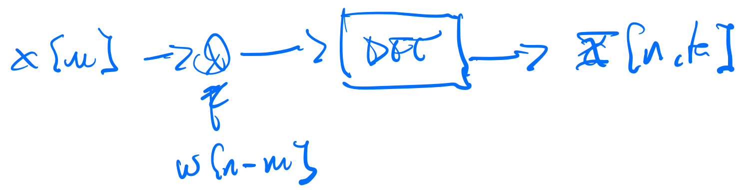
4/19/23 INTRO TO SHORT-TIME FOURIER TRANSFORMS (STFTs)

SOURCES:

- ★ ADSP CLASS NOTES
- ★ NAWAB/QUATIERI CHAPTER IN LIM + OPPENHEIM
- ★ DTSP 10.3-10.4 (SLIGHTLY DIFFERENT NOTATION)

FOURIER TRANSFORM IMPLEMENTATION OF THE STFT



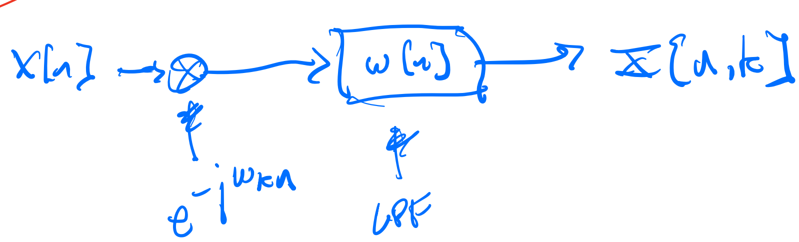
OTHER IMPLEMENTATIONS ARE POSSIBLE...

$$X[n,k] = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m} \qquad \omega_k = \frac{2\pi k}{N}$$

$$= \sum_{m=-\infty}^{\infty} w[n-m] \left(x[m]\, e^{-j\omega_k m} \right)$$

$$= w[n] * x[n]\, e^{-j\omega_k n}$$

LOW PASS FILTER IMPLEMENTATION



$$X[n,k] = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}$$

$$= e^{-j\omega_k n} \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_k m}\, e^{+j\omega_k n}$$

$$= e^{-j\omega_k n} \sum_{m=-\infty}^{\infty} x[n]\, w[n-m]\, e^{j\omega_k (n-m)}$$

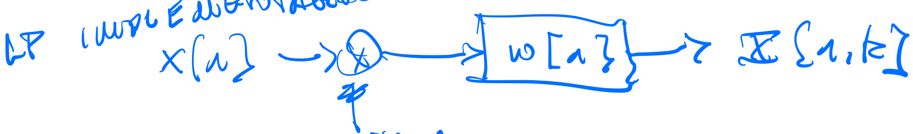
BANDPASS FILTER IMPLEMENTATION

$$X[n,k] = e^{-j\omega_k n} \left(x[n] * w[n]\, e^{j\omega_k n} \right)$$

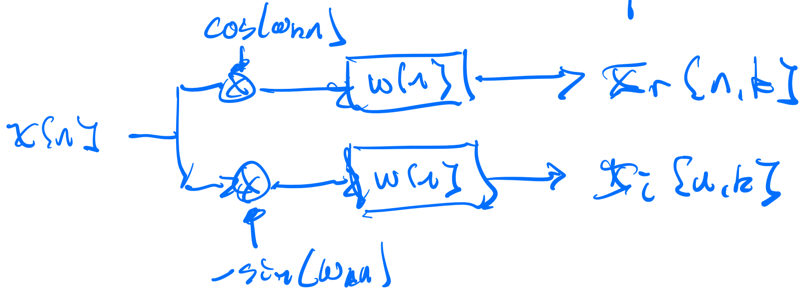
$$x[n] \longrightarrow \boxed{w[n]\, e^{j\omega_k n}} \longrightarrow \underset{\uparrow \, e^{-j\omega_k n}}{\otimes} \longrightarrow X[n,k]$$

