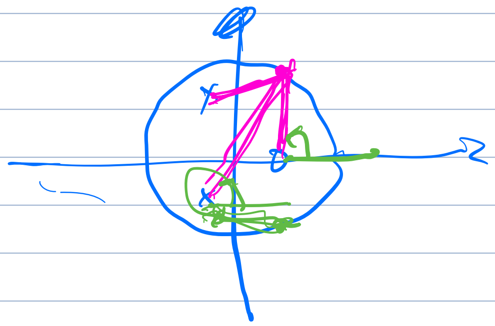
2/23/24 RECITATION Q1 B



$$|H(e^{j\omega})| = G \frac{\prod_{l=1}^{M} |z - c_l|}{\prod_{k=1}^{N} |z - d_k|}$$

$$\measuredangle H(e^{j\omega}) = \measuredangle G + \sum_{l=1}^{M} \measuredangle (z - c_l) - \sum_{k=1}^{N} \measuredangle (z - d_k)$$

QUIZ 1, 2013

Q1 $x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

★ $h[n]$ STABLE

★ $H(z) = \frac{z^{-3}}{1 - \frac{1}{2}z^{-1}} = \frac{1}{z^2(z - \frac{1}{2})}$

$|z| > 1/2$

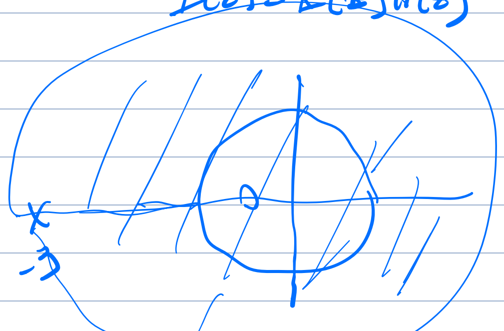
★ $x[n]$ FINITE ENERGY

★ $Y(z) = \frac{z^{-3}(1 + \frac{1}{2}z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 + 3z^{-1})}$

(a) FIND $h[n] = \left(\frac{1}{2}\right)^{n-3} \cdot u[n-3]$

(b) FIND $x[n]$ $X(z) = \frac{Y(z)}{H(z)} = \frac{z^{-3}(1 + \frac{1}{2}z^{-1})}{\cancel{(1 - \frac{1}{2}z^{-1})}(1 + 3z^{-1})} \cdot \frac{\cancel{1 - \frac{1}{2}z^{-1}}}{\cancel{z^{-3}}}$

$Y(z) = X(z) H(z)$ $Y(z)$ $\frac{1}{H(z)}$



$X(z) = \frac{1 + \frac{1}{2}z^{-1}}{1 + 3z^{-1}} = \frac{z + \frac{1}{2}}{z + 3}$

$$
\begin{array}{r}
\frac{1}{6} \\
3+z\,\overline{\big)\,\frac{1}{2}+z+0z^2} \\
\underline{\frac{1}{2}+\frac{1}{6}z} \\
\frac{5}{6}z
\end{array}
$$

$$= \frac{1}{6} + \frac{\frac{5}{6}z}{3+z} = \frac{1}{6} + \frac{5/6}{1+3z^{-1}}$$

$$-\alpha^n u[-n-1] \mapsto \frac{1}{1-\alpha z^{-1}}, \quad |z|<|\alpha|$$

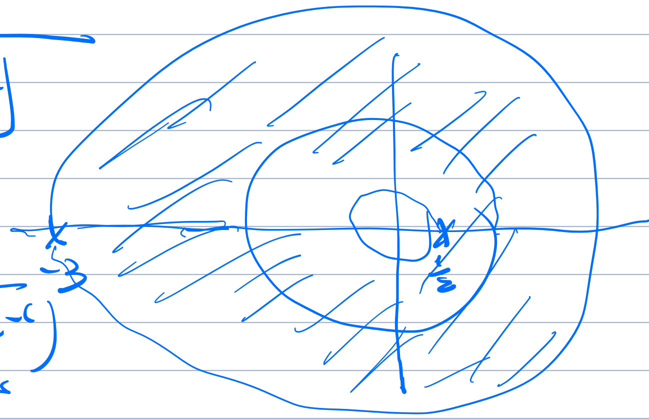
$$X(z) = \frac{1}{6} + \frac{5/6}{1+3z^{-1}}$$

$$x[n] = \frac{1}{6}\delta[n] - \frac{5}{6}(-3)^n u[-n-1]$$

(c) FIND $y[n]$

$$Y(z) = \frac{z^{-3}\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+3z^{-1}\right)}$$

