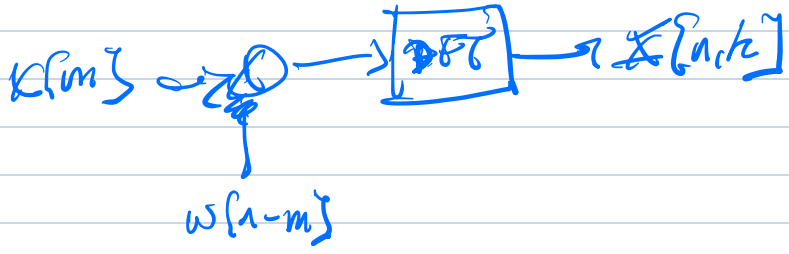
4/15/24 INTRO TO SHORT-TIME FOURIER TRANSFORMS (STFTs)

SOURCES:

- ★ ADSP CLASS NOTES
- ★ NAWAB/QUATIERI
- ★ OSAP 10.3 - 10.4 (SLIGHTLY DIFFERENT NOTATION)

FOURIER TRANSFORM IMPLEMENTATION OF STFT

$$X[n,k] = \sum_m w[n-m]\, x[m]\, e^{-j\omega_k m}, \quad \omega_k = \frac{2\pi k}{N}$$



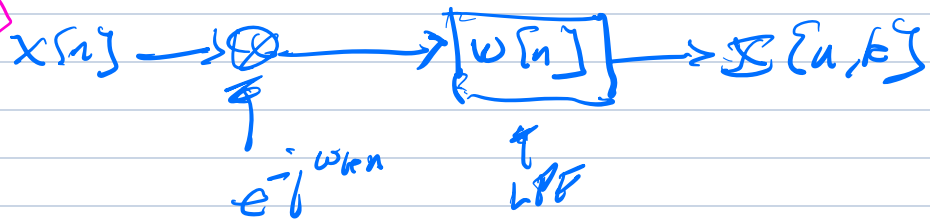
OTHER "IMPLEMENTATIONS" OF THE STFT

$$X[n,k] = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}$$

$$= \sum_{m=-\infty}^{\infty} w[n-m]\left(x[m]\, e^{-j\omega_k m}\right)$$

$$X[n,k] = w[n] * x[n]\, e^{-j\omega_k n}$$

LOWPASS FILTER IMPLEMENTATION



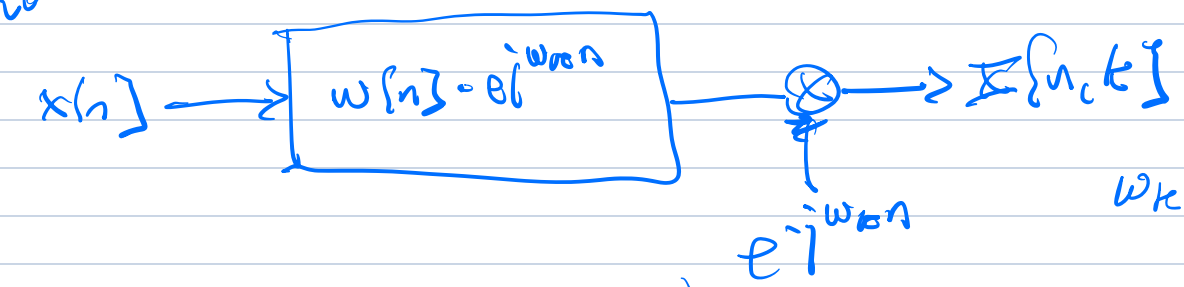
$$X[n,k] = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}$$

$$= e^{-j\omega_k n} \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}\, e^{+j\omega_k n}$$

