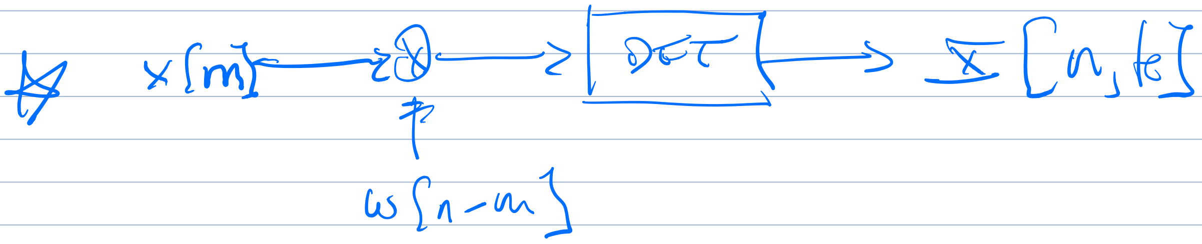
4/26/21

INTRO TO SHORT-TIME FOURIER TRANSFORMS (STFT)

→ NAWAB/QUATIERI CHAPTER IN CLASS NOTES, ADSP NOTES

ALSO OS4P 10.3–10.4

FOURIER TRANSFORM of STFT

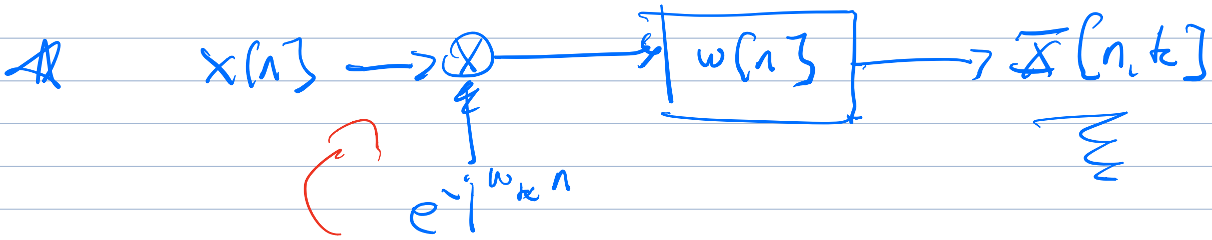


OTHER IMPLEMENTATIONS ARE POSSIBLE...

$$X[n,k] = \sum_{m=-\infty}^{\infty} x[m] w[n-m] e^{-j\omega_k m} \qquad \omega_k = \frac{2\pi k}{N}$$

$$= \sum_{m=-\infty}^{\infty} w[n-m] \left(\underline{x[m] e^{-j\omega_k m}} \right)$$