$$= z^{-3}\underbrace{\frac{1+\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+3z^{-1}\right)}}_{Y'(z)}$$



$$\frac{1}{2} < |z| < 3$$

$$Y'(z) = \frac{A_1}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{A_2}{\left(1+3z^{-1}\right)}$$

$$A_1 = Y'(z)\left(1-\frac{1}{2}z^{-1}\right)\Big|_{z=1/2} = \frac{1+\frac{1}{2}z^{-1}}{1+3z^{-1}}\Big|_{z=\frac{1}{2}} = \frac{1+1}{1+(3)(2)} = \frac{2}{7}$$

$$A_2 = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}\Big|_{z=-3} = \frac{1-\frac{1}{6}}{1+\frac{1}{6}} = \frac{5/6}{7/6} = \frac{5}{7}$$

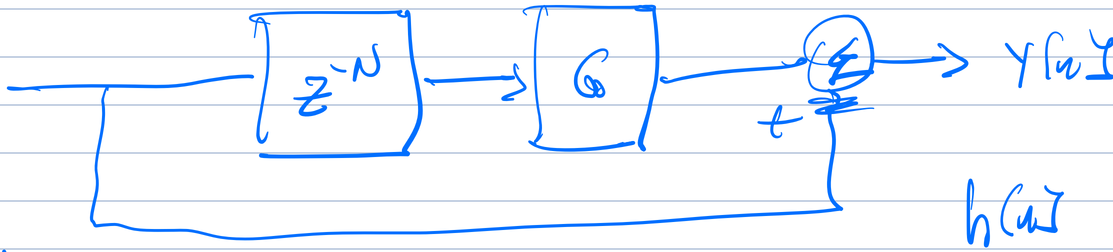
$$Y'(z) = \frac{2}{7}\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{5}{7}\cdot\frac{1}{1+3z^{-1}}$$

$$y'[n] = \frac{2}{7}\left(\frac{1}{2}\right)^n u[n] - \frac{5}{7}(-3)^n u[-n-1]$$

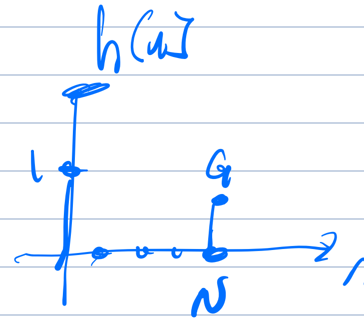
$y[n] = y'[n-3] = \frac{2}{7}\left(\frac{1}{2}\right)^{n-3} u[n-3] - \frac{5}{7}(-3)^{n-3}\, u[\cancel{-(n-3)}-1]$

$u[-n+2]$

Q2 $x[n]$



(a) FIND $h[n] = \delta[n] + G\,\delta[n-N]$



(b) $y[n] = x[n] + G\,x[n-N]$

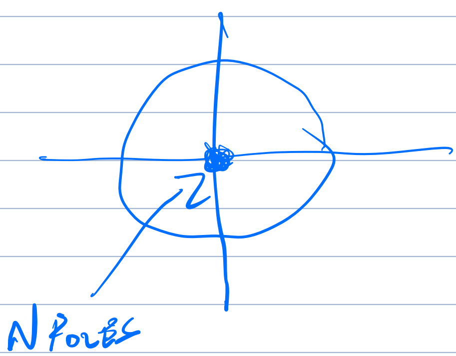
(c) $H(e^{j\omega}) = 1 + G\,e^{-j\omega N}$

(d) 1. $H(z) = 1 + G z^{-N}$

z_0

$\frac{z^N + G}{z^N}$

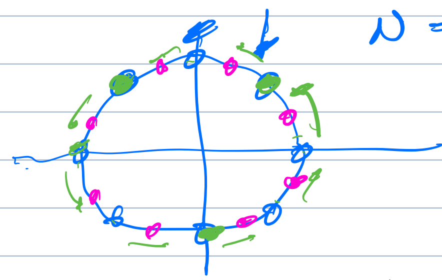
$z^N = G \Rightarrow z = G^{1/N} e^{j2\pi k/N}$