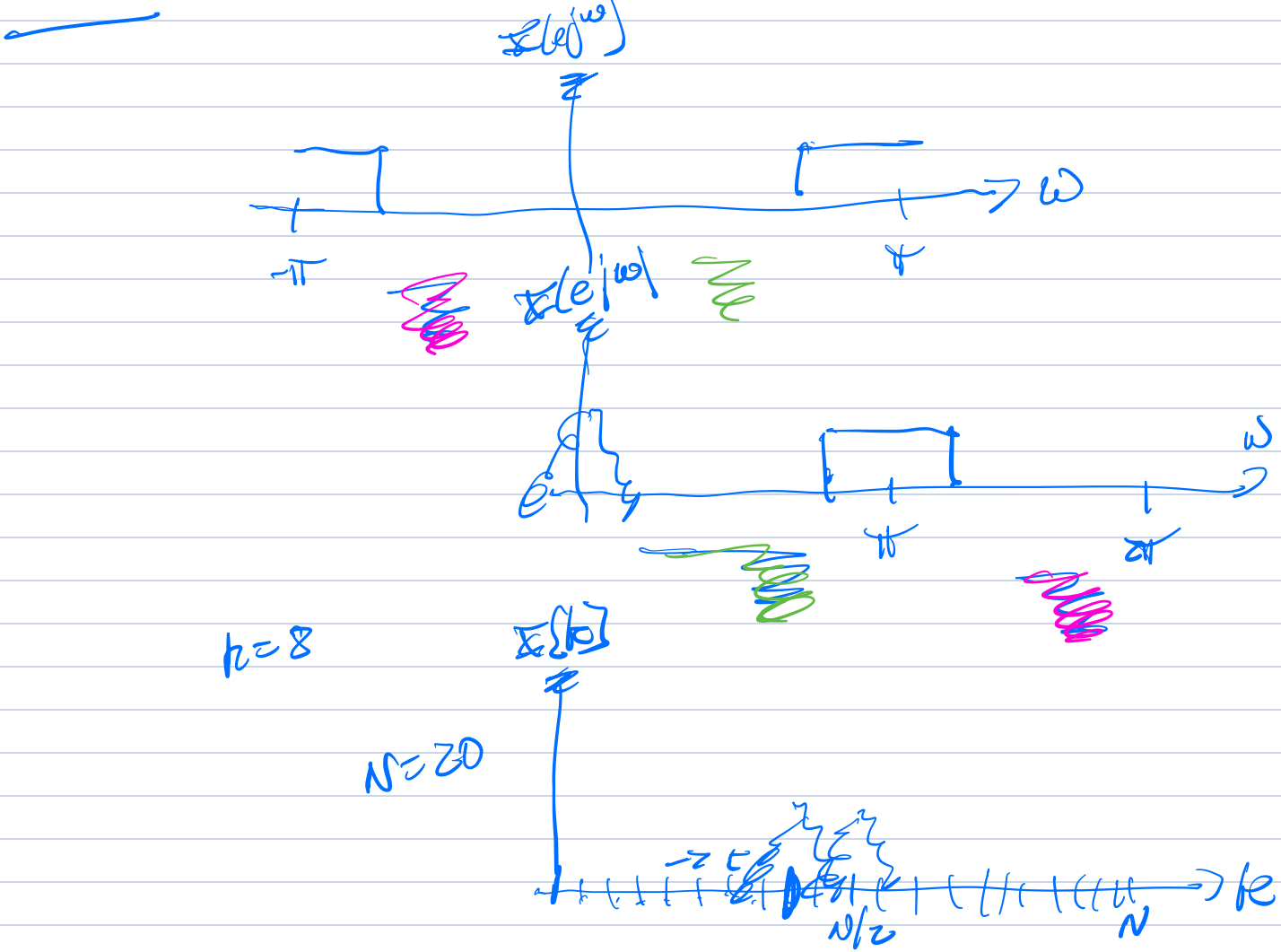
$$= e^{-j\omega_k n} \sum_{m=-\infty}^{\infty} x[m] \left(w[n-m]\, e^{j\omega_k (n-m)} \right)$$

