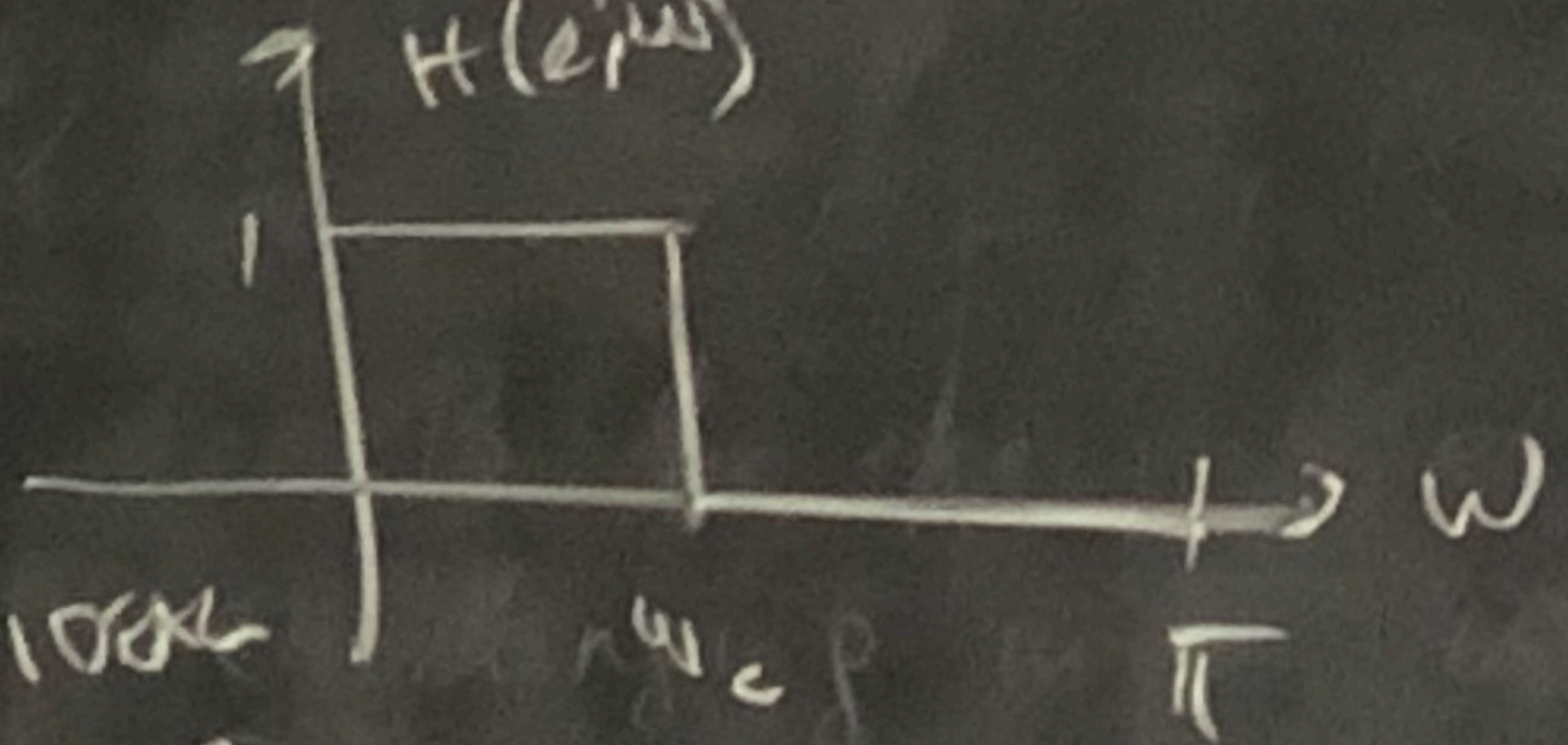
IIR FILTER DESIGN EXAMPLES (OSYP 7.xx)

