DTFT:

$$X(\omega) = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

STFT:

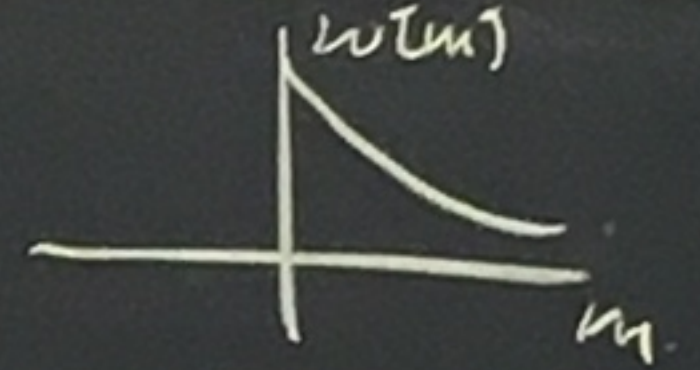
$w[-(m-n)]$

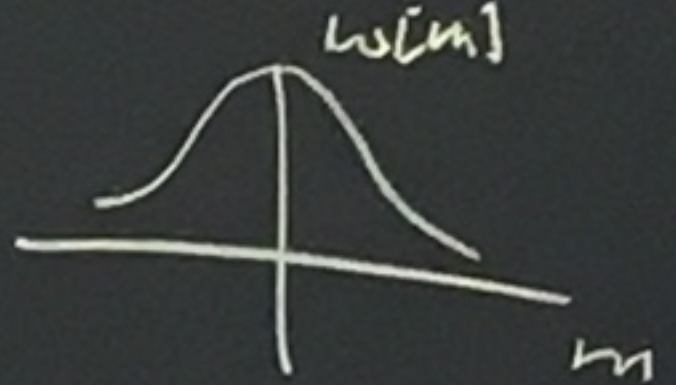
$$S[n,\omega) = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega m}$$

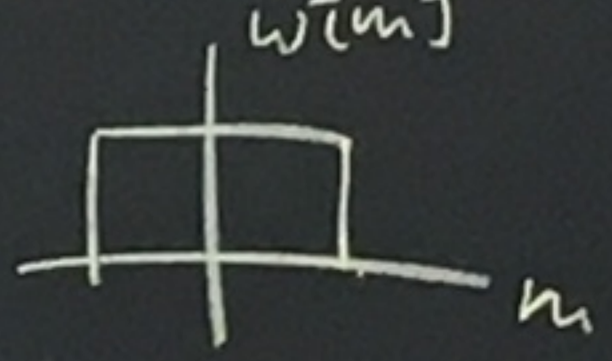
$$\rightarrow S[0,\omega) = S_0(\omega) = \sum_{m=-\infty}^{\infty} \big(x[m]\, w[-m]\big) e^{-j\omega m}$$

$$w[n-m] \xleftrightarrow{\text{DTFT}} \sum_{m=-\infty}^{\infty} w[n-m] e^{-j\omega m} \underset{l=n-m}{=} \sum_{l=-\infty}^{\infty} w[l] e^{-j\omega(n-l)}$$

$$e^{-j\omega n} \cdot W(e^{-j\omega}) =$$



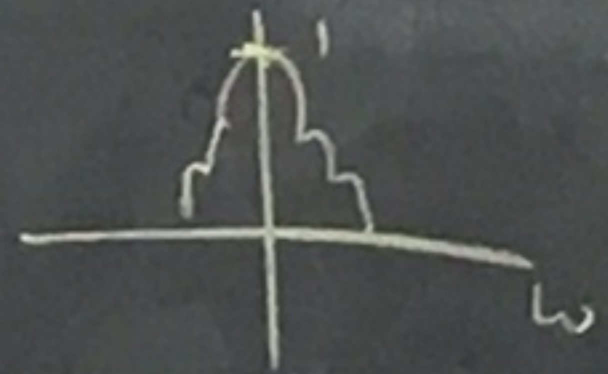




$$S[n,\omega) = \mathrm{DTFT}\{x[m]\, w[n-m]\}$$

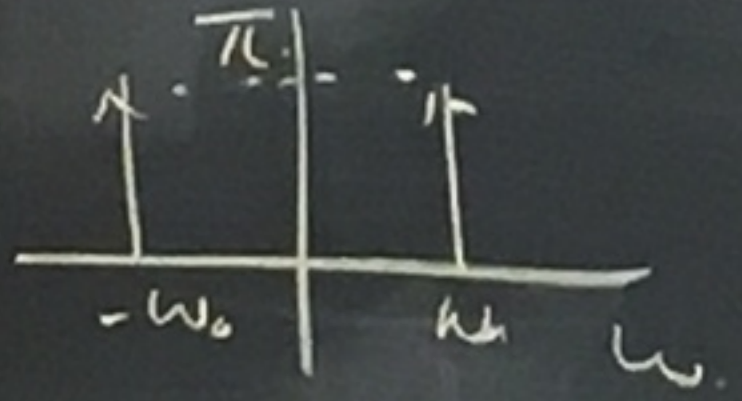
$$= \frac{1}{2\pi} e^{-j\omega n} \circledast X(e^{j\omega})\, W(e^{-j\omega})$$

