac{1}{3}z^{-1}} + 44.7 + 4z^{-1}$$



Zeros

poles FEEDBACK

go FORWARDS

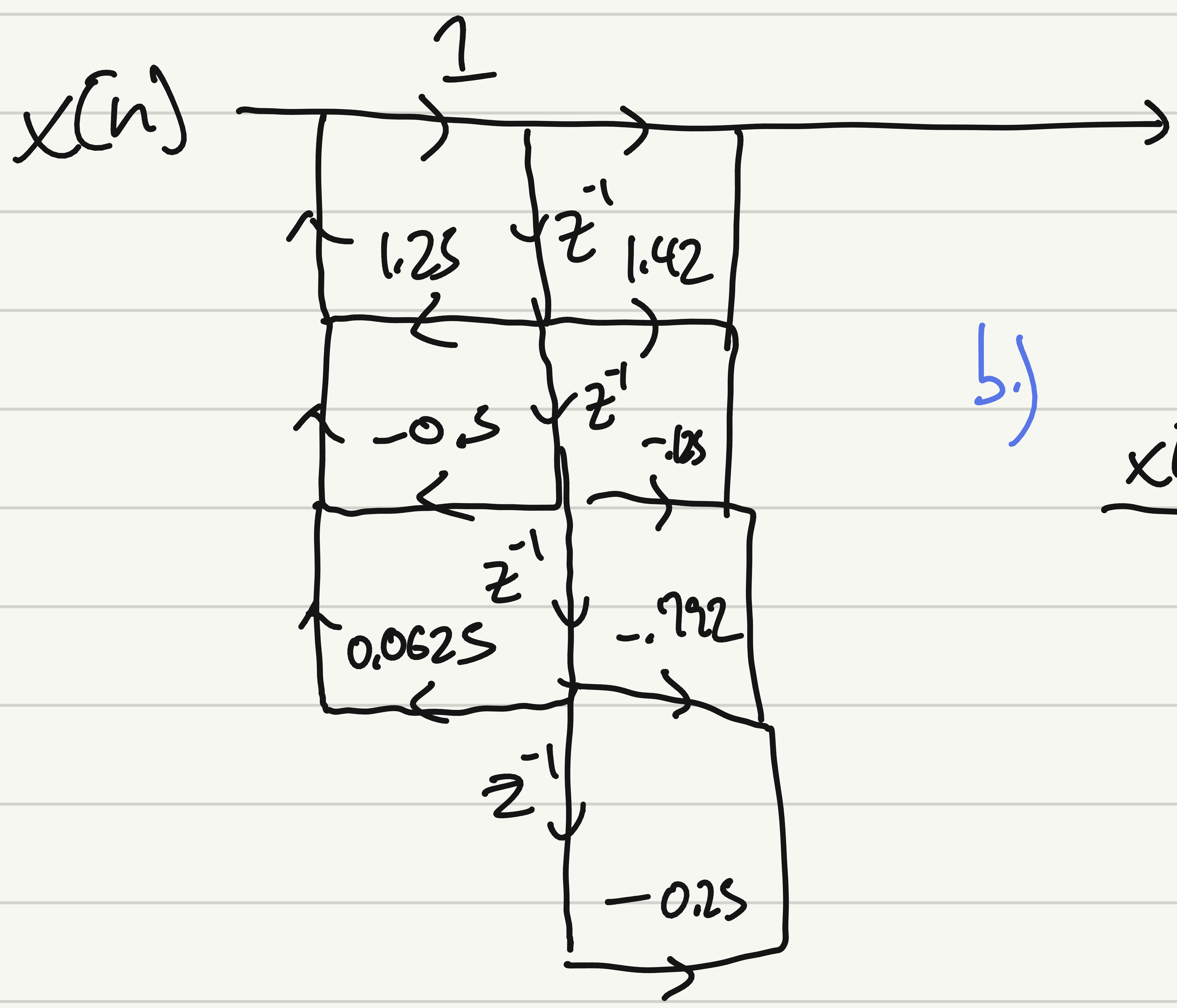
from:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{\underline{1 - \sum_{k=1}^{N} a_k z^{-k}}}$$

