INTRODUCTION TO
FILTER DESIGN BY
MODELING:
LINEAR PREDICTION +
LATTICE FILTERS

FILTER DESIGN BY MODELLING

1. $H(z) = B(z)$ MOVING AVERAGE MODEL (MA)
   ZEROS ONLY

2. $H(z) = \dfrac{G}{A(z)}$ AUTO-REGRESSIVE MODEL (AR)

3. $H(z) = \dfrac{B(z)}{A(z)}$ ARMA MODEL

DETERMINISTIC FORMULATION OF THE ALL-POLE (AR) MODEL

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$$

ASSUME $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{G}{1 - \sum_{k=1}^{P} \alpha_k z^{-k}}$; $y[n] - \sum_{k=1}^{P} \alpha_k y[n-k] = G x[n]$

$$y[n] = \sum_{k=1}^{P} \alpha_k y[n-k] + G x[n]$$

$$h[n] = \sum_{k=1}^{P} \alpha_k h[n-k] + G \delta[n]$$

FIND ALL-POLE $H(z)$ THAT HAS $H(e^{j\omega})$ THAT MOST CLOSELY RESEMBLES $H_d(e^{j\omega})$

ONE APPROACH:

FIND $H(z)$ THAT MINIMIZES $\xi^2 = \dfrac{1}{2\pi} \displaystyle\int_{-\pi}^{\pi} \left| H(e^{j\omega}) - H_d(e^{j\omega}) \right|^2 d\omega$

PARSEVAL'S TH. MIN $\xi^2 = \sum_{n=-\infty}^{\infty} \left(h[n] - h_d[n] \right)^2$ $\quad 1 \le k \le P$

$$\text{MIN } \xi^2 = \sum_{n=-\infty}^{\infty} \left(\left(G\delta[n] + \sum_{l=1}^{P} \alpha_l^{(P)} h[n-l] \right) - h_d[n] \right)^2$$

$$\frac{\partial \xi^2}{\partial \alpha_k} = \sum_{n=-\infty}^{\infty} 2 \left(\left(G\delta[n] + \sum_{l=1}^{P} \alpha_l^{(P)} h[n-l] \right) - h_d[n] \right) h[n-k] = 0$$

$$\sum_{n=-\infty}^{\infty} \left(\cancel{G\delta[n] h[n-k]} + \sum_{l=1}^{P} \alpha_l^{(P)} h[n-l] h[n-k] \right) = \sum_{n=-\infty}^{\infty} h_d[n] h[n-k]$$

$$\boxed{\sum_{l=1}^{P} \alpha_l^{(P)} \sum_{n=0}^{N-1} h[n-l] h[n-k] = \sum_{n=0}^{N-1} h_d[n] h[n-k]}$$

YULE-WALKER EQ.

WHITE NOISE

$$w[n] \rightarrow \boxed{H(z)} \rightarrow x[n]$$

$$y[n] = \underbrace{\sum_{k=1}^{p} y[n-k]\,\alpha_k^{(p)}}_{\hat{y}[n]} + G\,x[n]$$

$$\xi^2 = \min E\left[\left(y[n] - \hat{y}[n]\right)^2\right]$$

FIND $\{\alpha_k\}$ THAT MIN $\xi^2 = E\left[\left(y[n] - \sum_{k=1}^{p} \alpha_k^{(p)}\, y[n-k]\right)^2\right]$

SOLUTION FIND

$$E\left[\sum_{\ell=1}^{p} \alpha_\ell^{(p)}\, y[n-\ell]\, y[n-k]\right] = E\left[y[n]\, y[n-k]\right]$$