$W_2(e^{j\omega})$ N_2-pt hamming, $N_2 \gg N_1$



$$x[n] = \cos(\omega_0 n)$$

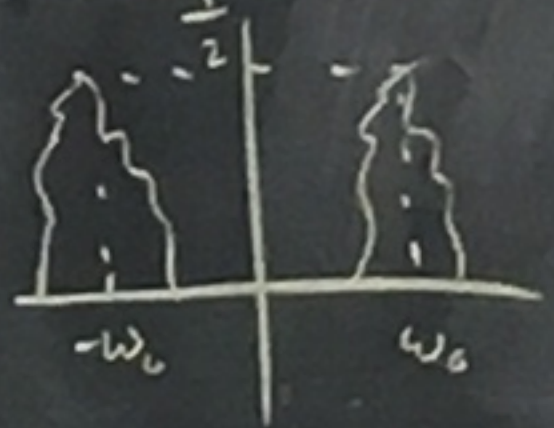
$X(e^{j\omega})$



$W_1(e^{j\omega})$ – N_1-hamming

$S_0(\omega) = S[0,\omega)$

$S_1(e^{j\omega})$



Computation Complexity - FT view

Each frame: N-pt FFT - $O(N \log_2 N)$

# of frames: M

Total: $O(MN \log_2 N)$

Discrete STFT

$$S[n,\omega)\Big|_{\omega=\omega_k} = S[n,k] = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}, \quad 0 \le k < N$$

N-pt DFT

$$\omega_k = \frac{2\pi k}{N}$$

$$S \; \begin{array}{c} \xrightarrow{\text{time}} \\ \left[\;\big|\;\big|\; \qquad \right] \end{array} \uparrow \text{freq}$$

$x[m] \neq 0, \; 0 \le m < M$

① Fourier Transform View

$$S[n,k] = \sum_{m=-\infty}^{\infty} x'[m]\, e^{-j\omega_k m},$$

$$x'[m] = x[m]\, w[n-m], \quad \text{for each } \underline{n}$$

FT – Each frame: N-pt FFT – $O(N\log_2 N)$

\# of frames: M

Total: $O(MN\log_2 N)$

LPF Each channel: Total: $O(MN\log_2 N)$

\# chan: $\frac{2\pi}{BW} = \frac{N}{4}$

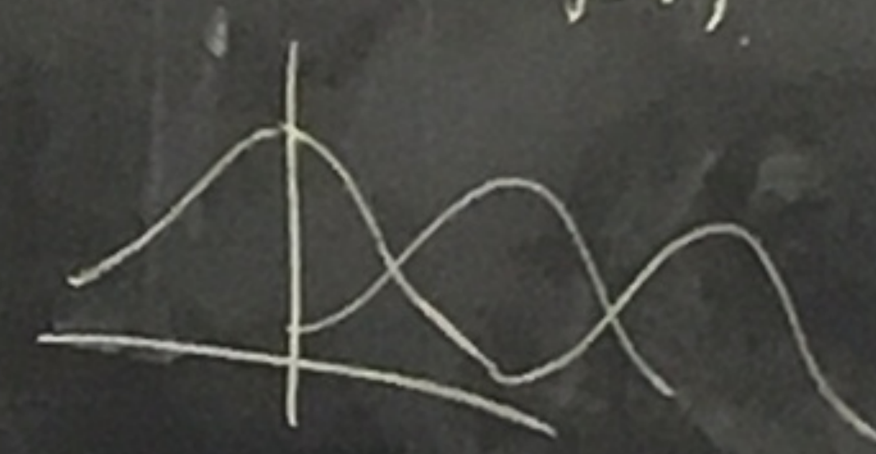
- Direct conv: $O((M+N-1)M) = O(M^2)$
- OLA/OLS $O\left(\frac{M}{N} N\log_2(N)\right) = O(M\log_2 N)$

BPF

Each channel: $O(M\log_2 N) + O(M) = O(M\log_2 N)$

\# channels: $\frac{N}{4}$

Total: $O(MN\log_2 N)$



Signal Type	Storage	Decimation
Time domain	M real	
DFT	M complex $\to \frac{M}{2}$ complex	
STFT-FT	MN complex $\to \frac{MN}{2}$ complex	$\to \boxed{\downarrow \frac{N}{2}} \to \frac{M}{N/2}\frac{N}{2} = 2M$
STFT-LPF/BPF	MK, $\tilde{K} = \frac{N}{4}$ ~ Hamming $\to \frac{MN}{4}$ complex	$\to \boxed{\downarrow \frac{N}{4}} \to \frac{M}{N/4}K = \frac{M}{N/4}\frac{N}{4} = M$

③ Bandpass filter view

$$\tilde{X}[n,k] = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}$$