$$= w[n] * x[n] e^{-j\omega_k n}$$



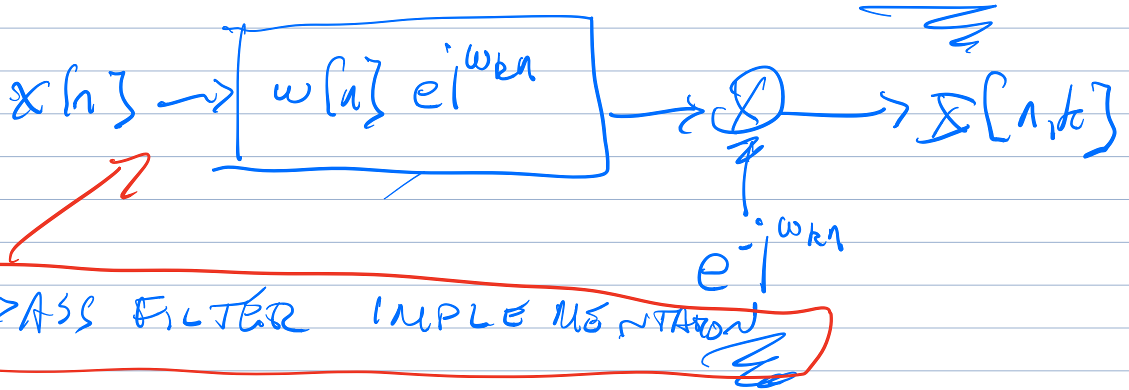
LOW PASS FILTER IMPLEMENTATION

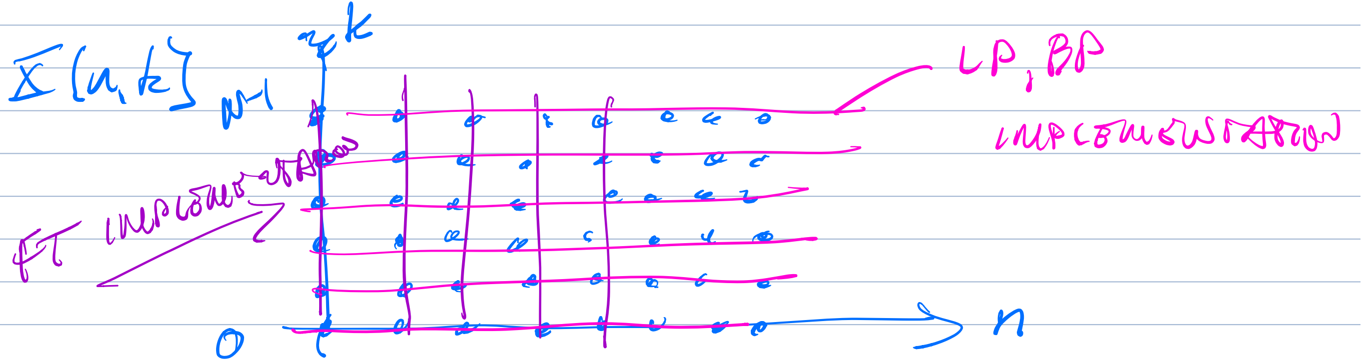
$$X[n,k] = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}$$

$$= e^{-j\omega_k n} \cdot \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m} \quad \underset{\equiv}{e^{+j\omega_k n}}$$

$$\equiv$$

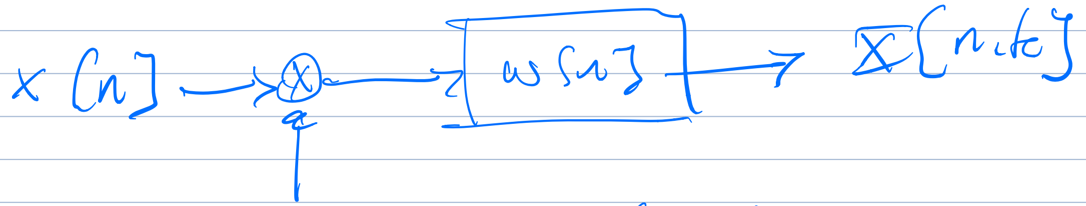
$$= e^{-j\omega_k n} \cdot \underbrace{\sum_{m=-\infty}^{\infty} x[m] \cdot w[n-m]\, e^{j\omega_k (n-m)}}_{x[n] * w[n] e^{j\omega_k n}}$$