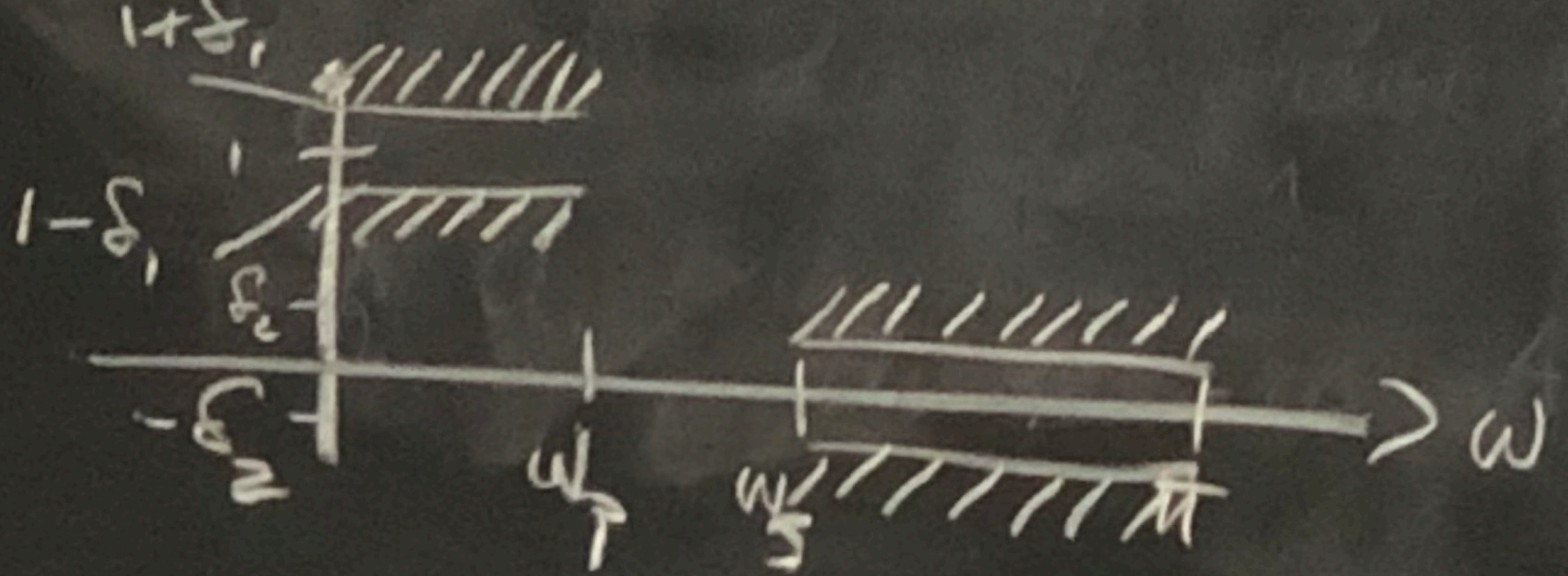
OSYP 7.3.1

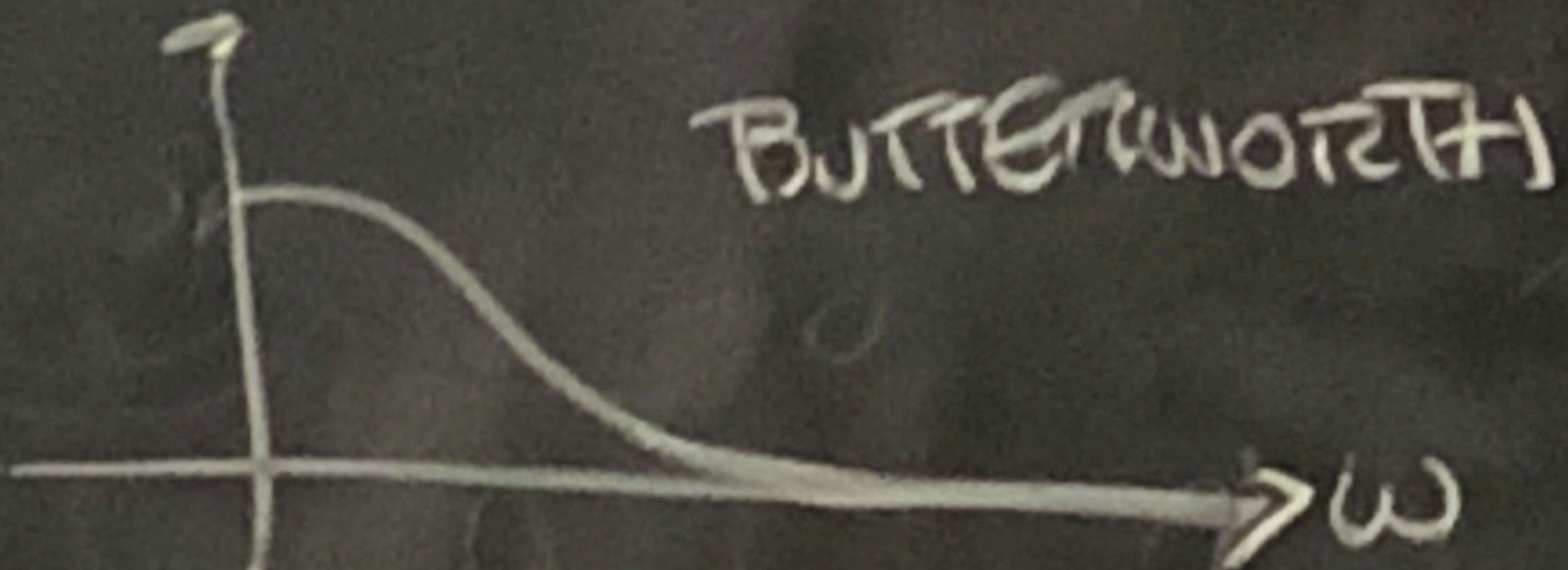
See also handout for this lecture

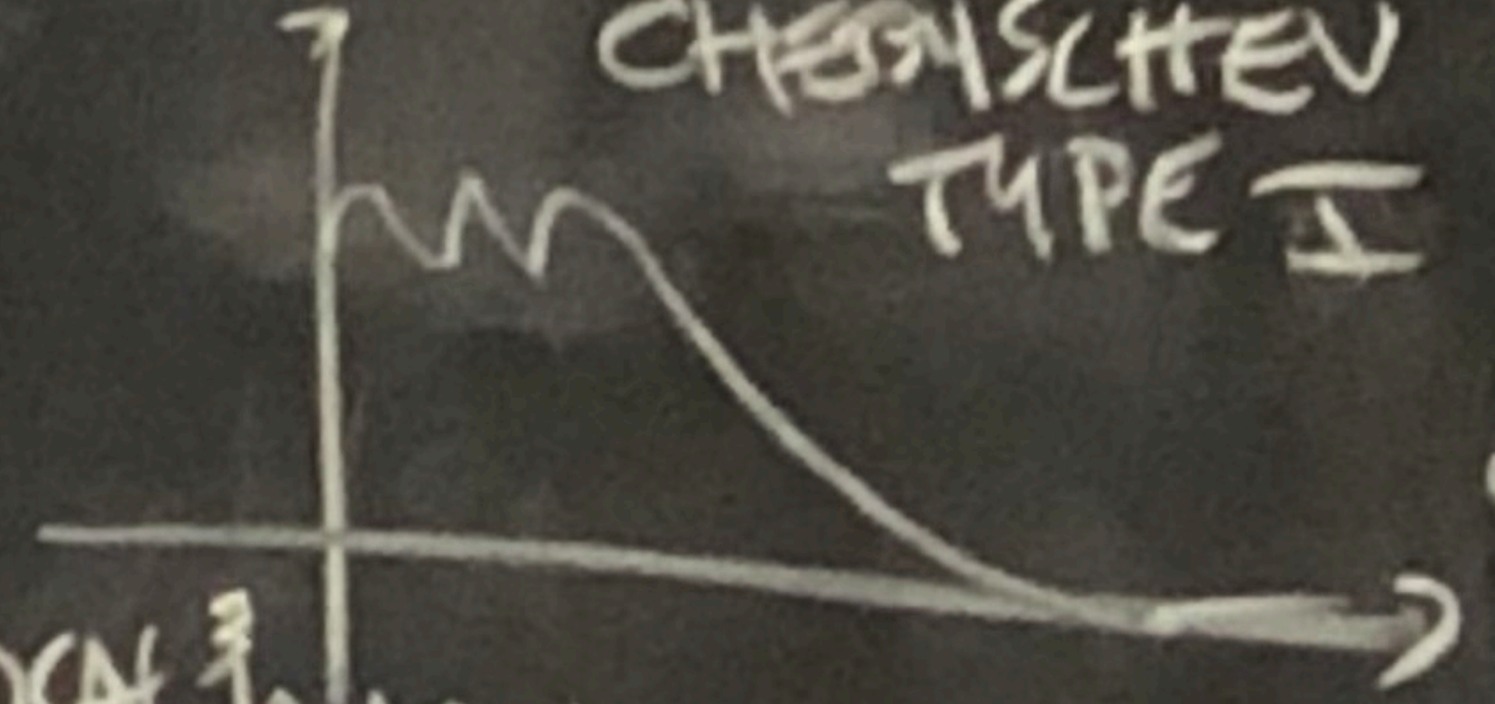
IDEAL LPF

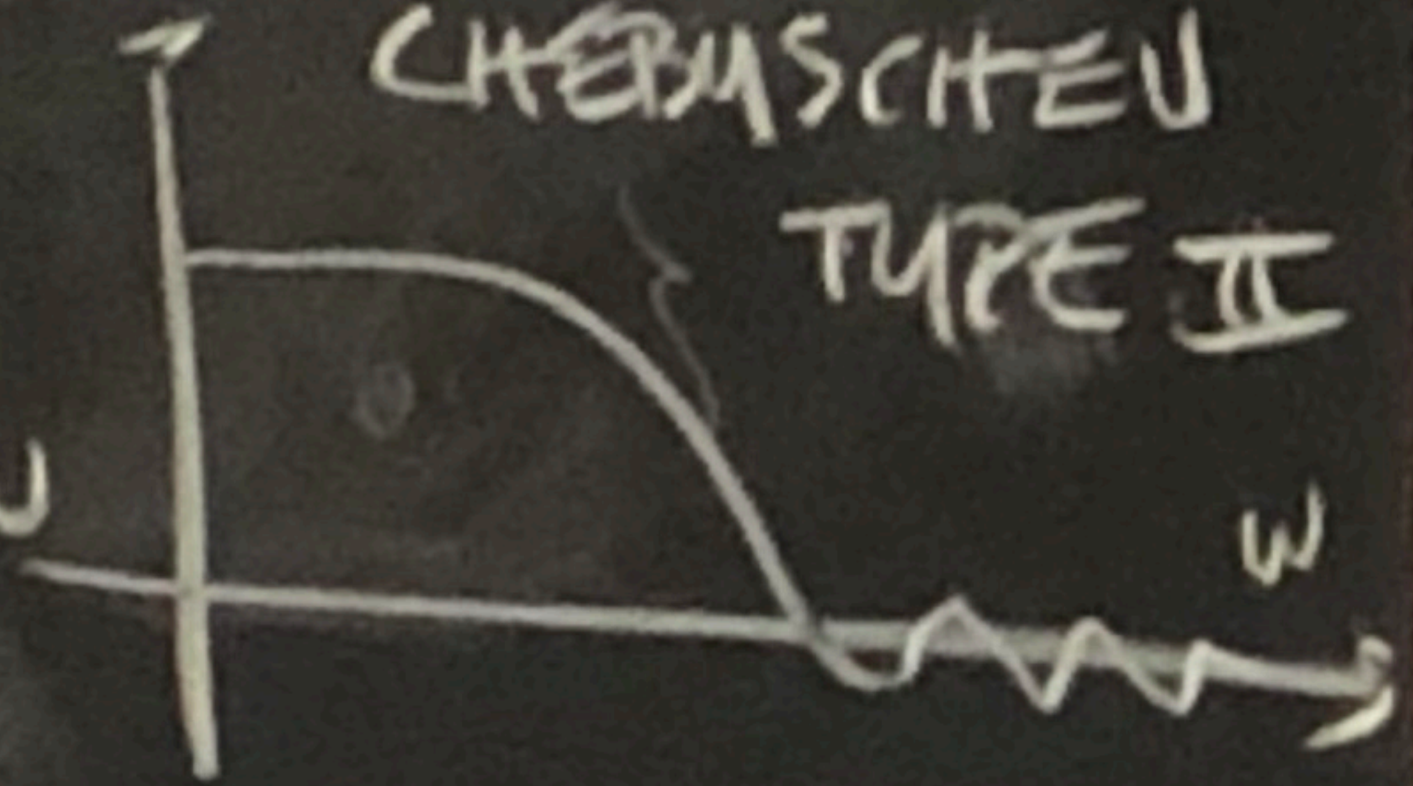


NON-IDEAL

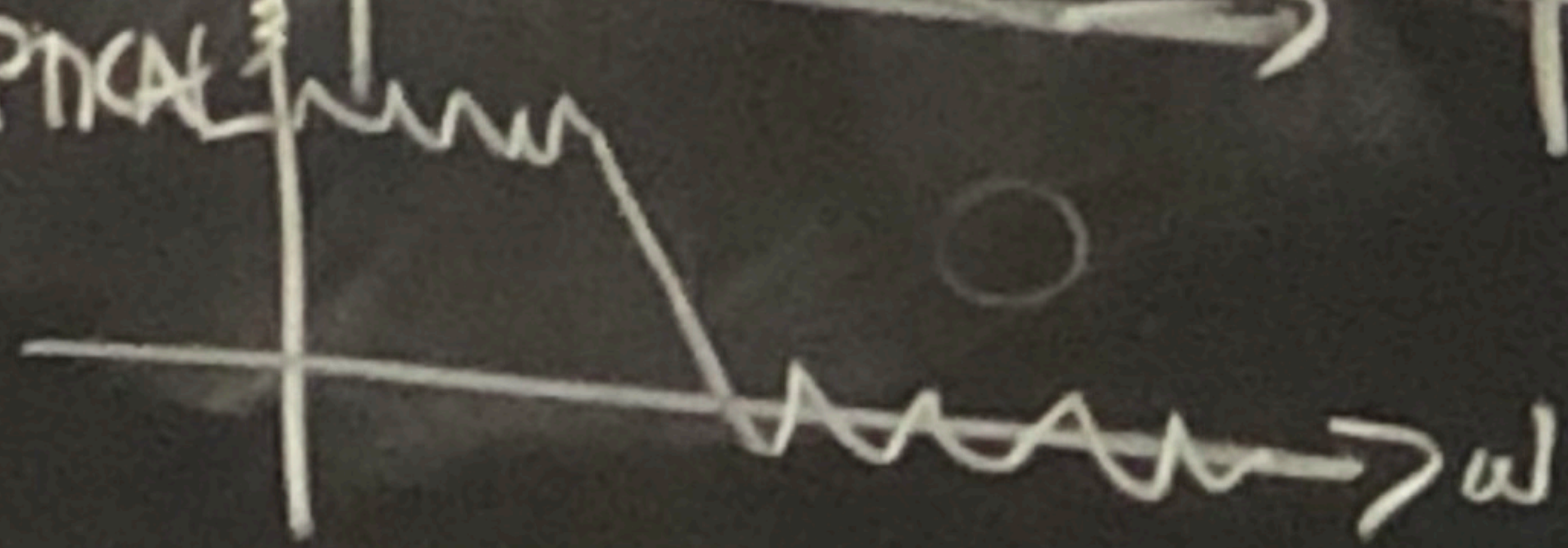






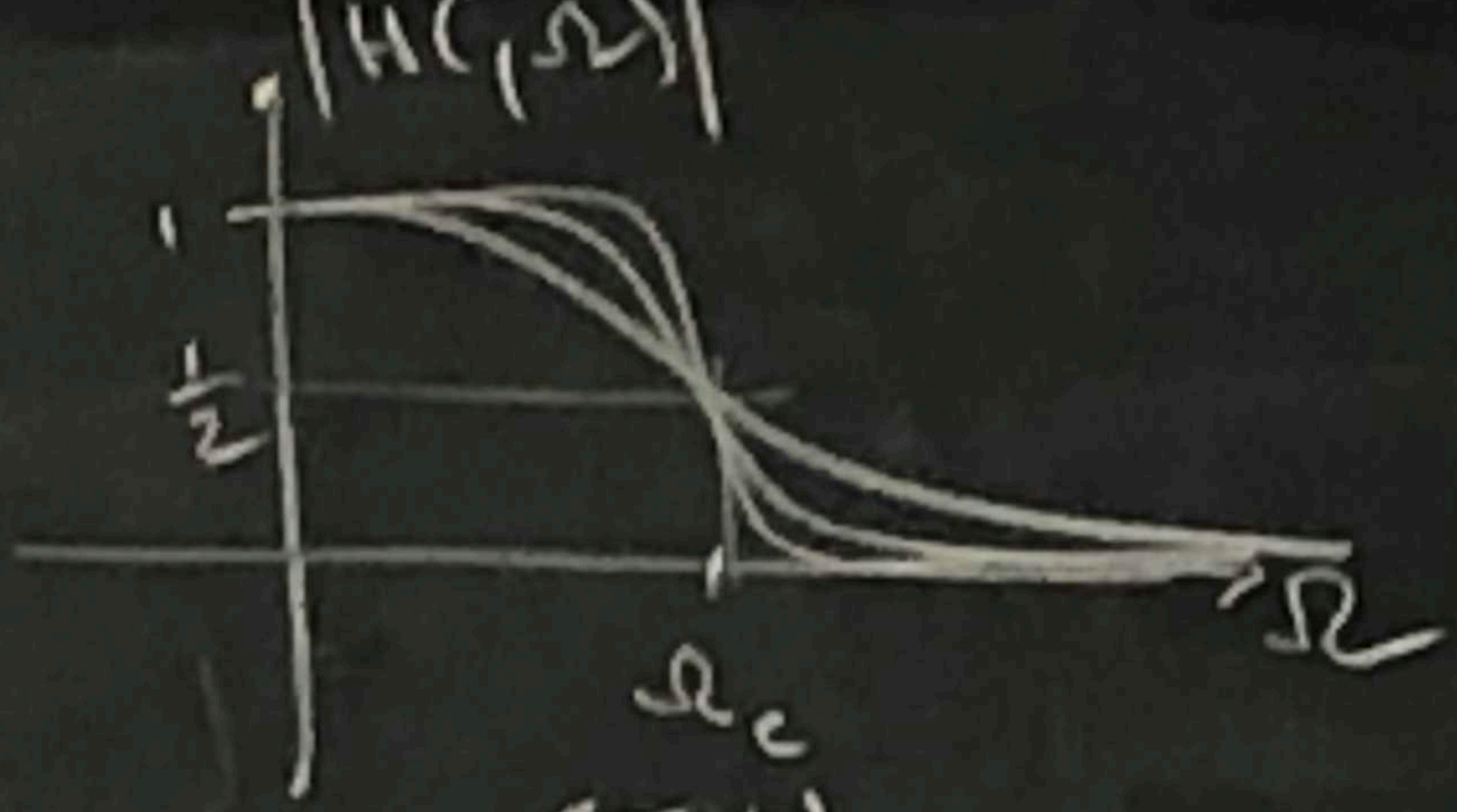


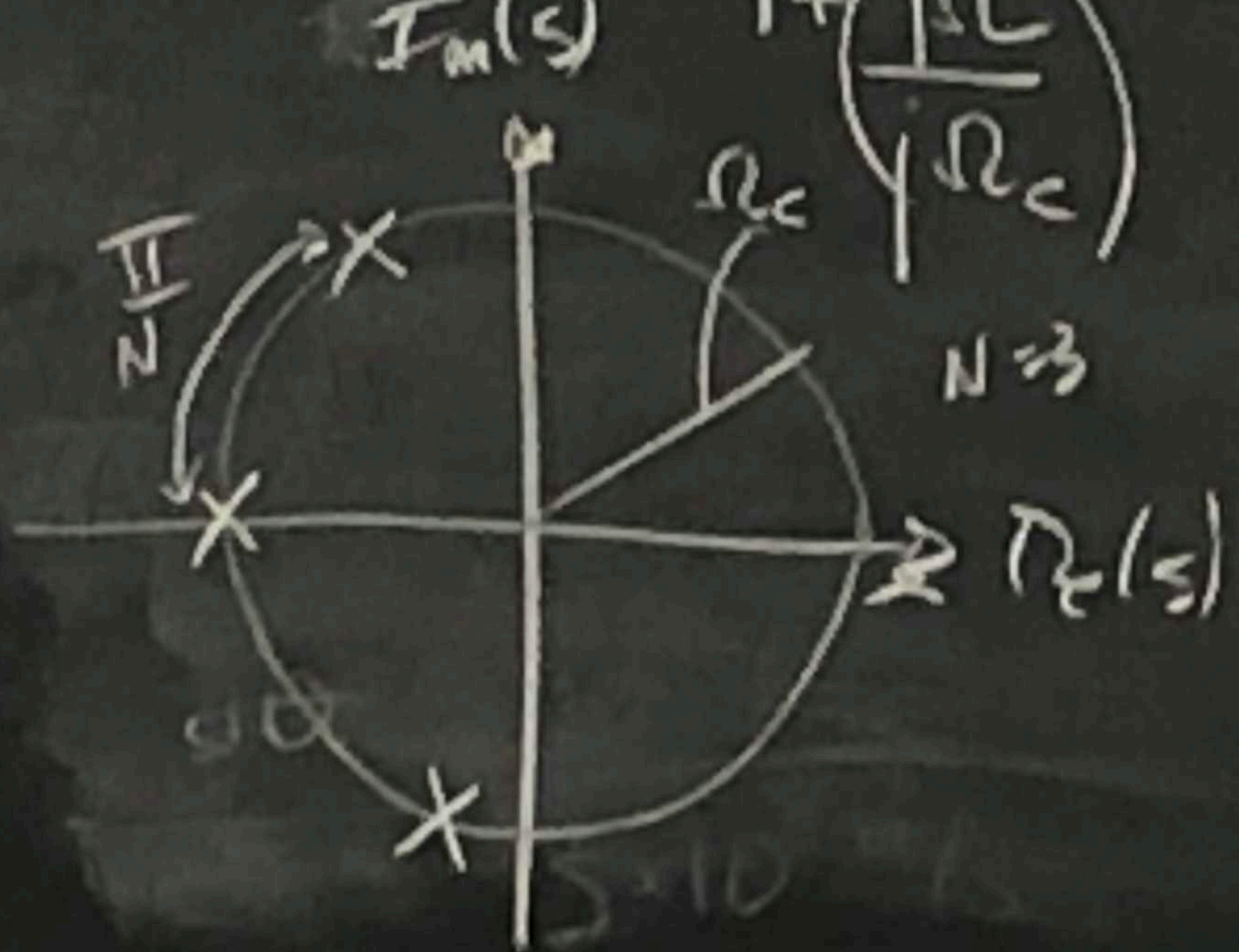
ELLIPTICAL

