4/8/23

OPTIMUM EQUIRIPPLE FILTERS:

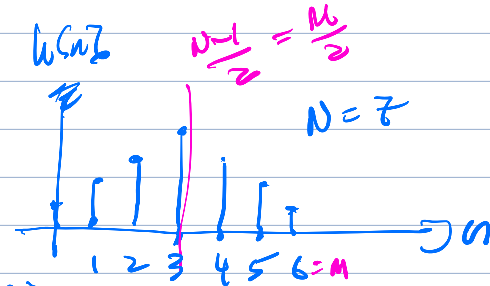
THE PARKS–McCLELLAN ALGORITHM

(OSYP 7.7–7.9)

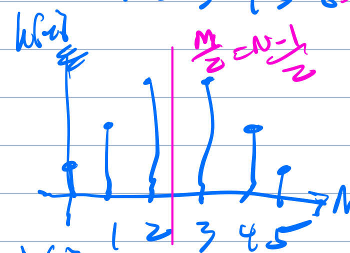
$M = N - 1$

TYPES OF SYMMETRY FOR FILTERS

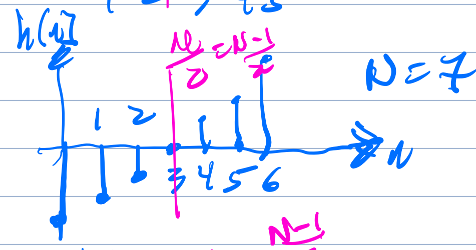
TYPE I N ODD, EVEN SYMMETRY



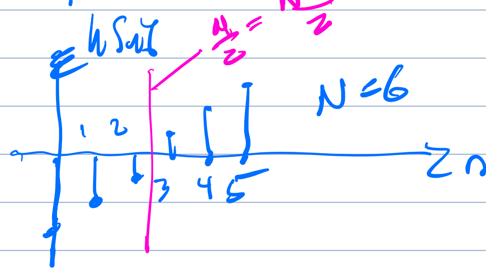
TYPE II N EVEN, EVEN SYMMETRY



TYPE III N ODD, ODD SYMMETRY



TYPE IV N EVEN ODD SYMMETRY



$h[n] = h'[n - \frac{M}{2}]$ (N ODD)

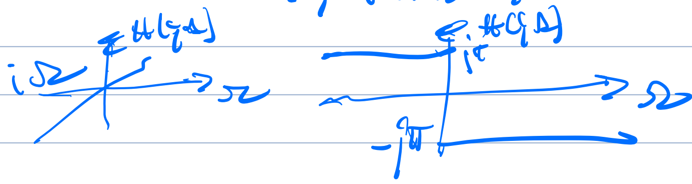
$H(e^{j\omega}) = H'(e^{j\omega})\, e^{-j\frac{\omega M}{2}}$ (ANY N)