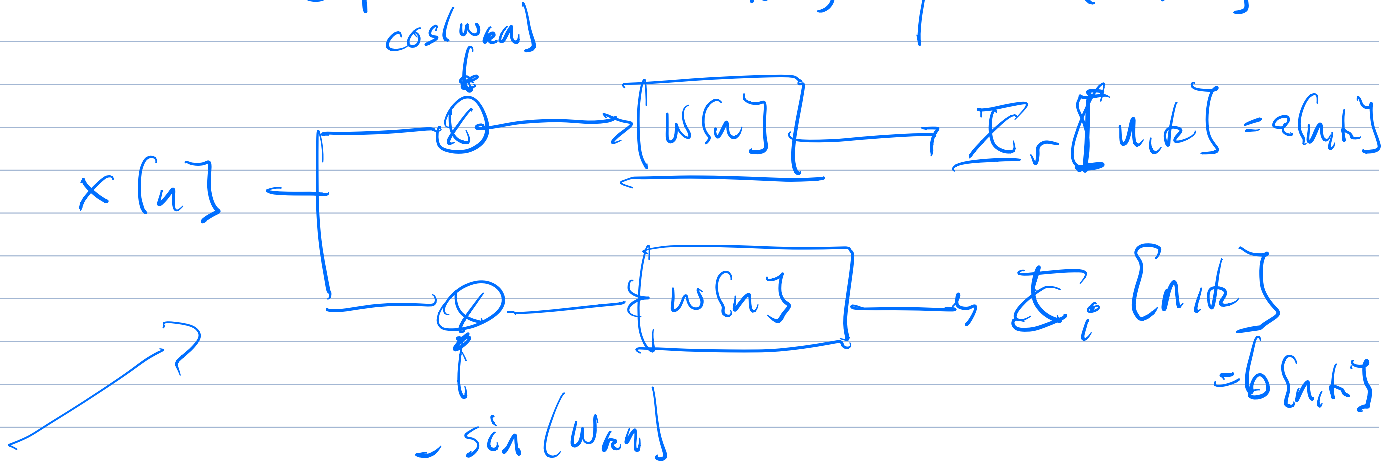


STFT IMPLEMENTATIONS w REAL COEFFS

LP IMPLEMENTATION:

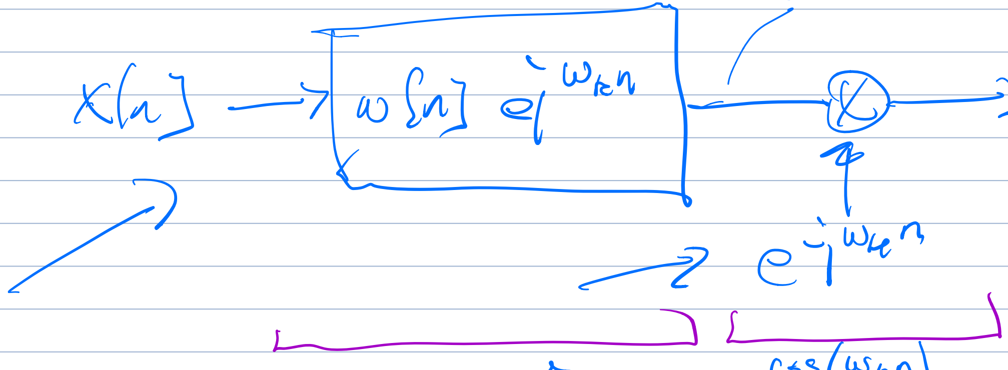


$$e^{-j\omega_k n} = \cos(\omega_k n) - j\sin(\omega_k n)$$

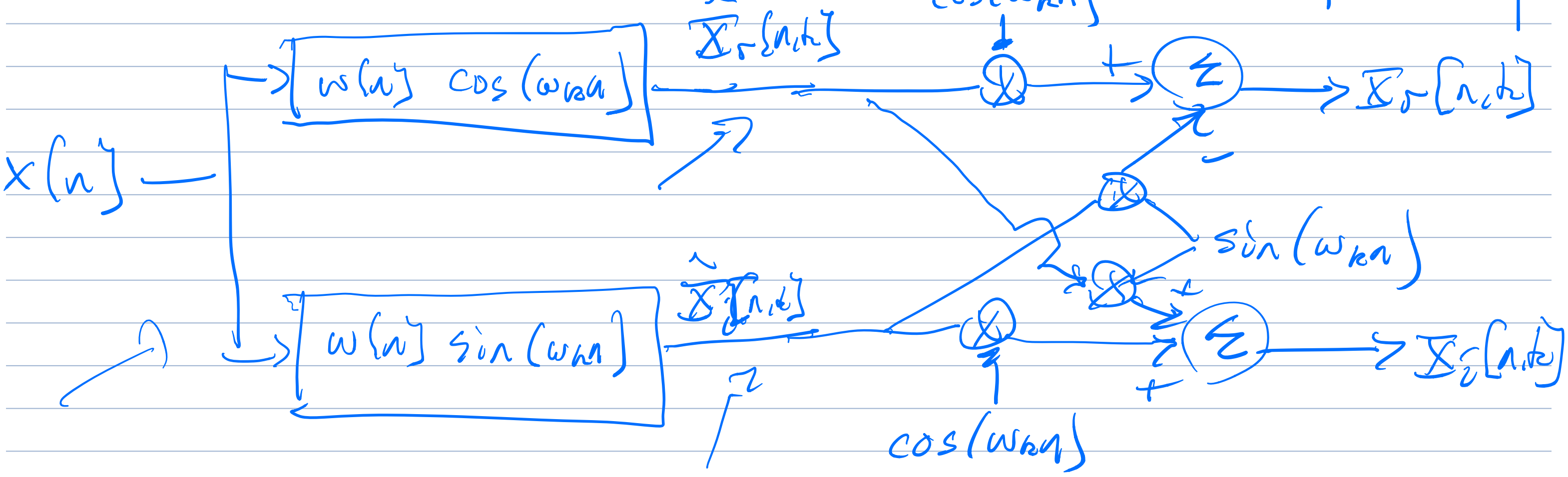


$$X[n,k] = X_r[n,k] + jX_i[n,k]$$

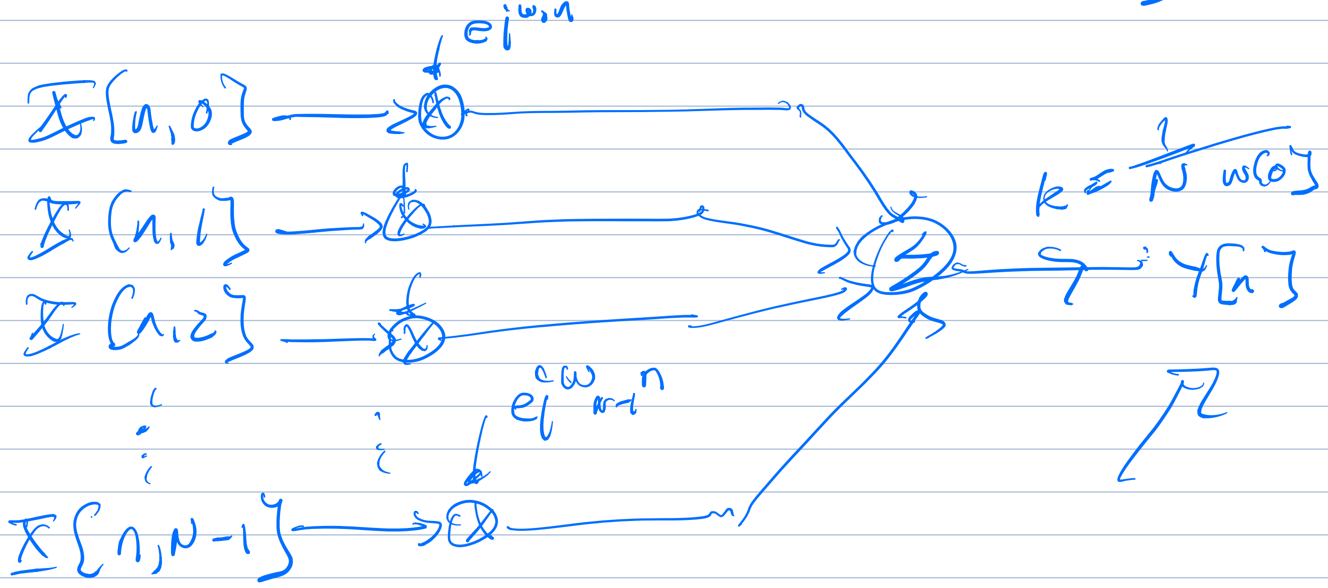
BP IMPLEMENTATION w $\tilde{X}[n,k]$



$$|\tilde{X}[n,k]| = |X[n,k]|$$



SHORT-TIME FOURIER SYNTHESIS



$$Y[n] = \sum_{k=0}^{N-1} \frac{1}{N\,w[0]} X[n,k]\, e^{j\omega_k n}$$

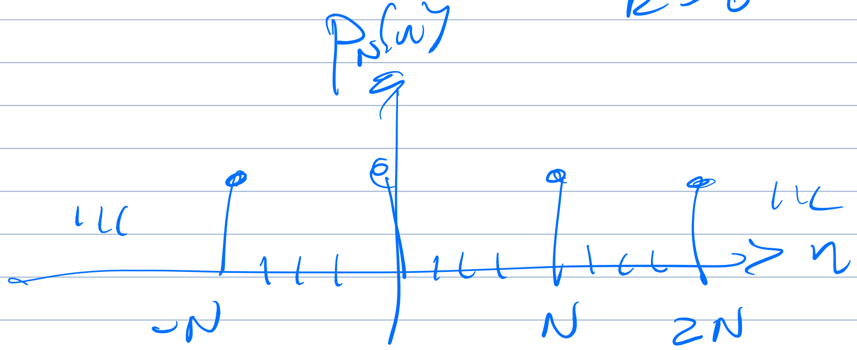
$$w[n-m]\,x[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[n,k]\, e^{j\omega_k n}$$

$$\Rightarrow \frac{1}{N\,w[0]} \sum_{k=0}^{N-1} \left(\underbrace{\sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{j\omega_k m}}_{X[n,k]} \right) e^{j\omega_k n}$$

$$Y[n] = \frac{1}{N\,w[0]} \sum_{k=0}^{N-1} \left(x[n] * w[n]\, e^{j\omega_k n} \right)$$

$$= \frac{1}{N\,w[0]} x[n] * \underbrace{\left[w[n] \sum_{k=0}^{N-1} e^{j\frac{2\pi n k}{N}} \right]}_{\text{MUST BE } \delta[n]}$$