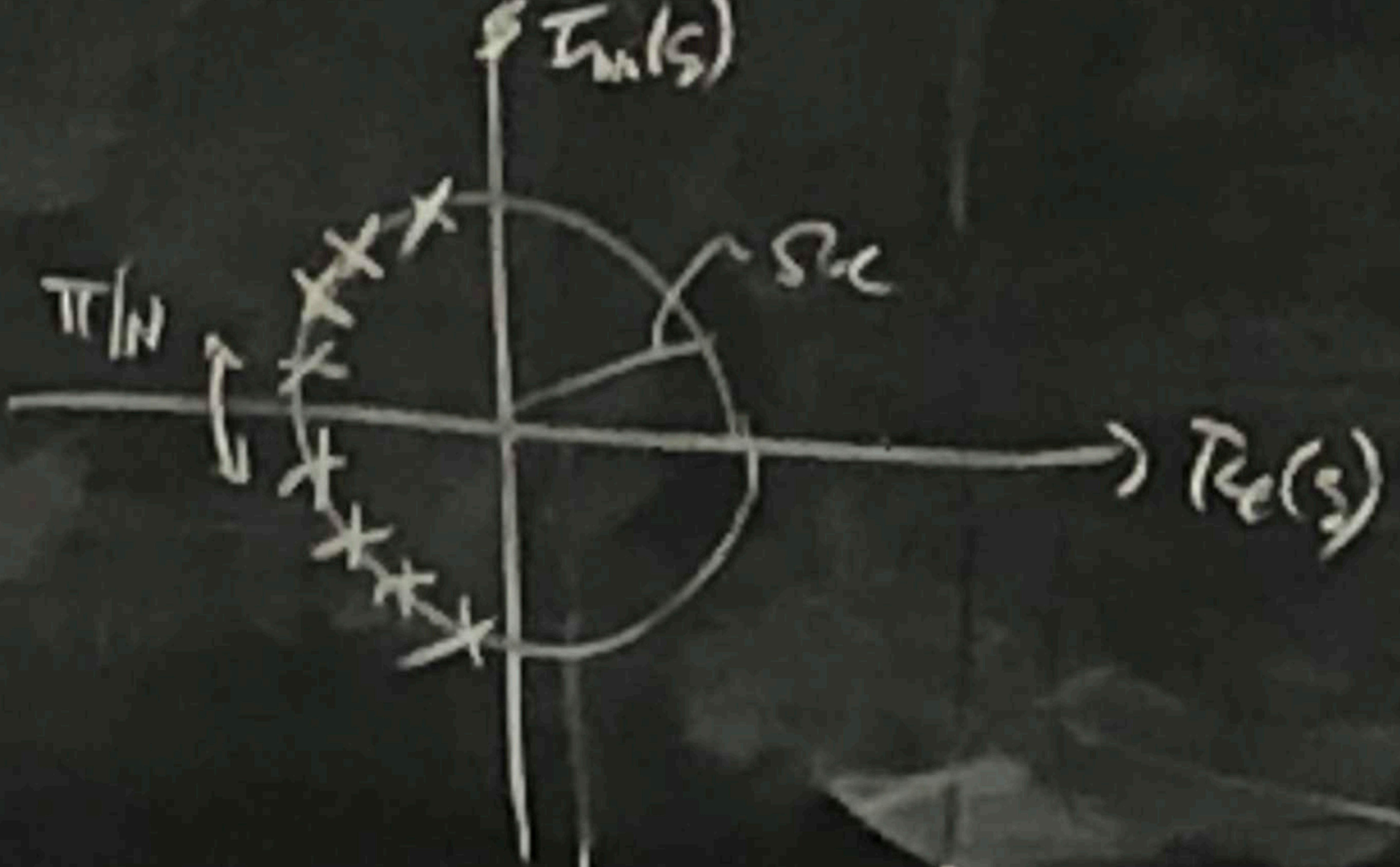


BUTTERWORTH FILTER

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$$







SPECS of DT LPF

$\omega_p = .15\pi$

$\omega_s = .35\pi$

PASSBAND RIPPLE $-3\,dB \le |H(e^{j\omega})| \le 0\,dB$, $0 \le |\omega| \le \omega_p$

STOPBAND RIPPLE $-20\,dB \ge |H(e^{j\omega})|$, $\omega_s \le |\omega| \le \pi$

$1 - \delta_1 = -3\,dB$