TYPE I, II FILTERS $H'(e^{j\omega})$ REAL, EVEN

LPF, HPF, BPF, NOTCH

TYPE III, IV FILTERS $H'(e^{j\omega})$ IMAGINARY, ODD

DIFFERENTIATORS, PHASE SHIFTERS



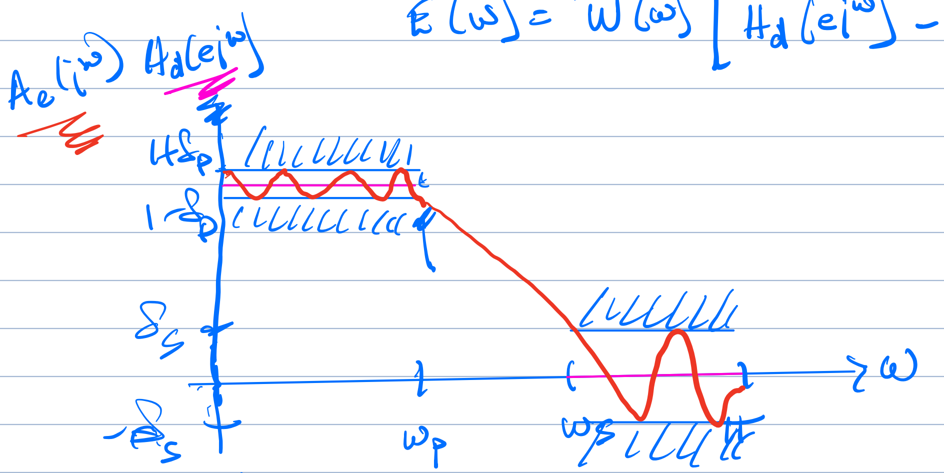
OPTIMUM EQUIRIPPLE DESIGN

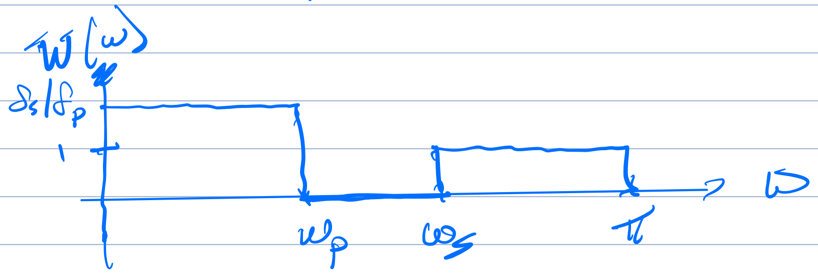
GIVEN ω_p, ω_s, N, δ_p / δ_s

FIND FILTER COEFFS $h[n]$ THAT MINIMIZE δ_p, δ_s

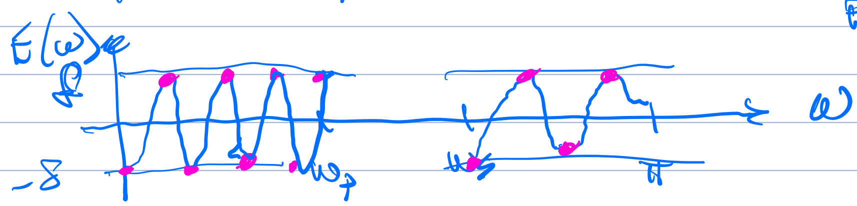
OR $\min \max |E(\omega)|$

$$E(\omega) = W(\omega)\left[H_d(e^{j\omega}) - A_e(e^{j\omega})\right]$$





$$E(\omega) = W(\omega)\left[H_d(e^{j\omega}) - A_e(e^{j\omega})\right]$$



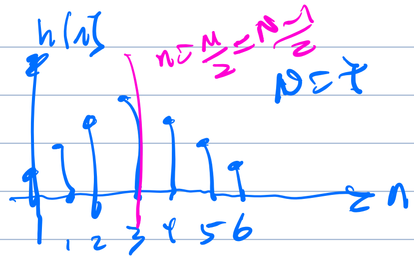
THE ALTERNATION THEOREM

"IN ORDER FOR $\max |E(\omega)|$ TO BE MINIMIZED, IT IS NECESSARY & SUFFICIENT FOR $E(\omega)$ TO EXHIBIT AT LEAST $L+2$ ALTERNATIONS."

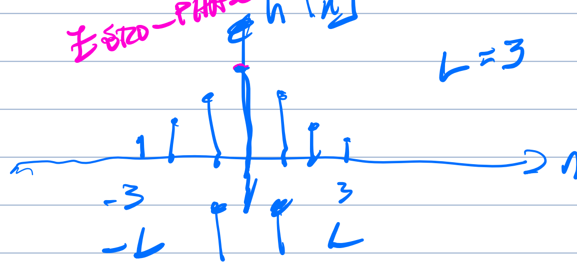
ALTERNATIONS ARE POINTS FOR WHICH $E(\omega) = \pm\delta$

CONSIDER TYPE I FIR FILTER

LINEAR PHASE



ZERO-PHASE



$L = \frac{N-1}{2}$; $N = 2L+1$

$$H'(e^{j\omega}) = \sum_{n=-L}^{L} h[n]\, e^{-j\omega n} = h[0] + \sum_{\ell=1}^{L} h[\ell] \underbrace{\left[e^{-j\omega \ell} + e^{+j\omega \ell} \right]}_{2\cos(\omega \ell)}$$

$$H'(e^{j\omega}) = h[0] + 2 \sum_{\ell=1}^{L} h[\ell] \cos(\ell \omega)$$

FOR ?

CHEBYSCHEV POLYNOMIALS

$$\sum_{\ell=0}^{L} a_\ell \left[\cos(\omega)\right]^{\ell}$$

CONSIDER:

1. $\cos(\omega)$

2. $\left(\cos(\omega)\right)^2 = \frac{1}{2} + \frac{1}{2}\cos(2\omega)$

3. $\left(\cos(\omega)\right)^3 = \left(\frac{1}{2} + \frac{1}{2}\cos(2\omega)\right)\cos(\omega)$

$$= \frac{1}{2}\cos(\omega) + \frac{1}{2}\underbrace{\cos(2\omega)\cos(\omega)}$$

$$= \frac{1}{2}\cos(\omega) + \frac{1}{4}\cos(\omega) + \frac{1}{4}\cos(3\omega)$$

$$= \frac{3}{4}\cos(\omega) + \frac{1}{4}\cos(3\omega)$$

IMPLICATION: FOR TYPE I FILTERS,

$$H'(e^{j\omega}) = h[0] + 2\sum_{\ell=1}^{L} h[\ell]\cos(\omega \ell) = \sum_{k=0}^{L} a_k (\cos\omega)^k$$

