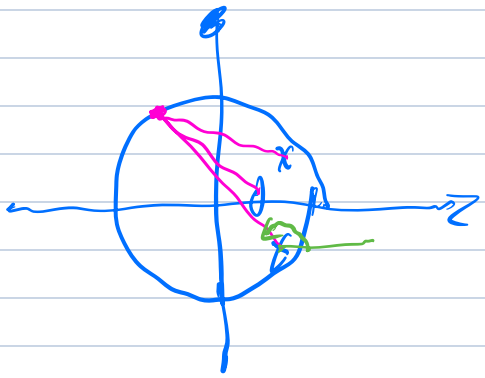
2/23/24 RECITATION Q1 A



$$|H(e^{j\omega})| = A \frac{\prod_{\ell=1}^{M} |z - c_\ell|}{\prod_{k=1}^{N} |z - d_k|}$$

$$\angle H(e^{j\omega}) = [\text{const}] + \sum_{\ell=1}^{M} \angle(z - c_\ell) - \sum_{k=1}^{N} \angle(z - d_k)$$

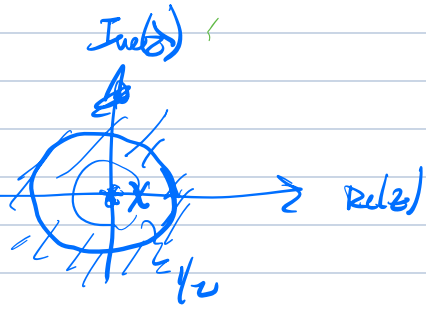
QUIZ 1 FROM 2023

Q1

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$$

* STABLE

* $H(z) = \dfrac{z^{-3}}{1 - \frac{1}{2}z^{-1}} = \dfrac{1}{z^2(z - \frac{1}{2})}$



* $X(z)$ FINITE ENERGY

* $Y(z) = \dfrac{z^{-3}(1 + \frac{1}{2}z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 + 3z^{-1})} = \dfrac{z^{-3}(1 + \frac{1}{2}z^{-1})}{1 + \frac{5}{2}z^{-1} - \frac{3}{2}z^{-2}}$

(a) FIND $h[n] = \left(\frac{1}{2}\right)^{n-3} u[n-3]$

(b) $H(z) = \dfrac{Y(z)}{X(z)}$; $X(z) = \dfrac{Y(z)}{H(z)}$

$$X(z) = \frac{z^{-2}\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+3z^{-1}\right)} \cdot \frac{\left(1-\frac{1}{2}z^{-1}\right)}{z^{-2}} = \frac{1+\frac{1}{2}z^{-1}}{1+3z^{-1}}$$

$X(z)$ $\quad$ $\frac{1}{H(z)}$

$$X(z) = 1+3z^{-1}\overline{\Big)\,1+\frac{1}{2}z^{-1}+0z^{-2}}$$

$$\frac{1+3z^{-1}}{-\frac{5}{2}z^{-1}+0z^{-2}}$$

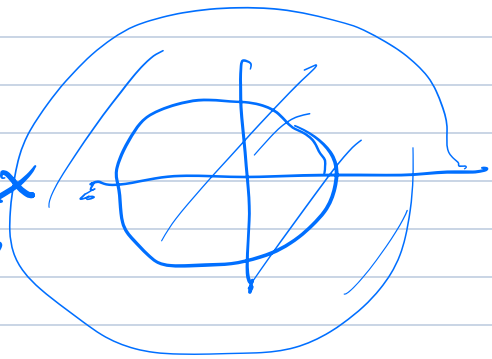
$$= 1 + \frac{-5/2\,z^{-1}}{1+3z^{-1}}$$