AMPLITUDE $\quad dB = 20 \log_{10}(1-\delta_1) = -3$

P

$\log_{10}(1-\delta_1) = -\frac{3}{20}$ $\quad -3/20$

$1-\delta_1 = 10^{-3/20} = .7079$

$-20 = 20 \log_{10} \delta_2; \quad \delta_2 = 10^{-20/20} = .1$

LET $K_1 = \frac{1}{(1-\delta_1)^2} - 1 = .9953$

$K_2 = \frac{1}{\delta_2^2} - 1 = 99$

MEETING SPEC @ Ω_S

$$|H(j\Omega_S)|^2 = \frac{1}{1+\left(\frac{\Omega_S}{\Omega_C}\right)^{2N}} = \delta_2^2$$

$$\boxed{\left(\frac{\Omega_S}{\Omega_C}\right)^{2N} = \frac{1}{\delta_2^2} - 1 = K_2}$$

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_C}\right)^{2N}}$$

MEETING SPEC AT $\Omega_P \Rightarrow$

$$\frac{1}{1+\left(\frac{\Omega_P}{\Omega_C}\right)^{2N}} = (1-\delta_1)^2$$

$$1+\left(\frac{\Omega_P}{\Omega_C}\right)^{2N} = \frac{1}{(1-\delta_1)^2}$$

