FIR FILTER DESIGN USING WINDOWING

OS 4e 7.5–7.6

WANT $\quad H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n]\, e^{-j\omega n}$

Let $h[n] = h_d[n] \cdot w[n]$

$w[n] \neq 0, \quad 0 \leq n \leq N-1$

$0 \leq n \leq M$

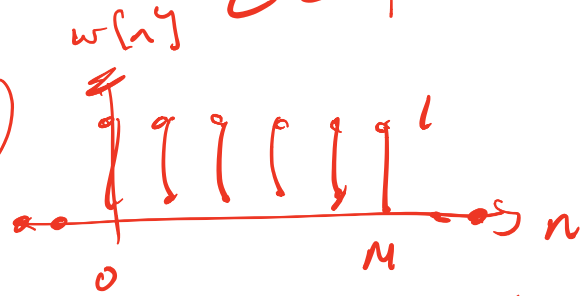
$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta})\, W(e^{j(\omega-\theta)})\, d\theta$$

$$= H_d(e^{j\omega}) \circledast W(e^{j\omega})$$

$$W(e^{j\omega}) = \sum_{n=0}^{M} w[n]\, e^{-j\omega n}$$

SIMPLE EXAMPLE: $\quad h[n] = \begin{cases} h_d[n], & 0 \leq n \leq M \\ 0, & \text{OTHERWISE} \end{cases}$

(RECTANGULAR OR BOXCAR WINDOW)

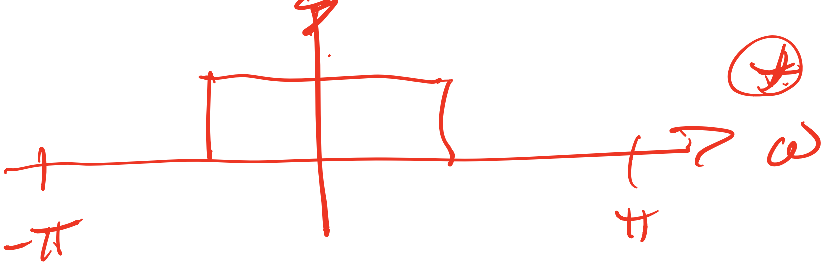


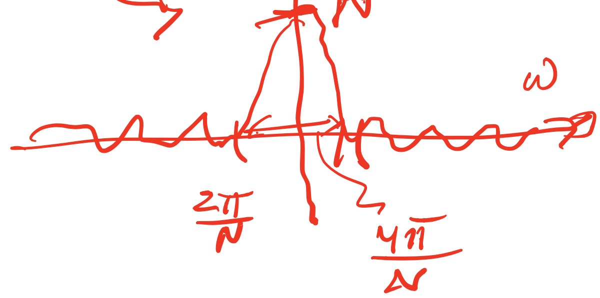
$$w[n] \Leftrightarrow W(e^{j\omega}) = \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\, e^{-j\omega\frac{(N-1)}{2}}$$

IDEAL LPF

$H_d(e^{j\omega})$

$W(e^{j\omega})$

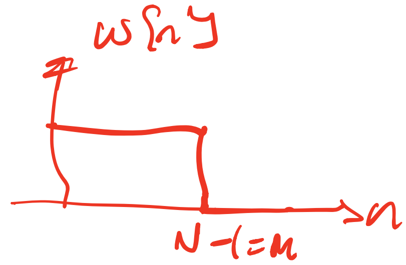




"CLASSIC WINDOWS"

RECTANGULAR WINDOW

$w[n] = 1, \quad 0 \le n \le M$



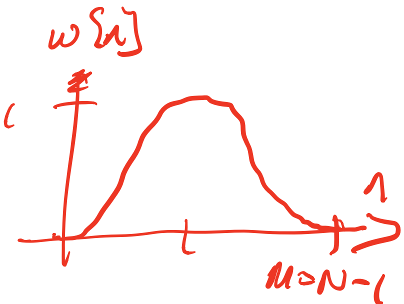
BARTLETT WINDOW

$$w[n] = \begin{cases} \frac{2n}{M}, & 0 \le n \le \frac{M}{2} \\ 2\left(1-\frac{n}{M}\right), & \frac{M}{2} \le n \le M \\ 0, & \text{ELSE} \end{cases}$$

$1 - \frac{\left|n - \frac{M}{2}\right|}{M}$?