$$= 1 + \frac{-\frac{5}{2}}{z+3}$$

$$X(z) = 1 - 5/2\,z^{-1} \cdot \frac{1}{1+3z^{-1}}$$

$\updownarrow$ $\quad$ $\updownarrow$

$|z| > 3$ $\quad$ -3



$$x[n] = \delta[n] + \frac{5}{2}\left(-3\right)^{n-1}u[-(n-1)-1]$$

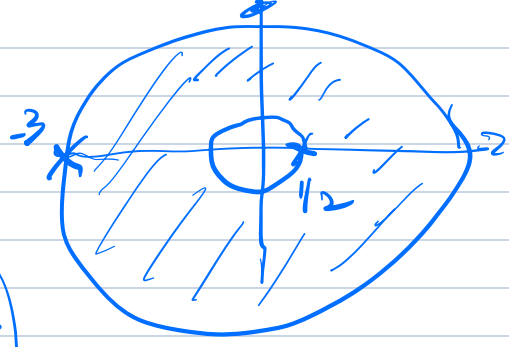
$$= \delta[n] + \frac{5}{2}(-3)^{n-1}u[-n]$$

METHOD II $\quad X(z) = \frac{1+\frac{1}{2}z^{-1}}{1+3z^{-1}} = \frac{1}{1+3z^{-1}} + \frac{\frac{1}{2}z^{-1}}{1+3z^{-1}}$

$$x[n] = (-3)^n u[-n-1] - \frac{1}{2}(-3)^{n-1}u[-n]$$

$$-\alpha^n u[-n-1] \rightleftharpoons \frac{1}{1-\alpha z^{-1}}, \quad |z| < |\alpha|$$

(c) $X(z) = \dfrac{z^{-3}\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+3z^{-1}\right)}$



$\frac{1}{2} < |z| < 3$

$X(z) = z^{-3}\left(\dfrac{A_1}{1-\frac{1}{2}z^{-1}} + \dfrac{A_2}{1+3z^{-1}}\right)$

$X'(z)$

$A_1 = X(z)\left(1-\frac{1}{2}z^{-1}\right)\Big|_{z=\frac{1}{2}} = \left.\dfrac{1+\frac{1}{2}z^{-1}}{1+3z^{-1}}\right|_{z=\frac{1}{2}} = \dfrac{1+\frac{1}{2}\cdot 2}{1+3\cdot 2} = \dfrac{2}{7}$

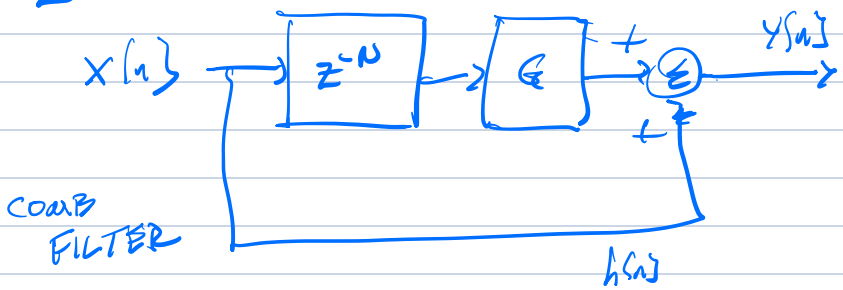
$A_2 = X'(z)\left(1+3z^{-1}\right)\Big|_{z=-3} = \left.\dfrac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}\right|_{z=-3} = \dfrac{1+\frac{1}{2}\left(-\frac{1}{3}\right)}{1-\frac{1}{2}\left(-\frac{1}{3}\right)} = \dfrac{\frac{5}{6}}{\frac{7}{6}} = \dfrac{5}{7}$

$y'[n] = \dfrac{2}{7}\left(\dfrac{1}{2}\right)^n u[n] - \dfrac{5}{7}(-3)^n u[-n-1]$

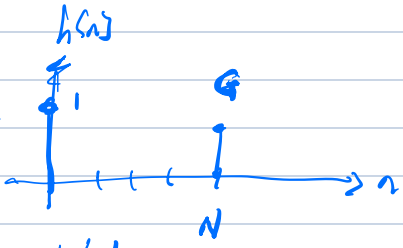
$X(z) = z^{-3}\cdot X'(z)$; $y[n] = y'[n-3] = \dfrac{2}{7}\left(\dfrac{1}{2}\right)^{n-3} u[n-3] - \dfrac{5}{7}(-3)^{n-3}$