$$\boxed{\left(\frac{\Omega_P}{\Omega_C}\right)^{2N} = \frac{1}{(1-\delta_1)^2} - 1 = K_1}$$

MEETING SPECS

@ Ω_P:

USING IMPULSE INV.

$$\omega = \Omega T \; ; \; \Omega = \frac{\omega}{T}$$

$$\Omega_p = \frac{.15\pi}{T} = \frac{.4716}{T}$$

$$\Omega_s = \frac{.35\pi}{T}$$

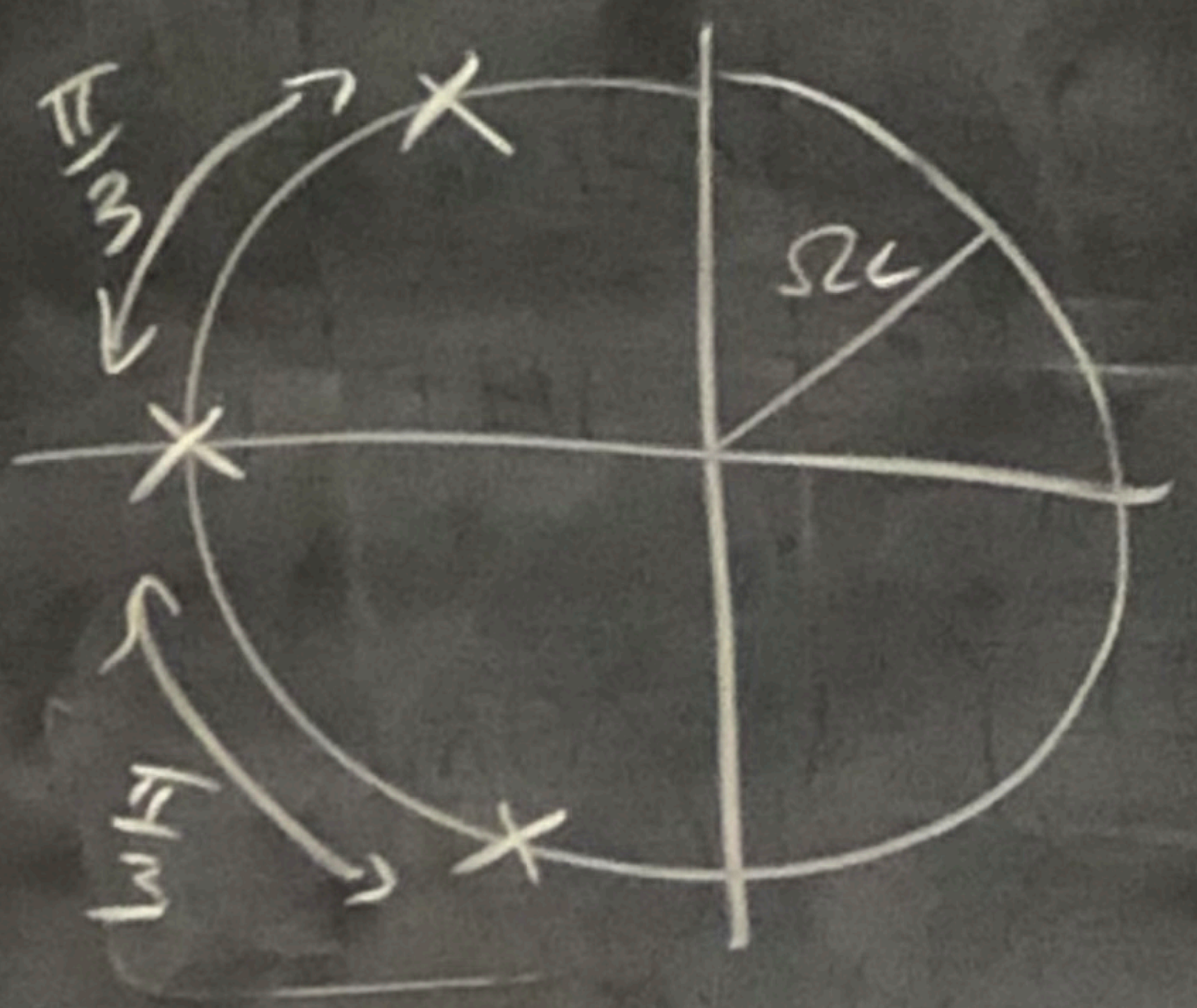
MEETING SPECS @ PASSBAND:

$$\left(\frac{\Omega_p}{\Omega_c}\right)^6 = K_1 \, ; \quad \Omega_c = \frac{\Omega_p}{K_1^{1/2N}} = \frac{.4716}{T}$$

MEETING SPEC @ STOPBAND

$$\left(\frac{\Omega_s}{\Omega_c}\right)^6 = K_2 \, ; \quad \Omega_c = \frac{\Omega_s}{K_2^{1/2N}} = \frac{.5112}{T}$$

$N = 3, \quad \Omega_c = \frac{.4806}{T}$



BUTTERWORTH DESIGN + IMPLEMENTATION

IMPULSE INV. POLE of $H(s)$ @ $s_k = s$

$\Rightarrow$ POLE of $H(z)$ @ $z = e^{s_k T}$

$$H(s) = \frac{\Omega_c^3}{(s - \Omega_c e^{j\pi})(s - \Omega_c e^{j2\pi/3})(s - \Omega_c e^{-j\frac{2\pi}{3}})}$$

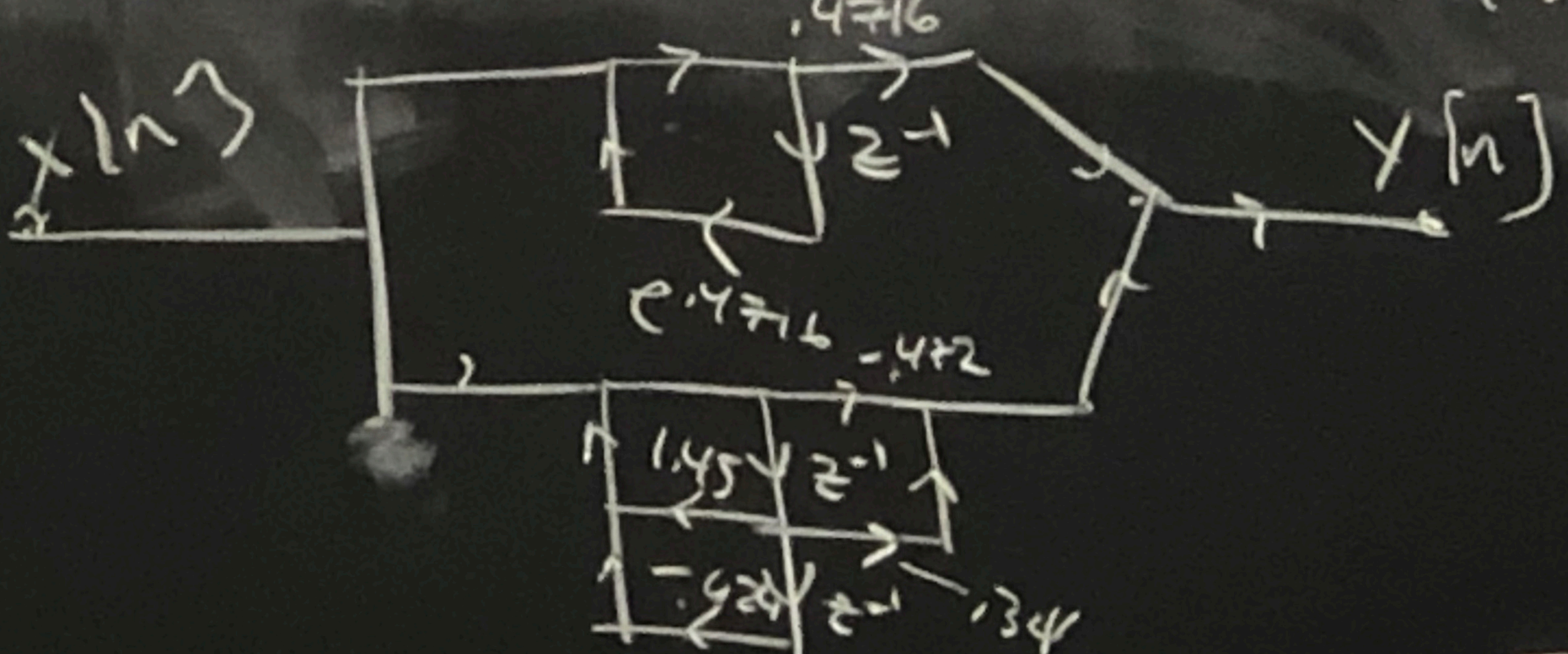
$$H(s) = \frac{(.4716/T)^3}{\left(s + \frac{.4716}{T}\right)\left(s - \left(\frac{-.24 + .41j}{T}\right)\right)\left(s - \left(\frac{-.24 - .41j}{T}\right)\right)}$$

$$H(s) = \sum_{k=1}^{N} \frac{A_k}{s - s_k} \quad \text{RESIDUE}$$

$$= \frac{.4716}{s + \frac{.4716}{T}} + \frac{-.2316 - .136j}{s - \left(\frac{-.236 + .408j}{T}\right)} + \frac{-.2316 + .136j}{s - \left(\frac{-.236 - .408j}{T}\right)}$$