ZEROS @ $z^N = -G$

let $G=1$

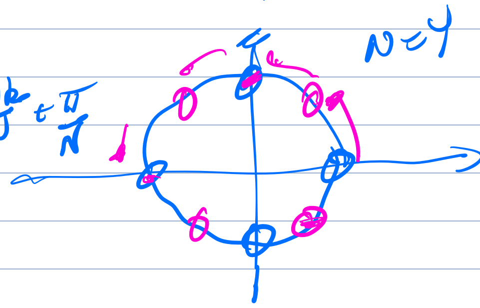
$N=8$



(d) 2 N POLES @ $z=0$

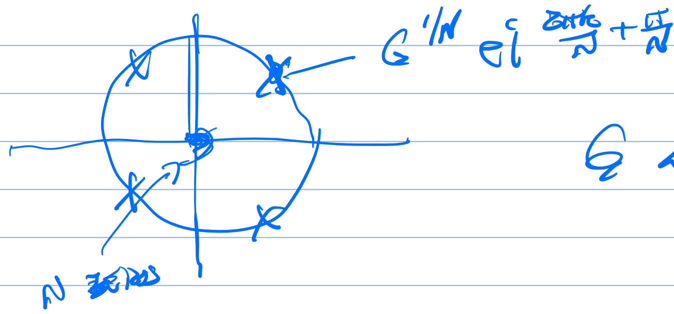
ZEROS $z = G^{1/N} \cdot e^{j\frac{2\pi k}{N} + \frac{\pi}{N}}$

$N=4$



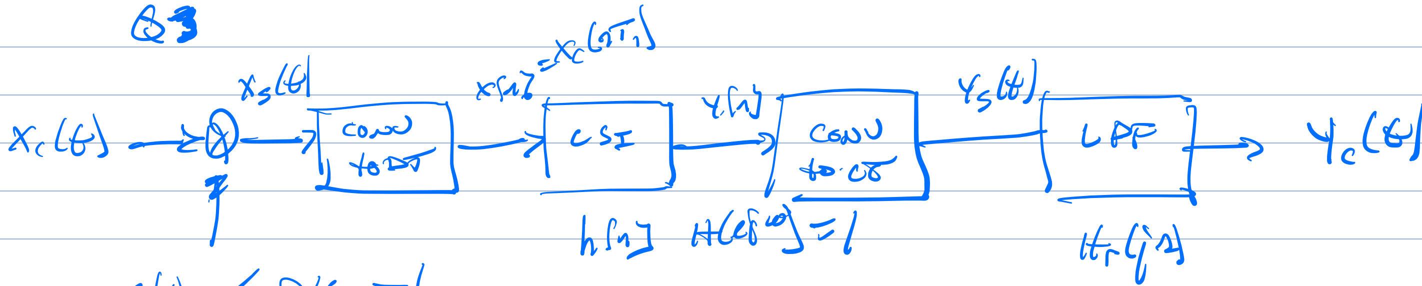
$\forall$ G OK FOR STABILITY

4.