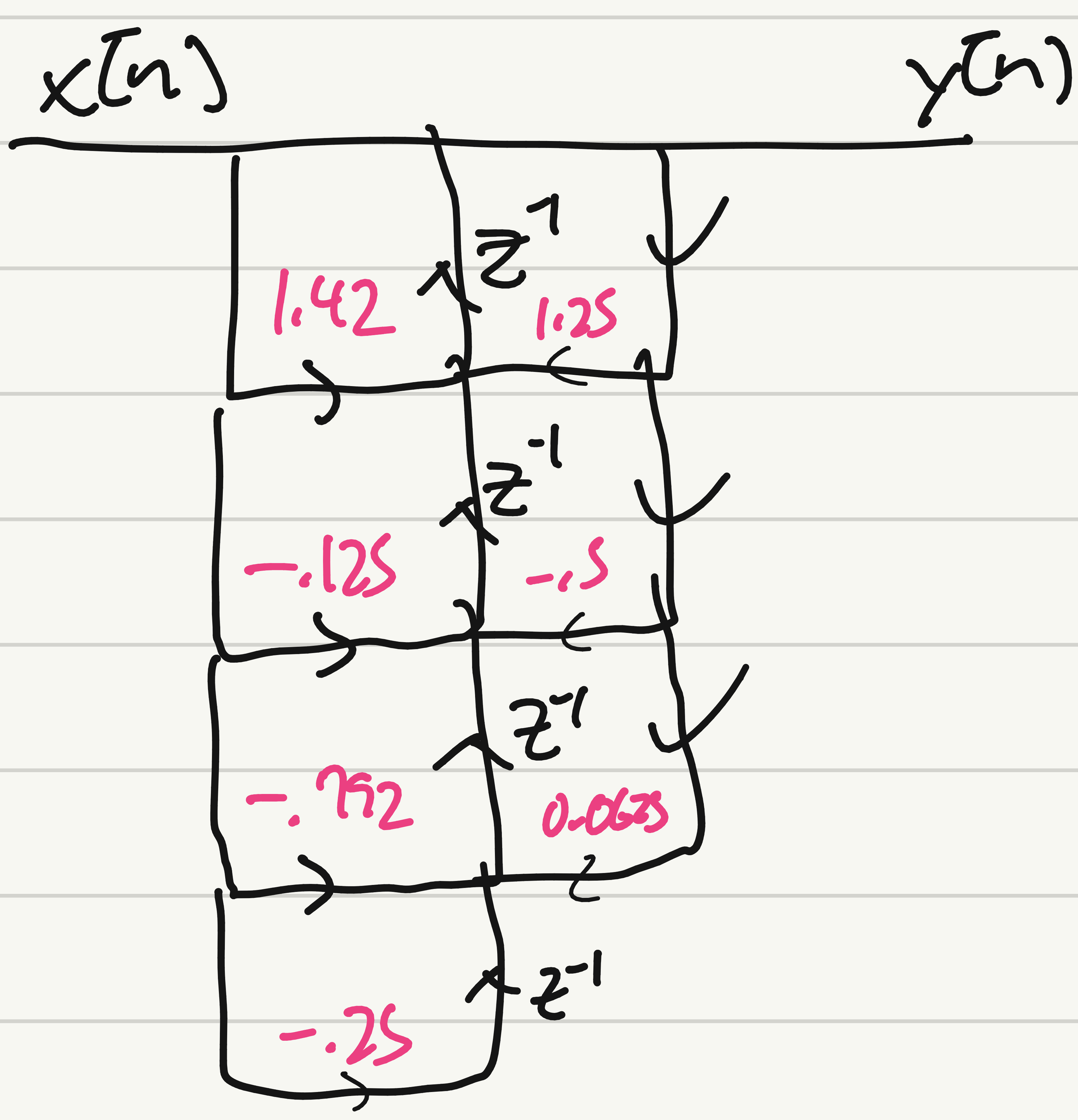
★ note sign flippage

$$Y[n] = \sum_{k=1}^{N} a_k Y[n-k] + \sum_{\ell=0}^{M} b_\ell x[n-\ell]$$

a.) Direct Form II



b.) TRANSPOSED



c.)