$$\sum_{k=0}^{N-1}\left(e^{i\frac{2\pi n}{N}}\right)^{k} = \frac{1-e^{i\frac{2\pi n N}{N}}}{1-e^{i\frac{2\pi n}{N}}}$$



$$= \begin{cases} 1, & n = rN \\ 0, & n \neq rN \end{cases}$$

FOR $y[n]$ TO EQUAL $x[n]$,

NEED

1. $w[0] \neq 0$
2. $w[n] = 0$, FOR $n = rN$

FBS CONSTRAINT

APPROACH IS FILTER BANK SUMMATION (FBS)

WORKS IF FBS CONSTRAINT SATISFIED

FBS CONSTRAINT

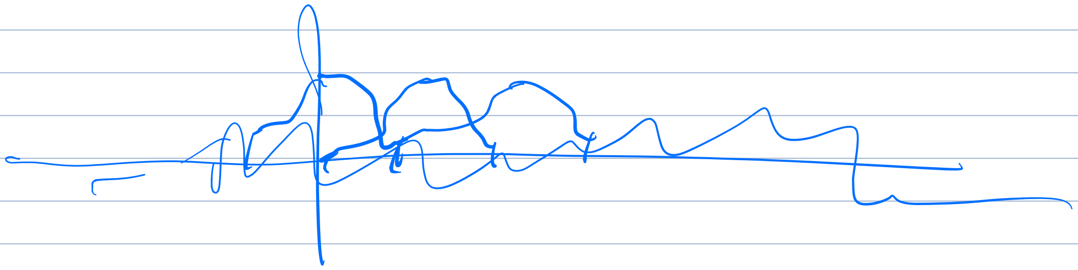
$$\frac{1}{N}\sum_{k=0}^{N-1} W\left(e^{i\left(\omega-\frac{2\pi k}{N}\right)}\right) = 1$$

FBS METHOD

OLA METHOD OF SYNTHESIS

$w[n-m] \cdot x[m] \Longleftrightarrow X[n,k]$

COMPUTE I STFT $\quad \frac{1}{N}\sum_{k=0}^{N-1} X[n,k]\, e^{j\omega_k m}$



OLA METHOD

1. COMPUTE $w[n-m]\, x[m]$ FROM STFT
FOR $m = rL$

2. ADD UP TIME-FUNCTIONS OVER TIME

$$\sum_{r=-\infty}^{\infty} w[n-rL]\, x[m]$$

$$= x[m] \underbrace{\sum_{r=-\infty}^{\infty} w[n-rL]}_{\text{MUST EQUAL CONST.}}$$