BAND-PASS FILTER IMPLEMENTATION

$$\left(x[n] * w[n] \cdot e^{j\omega_k n} \right) e^{-j\omega_k n}$$

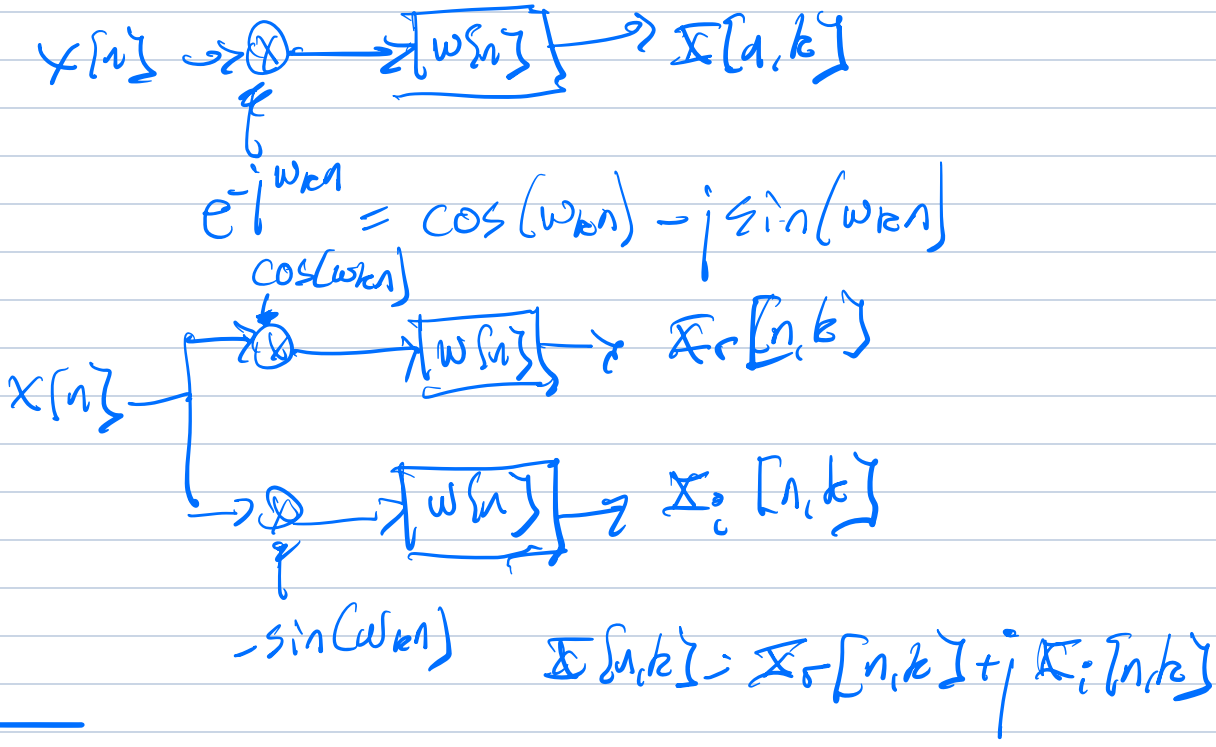


$$\omega_k = \frac{2\pi k}{N}$$

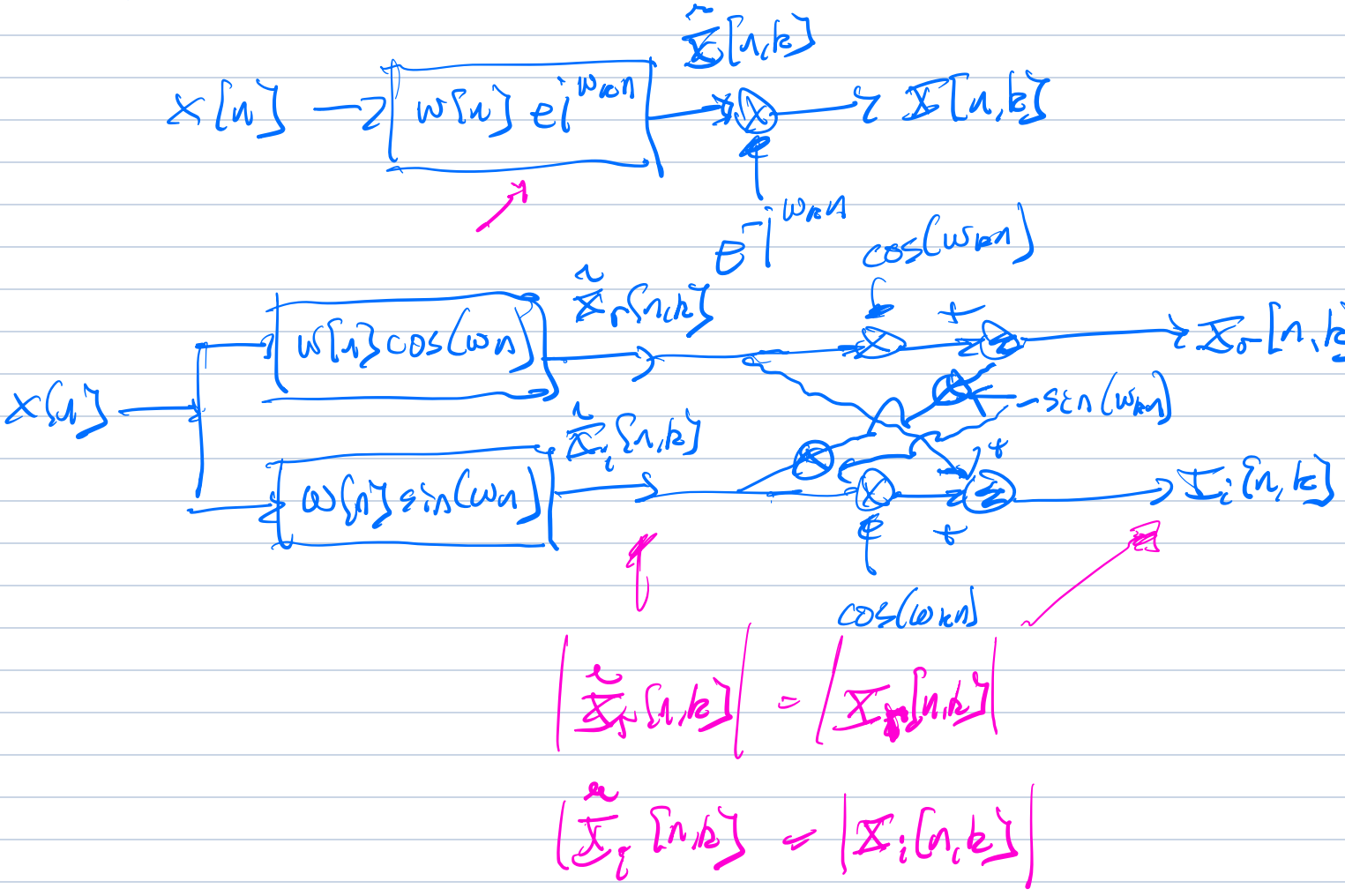


STFT IMPLEMENTATIONS WITH REAL COEFFICIENTS

LP IMPLEMENTATION



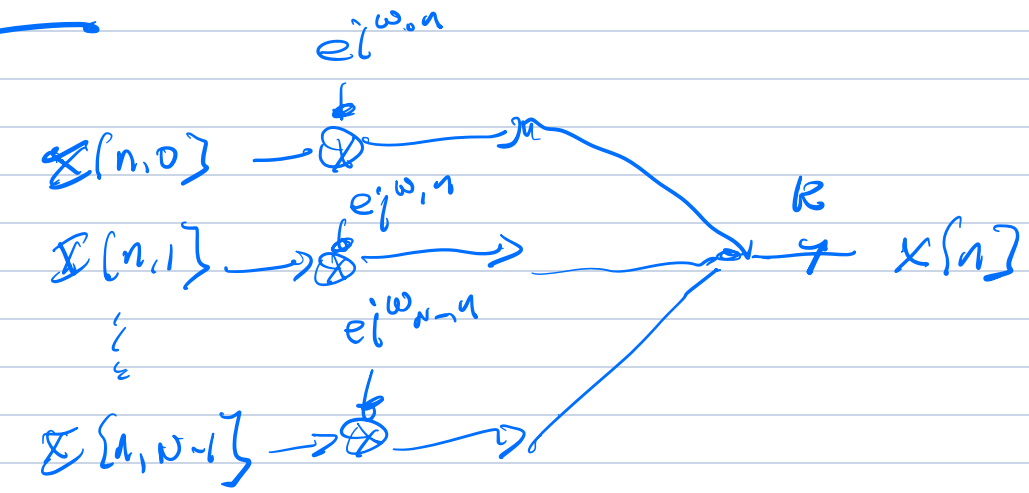
BP IMPLEMENTATION



INVERSION of STFT

GIVEN $X[n,k]$ AND $x[n]$

FBS



$$x[n] \stackrel{?}{=} A \sum_{k=0}^{N-1} X[n,k]\, e^{j\omega_k n}$$

$$x[n] \stackrel{?}{=} A \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}\, e^{j\omega_k n}$$

$$= A \sum_{m=-\infty}^{\infty} x[m] \sum_{k=0}^{N-1} w[n-m]\, e^{j\omega_k (n-m)}$$

$$e^{j\frac{2\pi}{N}(n-m)k}$$

$$= A \sum_{m=-\infty}^{\infty} x[m]\, w[n-m] \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(n-m)k}$$

$$\frac{1 - e^{j2\pi(n-m)N}}{1 - e^{j2\pi(n-m)}} \equiv \begin{cases} N & n = m \\ 0 & \text{ELSE} \end{cases}$$

CONSTRAINT:

OBTAIN $x[n]$ USING FBS

PROVIDED THAT FOR N CHANNELS

$$w[n] \neq 0, \quad n = 0$$

$$w[n] = 0, \quad n = rN, \quad r \neq 0$$

ALT. STATEMENT

CAN OBTAIN $X(\omega)$ IF $\sum_{k=0}^{N-1} w(k) e^{j\left[\omega - \frac{2\pi k}{N}\right]}$

$= \sum_{k=0}^{N-1} \tilde{w}\left[e^{j(\omega - \omega_k)}\right] = \text{CONST}$

OLA

[SEE ADSP NOTES + PPT]