Combine these to make 2nd order term

$$H(z) = \frac{80.3}{1-\frac{1}{2}z^{-1}} + \frac{-0.7}{(1-\frac{1}{2}z^{-1})^2} + \frac{-110}{1-\frac{1}{3}z^{-1}} + 44.7 + 4z^{-1}$$

$$\frac{80.3}{1-\frac{1}{2}z^{-1}} + \frac{-110}{1-\frac{1}{3}z^{-1}}$$

$$= \frac{80.3(1-\frac{1}{3}z^{-1}) - 110(1-\frac{1}{2}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$$

$$= \frac{80.3 - \frac{80.3}{3}z^{-1} - 110 + 55z^{-1}}{1 - \frac{1}{3}z^{-1} - \frac{1}{2}z^{-1} + \frac{1}{6}z^{-2}}$$

$$= \frac{-29.7 - 28.2z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

hence,

$$H(z) = \frac{-29.7-28.2z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} + \frac{-0.7}{1-z^{-1}+\frac{1}{4}z^{-2}} + 44.7 + 4z^{-1}$$

each term ~ branch!

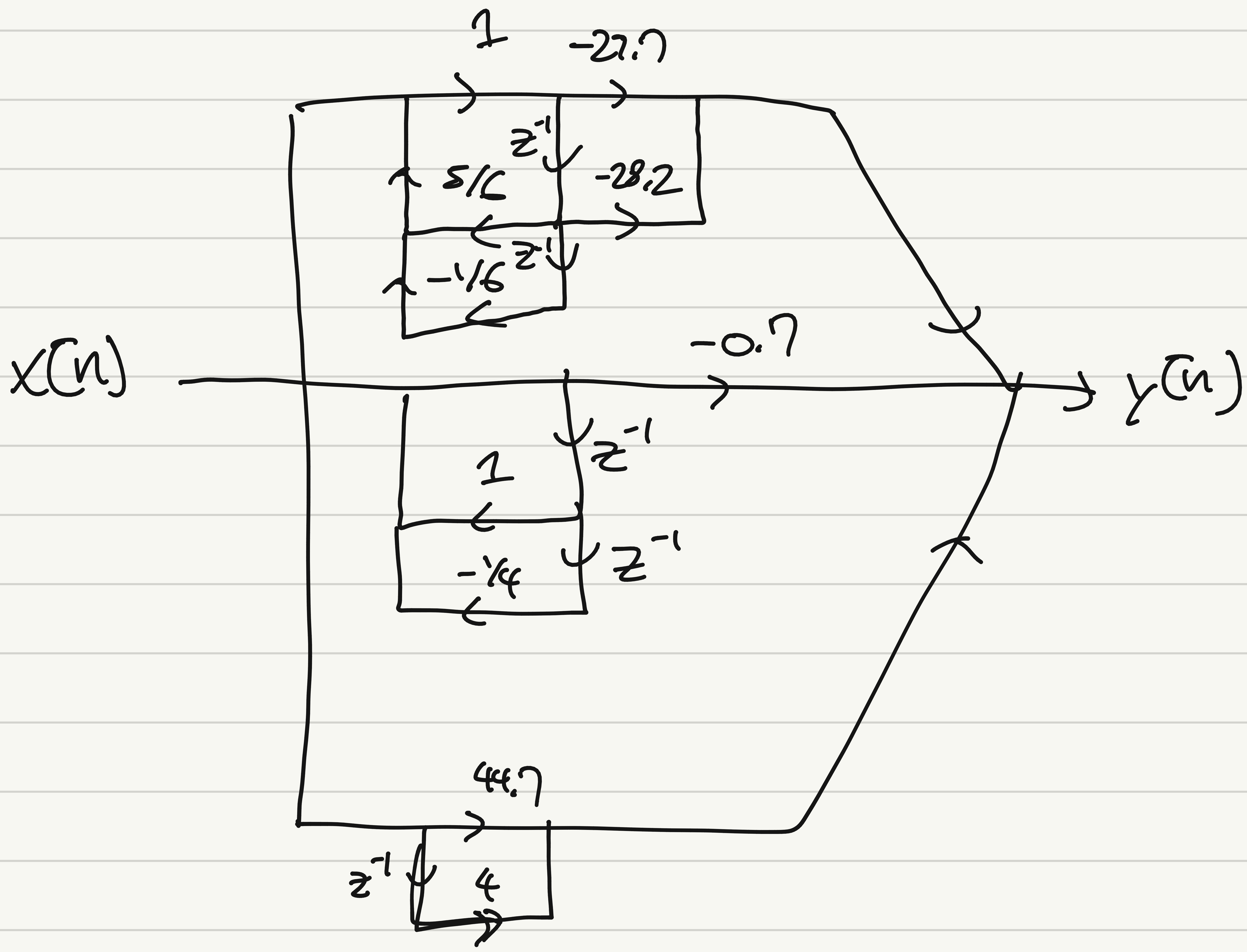
(see next page)

$$H(z) = \frac{-29.7 - 28.2z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} + \frac{-0.7}{1 - z^{-1} + \frac{1}{4}z^{-2}} + 44.7 + 4z^{-1}$$

1st branch

2nd branch

3rd branch



Cheatsheet items!