VON HANN "HANNING" WINDOW

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{M}\right)\right], & 0 \le n \le M \\ 0, & \text{ELSE} \end{cases}$$



HAMMING WINDOW

$$w[n] = \begin{cases} .54 - .46\cos\left(\frac{2\pi n}{M}\right) & 0 \le n \le M \\ 0, & \text{ELSE} \end{cases}$$

BLACKMAN WINDOW

$$w[n] = \begin{cases} .42 - .5\cos\left(\frac{2\pi n}{M}\right) + .18\cos\left(\frac{4\pi n}{M}\right) & 0 \le n \le M \\ 0, & \text{ELSE} \end{cases}$$

DESIGN EXAMPLE

LPF $\omega_p = .55\pi$, $\omega_s = .6\pi$ $\rightarrow \omega_c = .575\pi$

$\Delta\omega = .05\pi$

PASSBAND RIPPLE $\leq$ 1 dB
STOPBAND RIPPLE $\leq$ -60 dB

1. FIND SHAPE OF WINDOW BASED ON STOPBAND ATTENUATION
2. FIND SIZE OF WINDOW BASED ON TRANSITION BAND SPEC
3. WRITE $h[n] = h_d[n - \frac{M}{2}] \cdot w[n]$

---

60 dB $\Rightarrow$ BLACKMAN

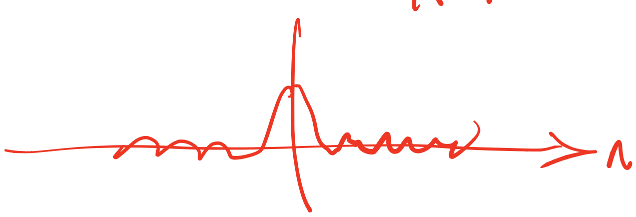
$$\Delta\omega = .05\pi = \frac{12\pi}{M}$$

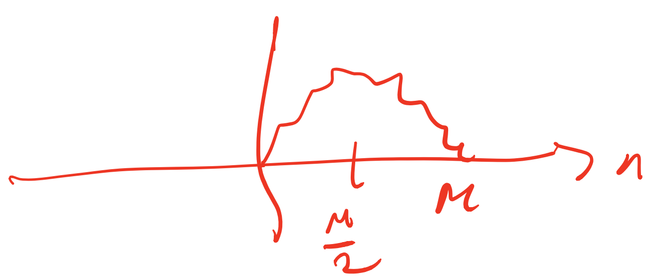
$$M = \frac{12}{.05} = 240, \quad \frac{M}{2}$$

$$h[n] = h_d[n - \frac{M}{2}] \cdot w[n]$$

$$= \frac{\sin\left(.575\pi(n-120)\right)}{\pi(n-120)} \cdot \left[.42 - .5\cos\left(\frac{2\pi n}{240}\right) + .08\cos\left(\frac{4\pi n}{240}\right)\right]$$

$$h_d[n] = \frac{\sin(\omega_c n)}{\pi n} \qquad w[n]$$





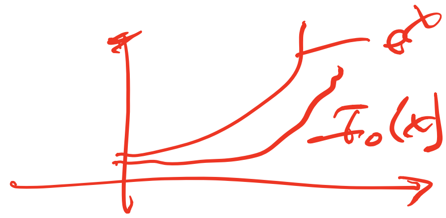
FOR LINEAR PHASE, $h[n]$ MUST BE SYMMETRIC ABOUT $n = \frac{M}{2}$

ABOUT $n = \frac{M}{2}$

KAISER WINDOW

$$w[n] = \frac{I_0\left[\beta\left(1-\left(\frac{n-\alpha}{\alpha}\right)^2\right)^{1/2}\right]}{I_0(\beta)} \qquad \alpha = \frac{M}{2}$$

I_0 IMAGINARY BESSEL FN. of ~~THE~~ 0^{th} KIND



DESIGN USING KAISER WINDOWS

$\Delta\omega = .05\pi$

$A = -20\log_{10}\delta_s = 60$

$$M = \frac{A-8}{\Delta\omega\,(2.285)} = \frac{52}{(.05\pi)(2.285)}$$

$144.8 \Rightarrow \cancel{149}\ 145$

$$\beta = \begin{cases} .1102(A-8.7), & A > 50 \\ .5842(A-21)^{.4} + .07886(A-21), & 21 \le A \le 50 \\ 0, & A < 21 \end{cases}$$