STFT IMPLEMENTATIONS W REAL COEFFICIENTS

LP IMPLEMENTATION

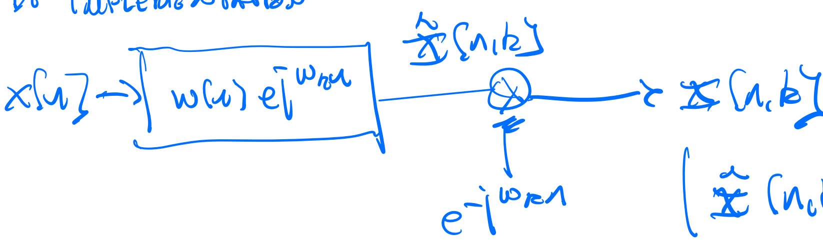


$$e^{-j\omega_k n} = \cos(\omega_k n) - j\sin(\omega_k n)$$

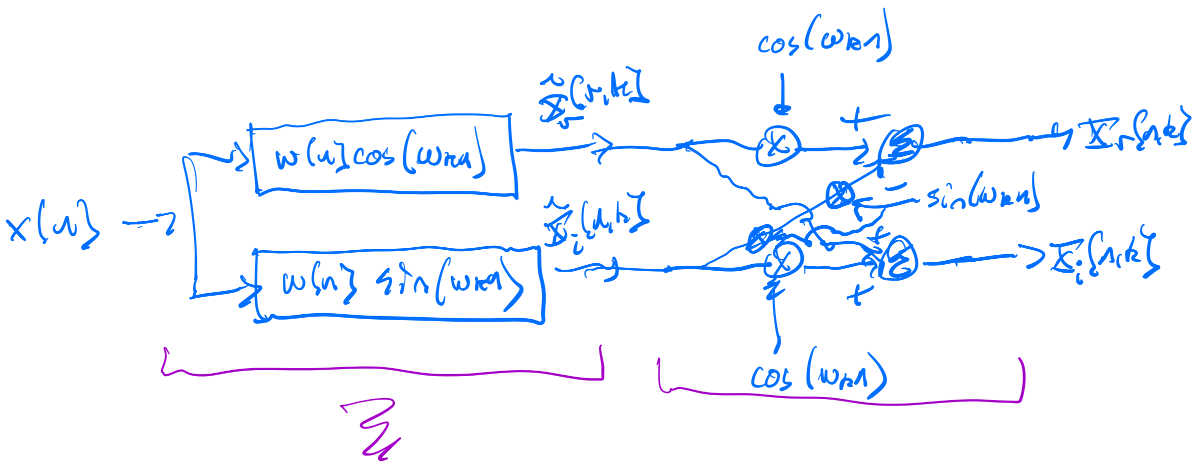


$$X[n,k] = X_r[n,k] + j X_i[n,k]$$

BP IMPLEMENTATION

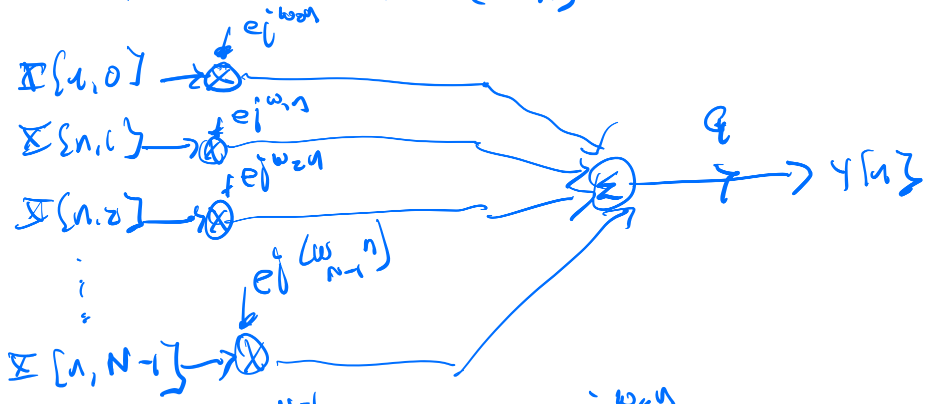


$$|\tilde{X}[n,k]| = |X[n,k]|$$



RECOVERING $x[n]$ FROM $X[n,k]$

- FILTERBANK SUMMATION (FBS)
- OVERLAP-ADD (OLA)



$$y[n] = \sum_{k=0}^{N-1} G\, X[n,k]\, e^{j\omega_k n}$$

RECALL

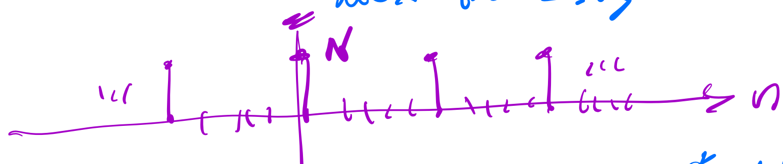
$$w[n-m]\,x[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[n,k]\, e^{j\omega_k m}$$

$$y[n] = \sum_{k=0}^{N-1} G \sum_{m=-\infty}^{\infty} w[n-m]\,x[m]\, e^{-j\omega_k m}\, e^{+j\omega_k n}$$

$$= G \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} \left(w[n-m]\, e^{j\omega_k (n-m)}\, x[m] \right)$$

$$= G \sum_{k=0}^{N-1} \left(x[n] * w[n]\, e^{j\omega_k n} \right)$$

$$y[n] = G \left(x[n] * \underbrace{w[n] \sum_{k=1}^{N-1} e^{j\omega_k n}}_{\text{MUST BE } \delta[n]} \right)$$



FBS CONSTRAINT

FOR $y[n] \stackrel{?}{=} x[n]$ REQUIRE $w[n] = 0,\ n = rN,\ r \neq 0$
$w[0] \neq 0$,

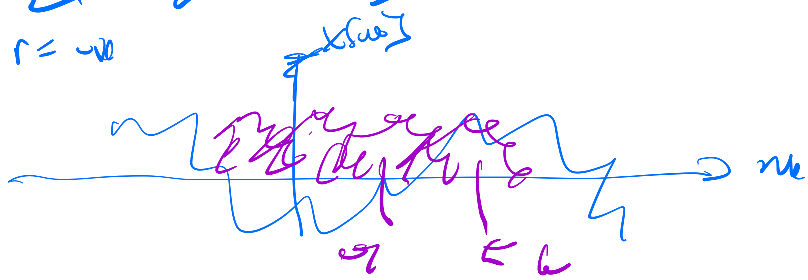
OR $\frac{1}{N}\sum_{k=0}^{N-1} W\left(e^{j(\omega-\frac{2\pi k}{N})}\right) = 1$

OLA SYNTHESIS

$w[n-m]\, x[m] \Longleftrightarrow X[n,k]$

COMPOSITE STFT

$$\sum_{r=-\infty}^{\infty} w[n-rL-m]\, x[m]$$



$$\sum_{r=-\infty}^{\infty} w[n-rL-m]\, x[m] = x[m] \sum_{r} w[n-rL-m]$$