☆ Residues, partial Fracs

☆ IIR (direct form I, II
parallel, poles feedback
zero feed FORWARD))

☆ complex conj formula

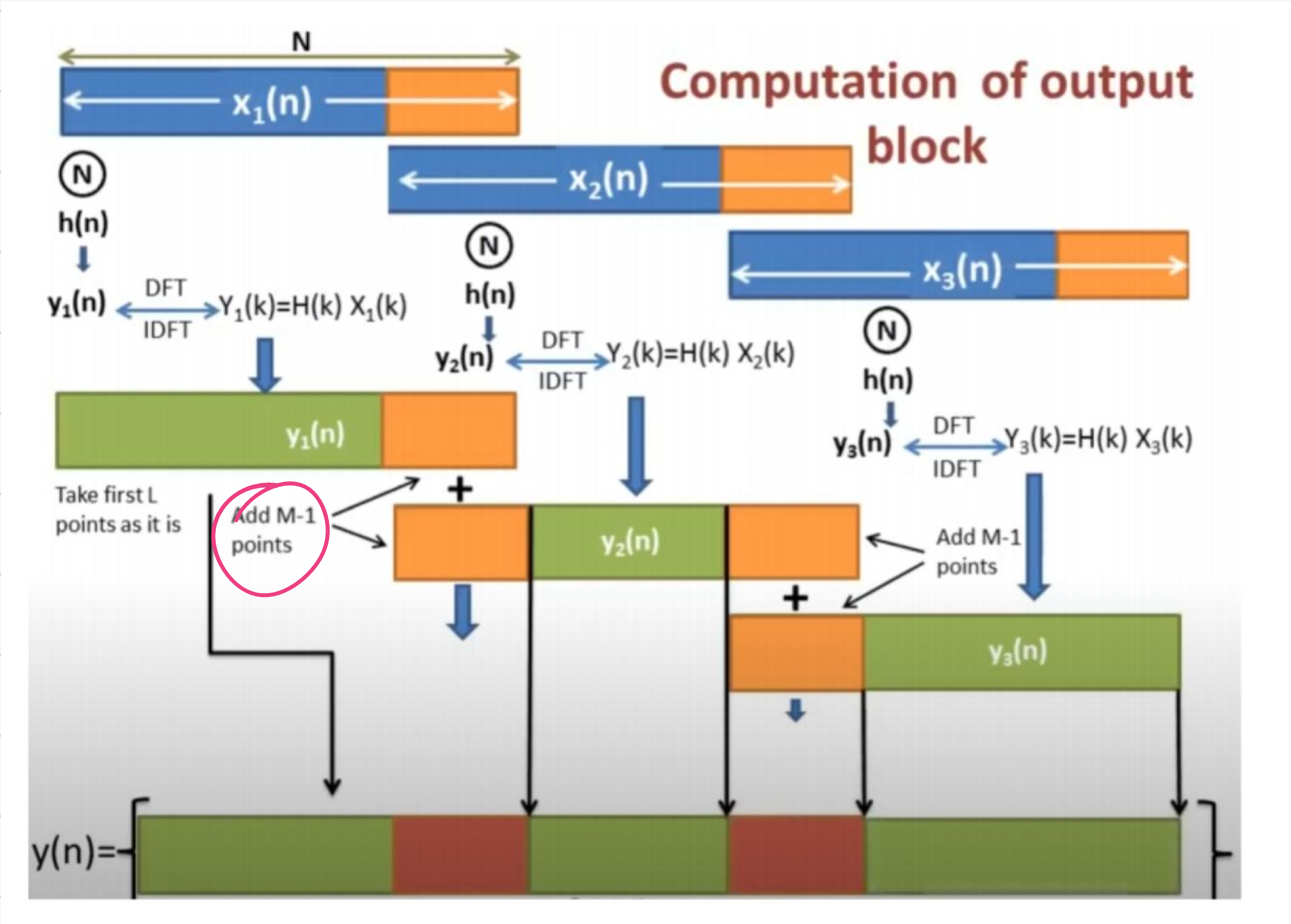
☆ FIR (linear phase)