For $H'(e^{j\omega}) = \sum_{k=0}^{L} a_k \left[\cos(\omega)\right]^k$

For which ω will $H'(e^{j\omega})$ have zero slope?

$$H'(e^{j\omega}) \approx A_e(e^{j\omega})$$

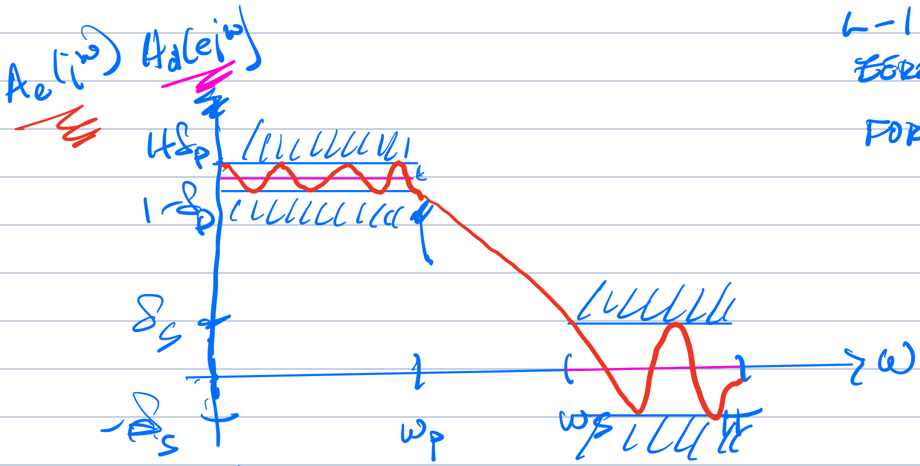
$$\frac{\partial H'(e^{j\omega})}{\partial \omega} = \frac{d}{d\omega} \cdot \sum_{k=0}^{L} a_k \left(\cos(\omega)\right)^k$$

$$= -\sum_{k=0}^{L} a_k\, k \left[\cos(\omega)\right]^{k-1} \cdot \sin(\omega) = 0$$

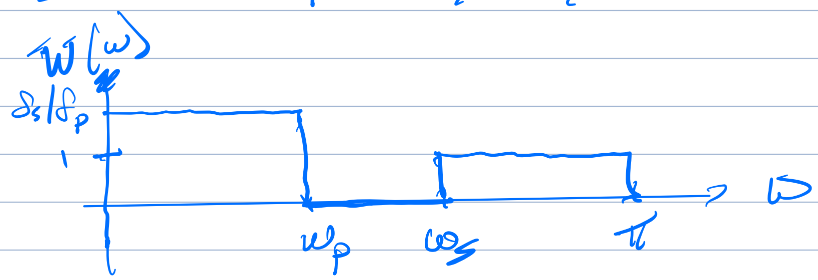
$\omega = 0, \pi$

$$= -\sin(\omega) \sum_{k=1}^{L} a_k \cos(\omega)^{k-1}$$

$L-1$ points of zero slope for $0 < \omega < \pi$

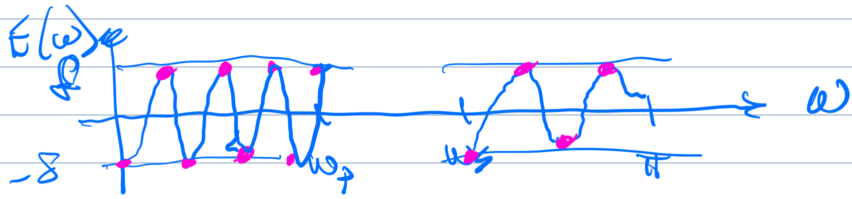


To satisfy alternation thm. need $L+2$ alts.



Zero-slope pts. $0, \pi$, $L-1$ more, ω_p, ω_s

Max possible # of alts is $L+3$



IMPLEMENTATION of PARKS – McCLELLAN

1. CHOOSE ALT POINTS @ ω_p, ω_s, L-1 MORE FOR $0 < \omega < \pi$
   PLUS $\omega = 0$ OR $\omega = \pi$

EX SUPPOSE L=10, N=21, ASSUME $W(\omega) = 1$

2. THREAD CHEBYSCHEV POLYNOMIAL THROUGH PTS
3. "REMEZ EXCHANGE" REPLACE ALT. FREQS BY BY ACTUAL EXTREMAL POINTS