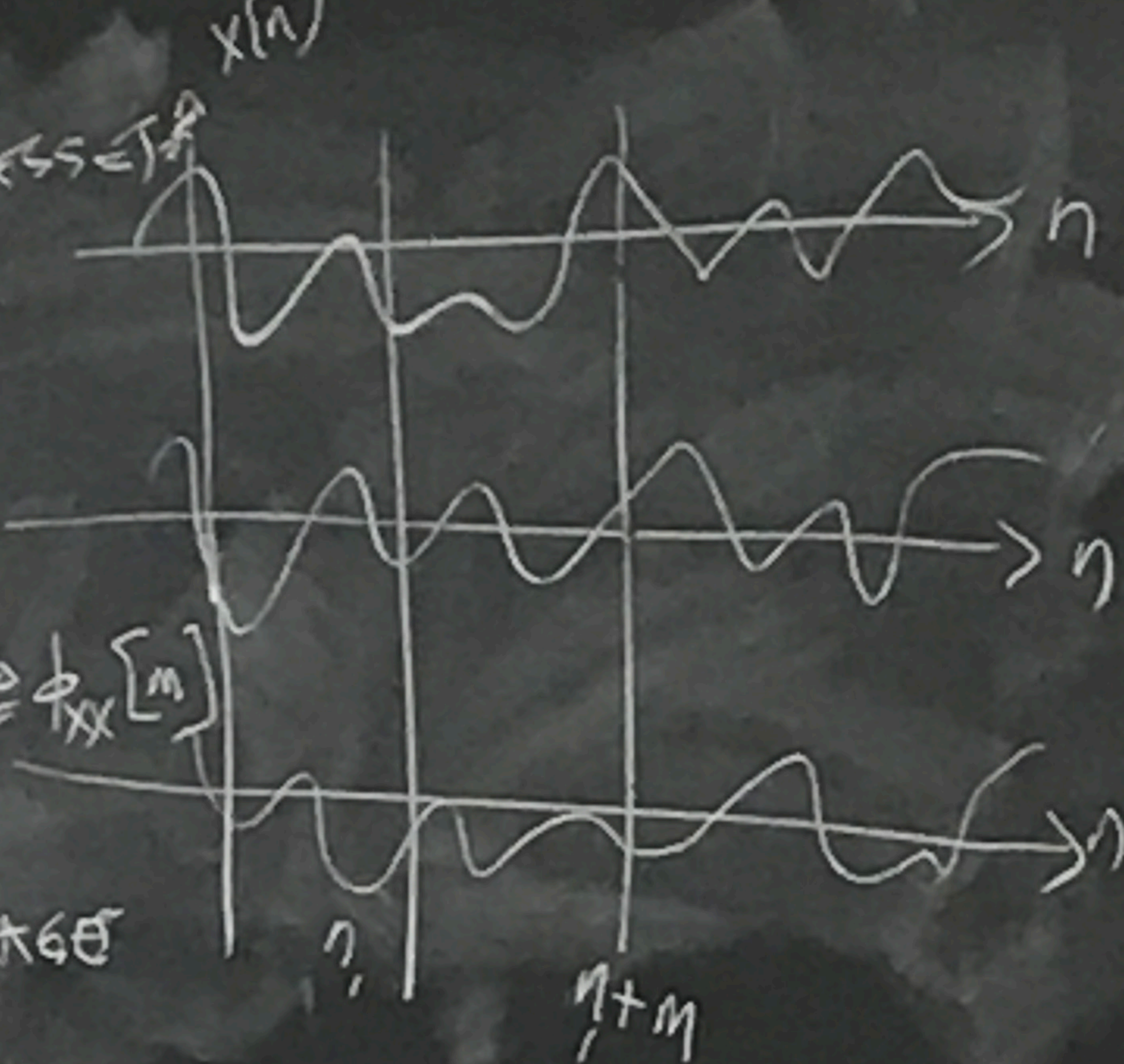
$$\sum_{\ell=1}^{p} \alpha_\ell^{(p)}\, E\left[y[n-\ell]\, y[n-k]\right] = E\left[y[n]\, y[n-k]\right]$$

YULE-WALKER

EQ.

$$\sum_{\ell=1}^{p} \alpha_\ell^{(p)}\, \phi_{xx}\left[|k-\ell|\right] = \phi_{xx}[k]$$

RANDOM PROCESSES



SAMPLE FUNCTIONS OF A RP

MEAN $E[x[n]]$

AUTOCORRELATION

$$E[x[n]\,x[n+m]] \triangleq \phi_{xx}[m]$$

ENSEMBLE AVERAGE

TIME AVERAGE MEAN $\langle x[n]\rangle = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x[n]$

AUTOCORRELATION $\langle x[n]\,x[n+m]\rangle = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x[n]\,x[n+m]$

RP $x[n]$ IS ERGODIC IF $E[g[x[n]]] = \langle g[x[n]]\rangle$

POWER SPECTRAL DENSITY

$$P_{xx}(\omega) = \text{DTFT}\{\phi_{xx}[m]\}$$

IF $P_{xx}(\omega) = \frac{N_0}{2}$ RP IS "WHITE"