☆ DFT, DTFS equations & time shift
props

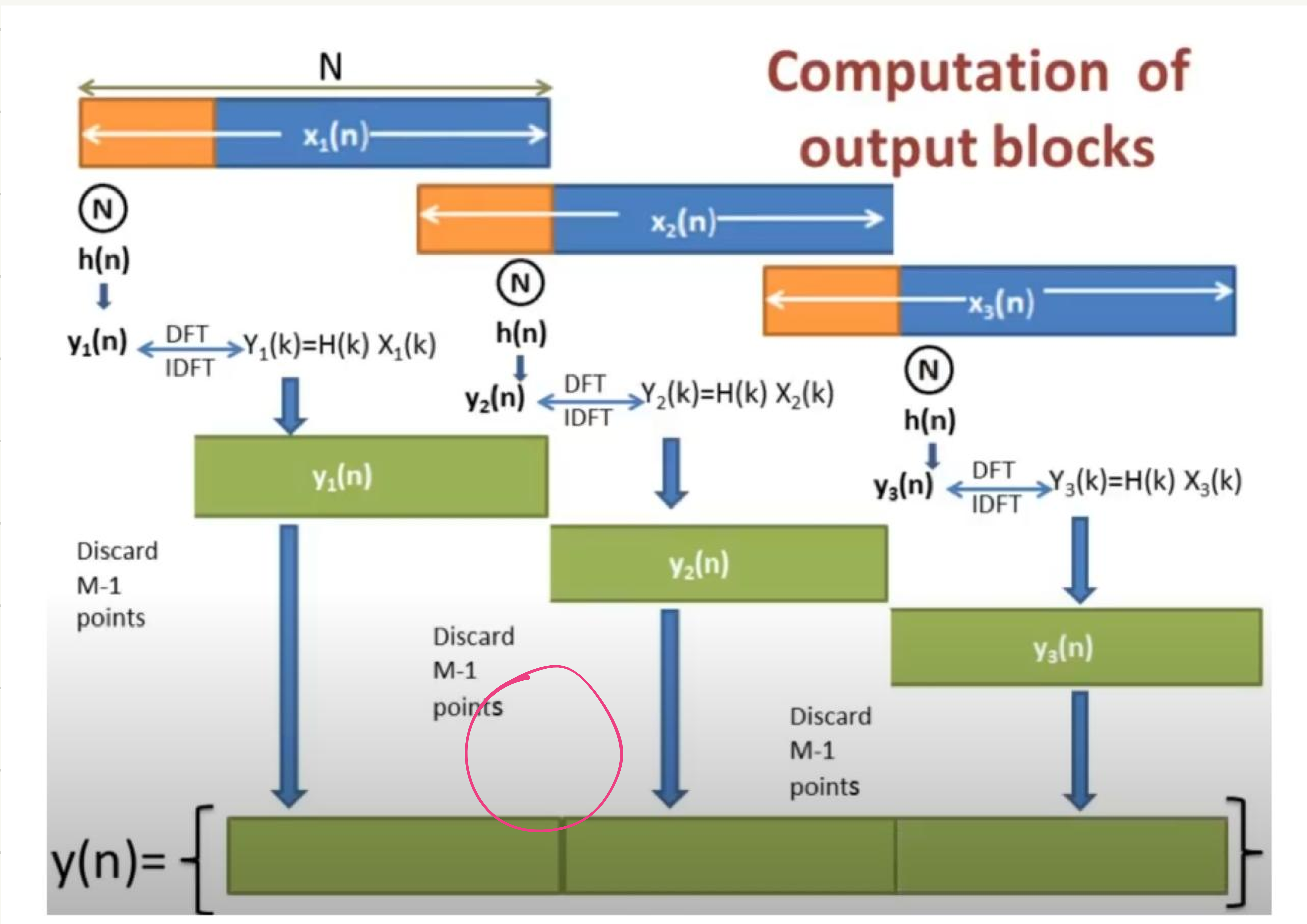
☆ Overlap Add, Save, # of multiplies (FFT Big O)

☆ FFT radix 2, non radix (p,q,s)
(butterfly diagram)

Overlp ADD



Overlp SAVE



From
https://www.youtube.com/watch?v=v50IMrrWG4E&t=174s → Natraj Wadhai's videos on overlap Add, Save