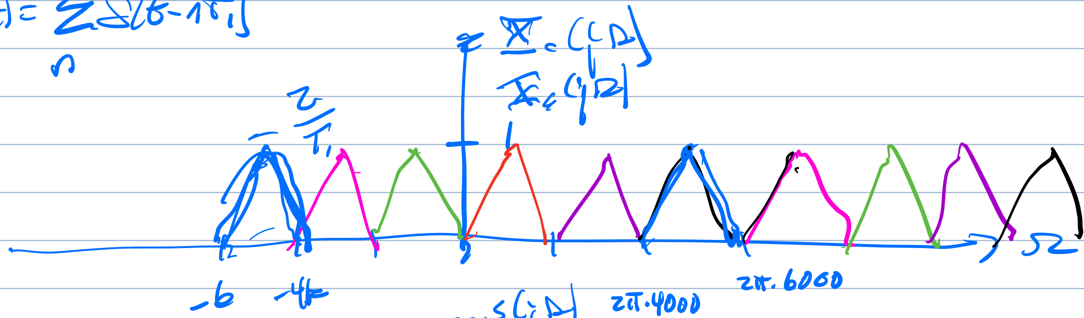


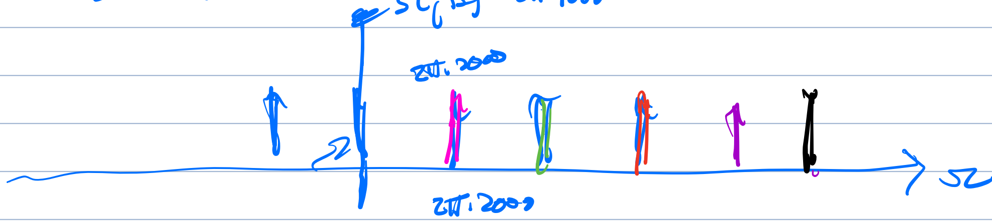
$G < 1$ FOR STABILITY

Q3



$s(t) = \sum_n \delta(t - nT_1)$





$x_s(t) = x_c(t)\, s(t)$

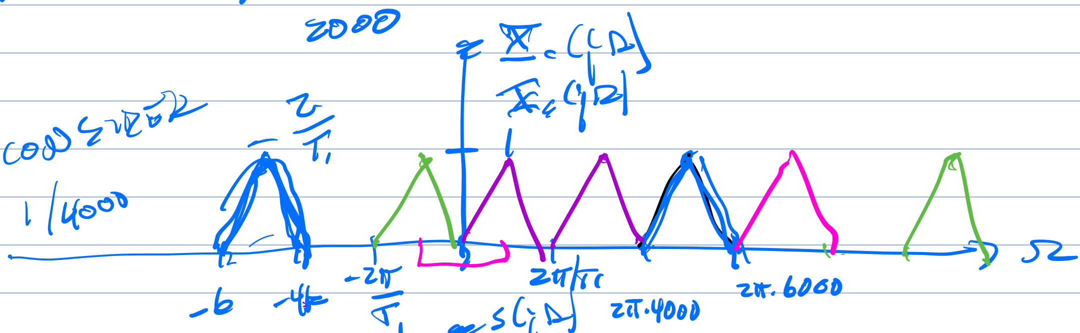
$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$

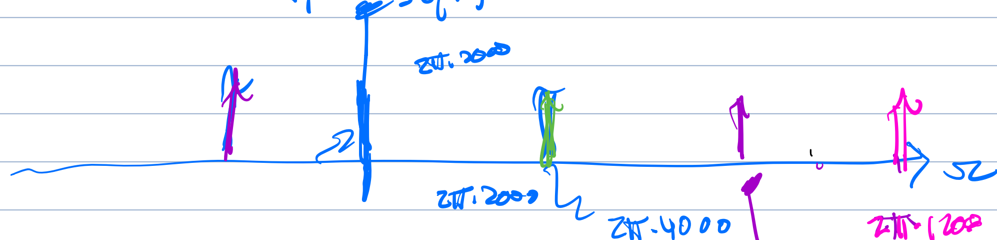
CONSIDER $T_1 = \frac{1}{2000}$

$T_1 = \frac{1}{2000}$ OK IF $X_c(j\Omega)$ HERMITIAN ABOUT CENTER

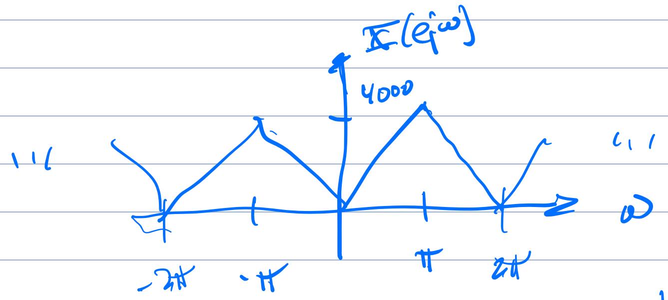
$T_1 = \frac{1}{4000}$

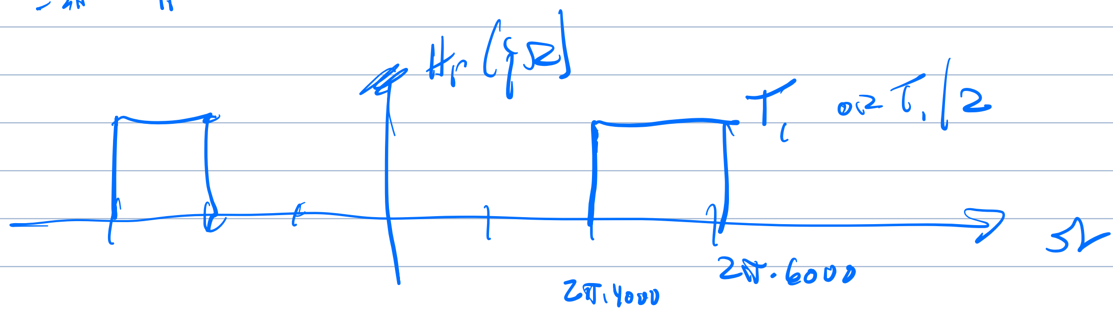
NOW CONSIDER $T_1 = 1/4000$





$2\pi \cdot 800$





e) $T_1 = T_2$

$H(e^{j\omega}) \neq 1$

$$H_{eff}(j\Omega) = \begin{cases} H(e^{j\Omega T_1}), & 2\pi \cdot 4000 \le |\Omega| \le 2\pi \cdot 6000 \\ 0, & \text{Else} \end{cases}$$

2. $T_1 \neq T_2$ $\qquad H_{eff}(j\Omega)$?