$u[-(n-3)-1] = u[-n+2]$

QUESTION 2



(a) $x[n] = \delta[n]$ $\quad h[n] =$



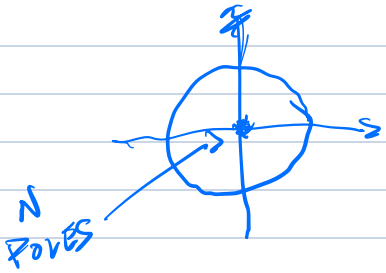
(b) $z^N = G \Rightarrow z = G^{1/N} e^{-2\pi i k/N}$

$h[n] = \delta[n] + G\,\delta[n-N]$

$G^{1/N} \cdot e^{i\left(\frac{\pi}{q} + i\frac{2\pi k}{N}\right)}$

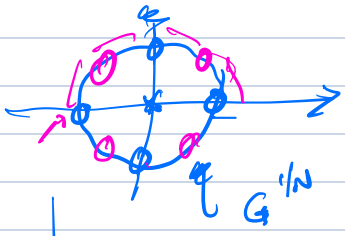
$k = 0, \ldots, N-1$

$H(z) = 1 + G z^{-N} = \frac{z^N + G}{z^N}$



ZEROS @ $z^N = -G$

SOLUTIONS FOR $z^N = 1$



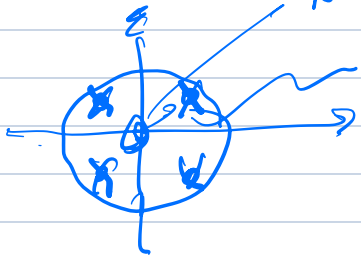
(c) ALWAYS STABLE!

(d) $H(z) = \left(1 + G z^{-N}\right) = \frac{Y(z)}{X(z)}$

$Y[n] = x[n] + G x[n-N]$

$H(e^{i\omega}) = 1 + G e^{-i\omega N}$

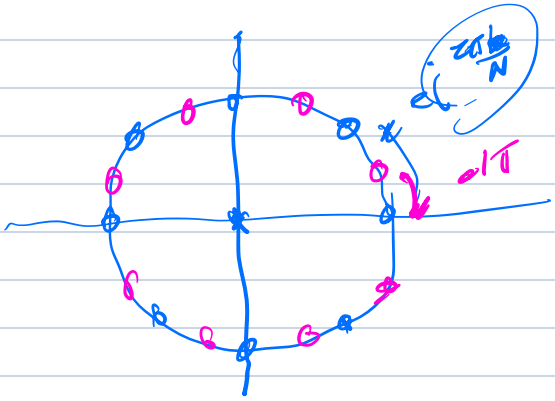
INVERSE SYSTEM



NEED $G^{1/N} < 1$

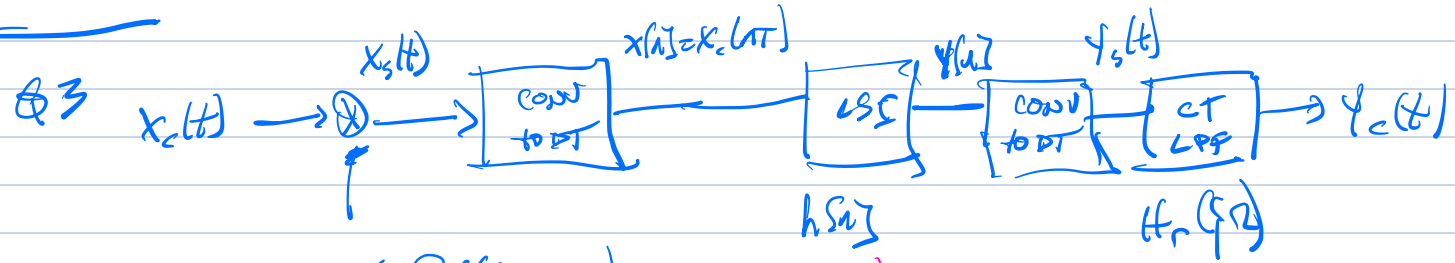
$G < 1$

(e)