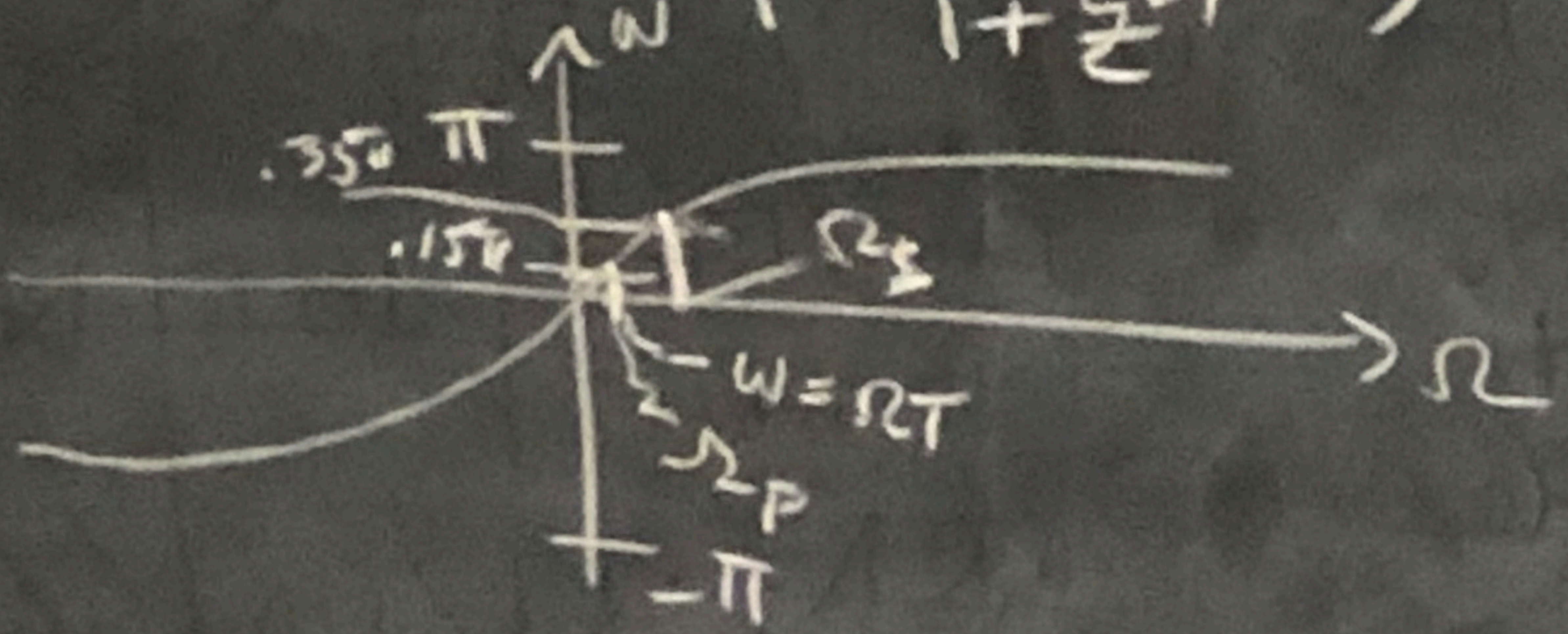
$$H(z) = \frac{A_k}{1 - e^{s_k T} z^{-1}} = \frac{.4716}{1 - e^{-.4716} z^{-1}} + \frac{-.2316 - .136j}{\left(1 - e^{-.236 + .408j} z^{-1}\right)} + \frac{-.2316 + .136j}{\left(1 - e^{-.236 + .408j} z^{-1}\right)}$$

$$H(z) = \frac{.4716}{1 - e^{.4716} z^{-1}} + \frac{-.472 + .341 z^{-1}}{1 - 1.45 z^{-1} + .624 z^{-2}}$$



BILINEAR TRANSFORMATION

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \; ; \; \Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$



$$\Omega_p = \frac{2}{T}\tan\left(\frac{.15\pi}{2}\right) = \frac{.4802}{T}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{.35\pi}{2}\right) = \frac{1.226}{T}$$

(BILINEAR)

$$N = \frac{1}{2}\,\frac{\log(k_1/k_2)}{\log(\Omega_s/\Omega_p)} = 2.4546$$

MATCHING @ PASSBAND

$$\Omega_c = \frac{\Omega_p}{k_1^{1/6}} = \frac{.4806}{T}$$

$$H(z) = .00854\,\frac{(1+3z^{-1}+3z^{-2}+z^{\ldots}}{1-2.064z^{-1}+1.517z^{-2}\ldots}$$

P

H

$$N = \frac{1}{2}\frac{\log(k_1/k_2)}{\log(\Omega_s/\Omega_p)} = 2.4546$$

MATCHING @ PASSBAND

$$) = \frac{.4802}{T}$$

$$\Omega_c = \frac{\Omega_p}{k_1^{1/6}} = \frac{.4806}{T}$$

$$) = \frac{1.226}{T}$$

$$H(s) = \frac{\Omega_c^3}{(s+\Omega_c)(s-\Omega_c e^{j\frac{2\pi}{3}})(s-\Omega_c e^{-j\frac{2\pi}{3}})}$$

Poly

$$H(s) = \frac{.110/T^3}{s^3 + .9612\frac{s^2}{T} + \frac{.462\,s}{T^2} + \frac{.110}{T^3}}$$

$$H(z) = \frac{.110/T^3}{\left(\frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}\right)^3 + \frac{.9612}{T}\left(\quad\right)^2 + \frac{.462}{T^2}\left(\quad\right) + \frac{.110}{T^3}}$$

$$H(z) = \frac{.00854(1+3z^{-1}+3z^{-2}+z^{-3})}{1-2.064z^{-1}+1.517z^{-2}-.386z^{-3}}$$