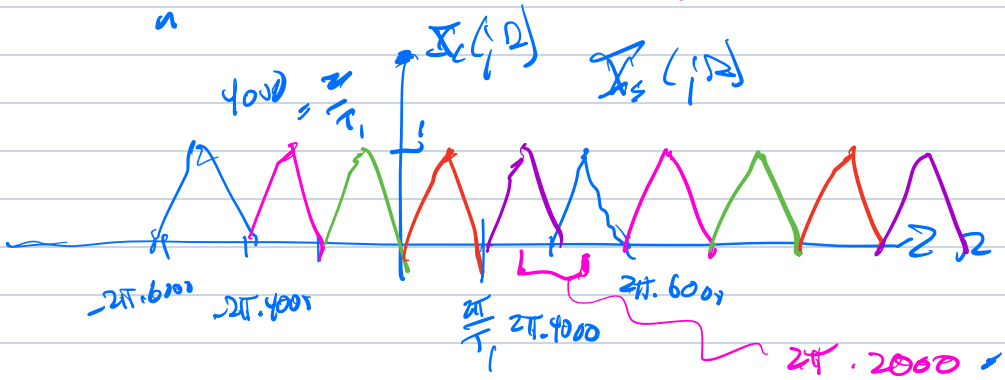
$\angle \frac{2\pi}{N} = -1\pi$

Q3



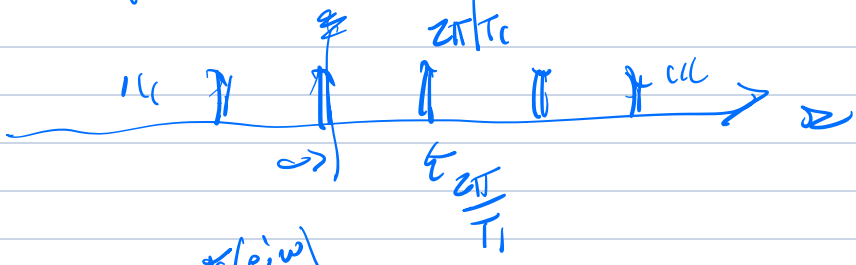
$s(t) = \sum_n \delta(t - nT_1)$

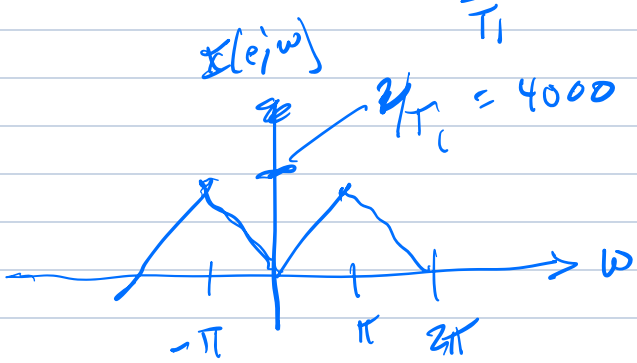
$H(e^{j\omega}) = 1$



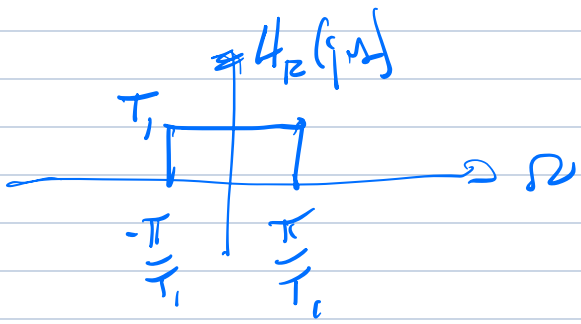
$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$

$T_1 = \frac{1}{2000}$





$T_1 = \frac{1}{2000}$



e 1 TRANSFER FUNCTION

$$H_{eff}(j\Omega) = \frac{Y_c(j\Omega)}{X_c(j\Omega)} = H(e^{j\omega})\Big|_{\omega=\Omega T} = H(e^{j\Omega T})$$

$2\pi \cdot 4000 \le \Omega \le 2\pi \cdot 6000$

0, ELSE