4/2/25

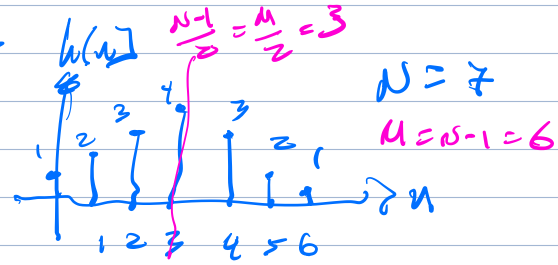
OPTIMUM EQUIRIPPLE FILTERS:

THE PARKS-McCLELLAN ALGORITHM

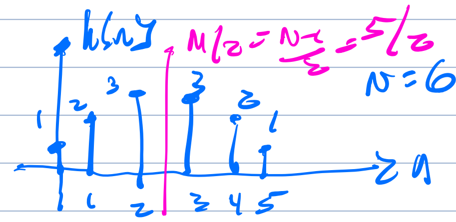
(DSP 7.7 - 7.9)

TYPES OF SYMMETRIC FIR FILTERS

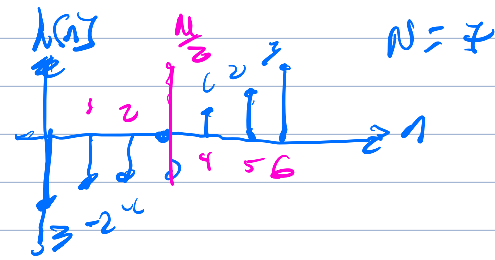
TYPE I N ODD, EVEN SYMMETRY



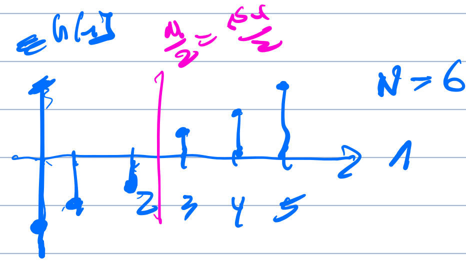
TYPE II N EVEN, EVEN SYMMETRY



TYPE III N ODD, ODD SYMMETRY



TYPE IV N EVEN, ODD SYMMETRY



FOR N ODD

$$h[n] = h'\left[n - \frac{M}{2}\right]$$

$$H(e^{j\omega}) = H'(e^{j\omega})\, e^{-j\omega \frac{M}{2}}, \quad \forall N$$

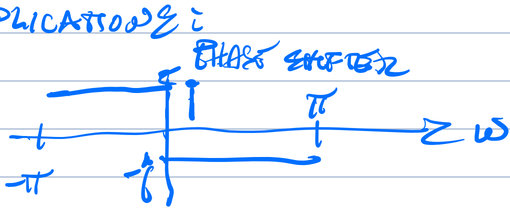
FOR TYPE I, II FILTERS $H'(e^{j\omega})$ REAL, EVEN

EX: LPF, HPF, BPF, NOTCH

FOR TYPE III, IV FILTERS $H'(e^{j\omega})$ IMAGINARY ODD

EXAMPLE APPLICATIONS:

PHASE SHIFTER



DIFFERENTIATOR

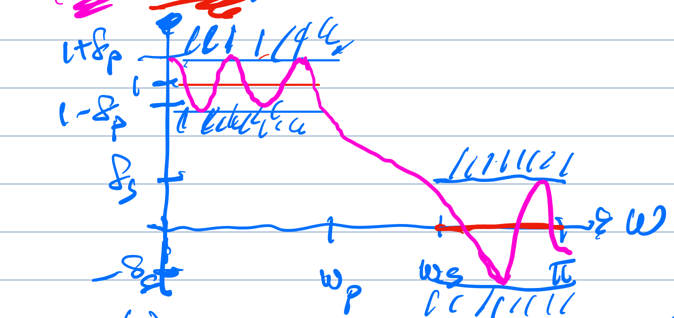
OPTIMUM EQUIRIPPLE DESIGN

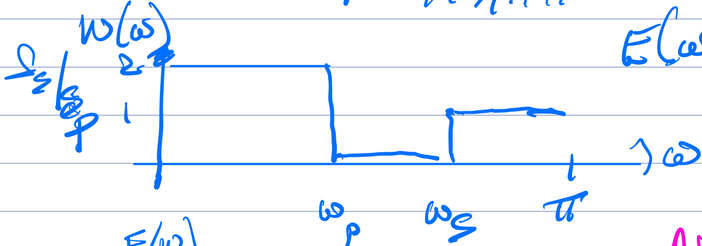
GIVEN $\omega_p, \omega_s, N, \delta_p / \delta_s$

FIND COEFFS $h[n]$ THAT MINIMIZE δ_p, δ_s

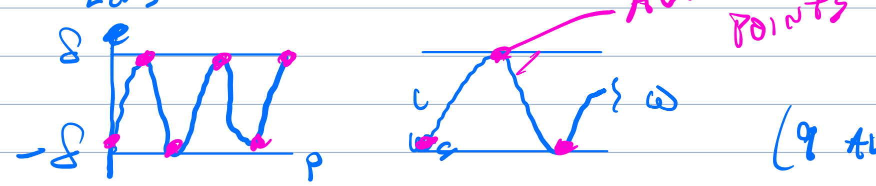
OR MIN MAX $|E(\omega)|$

$$E(\omega) = W(\omega)\left[H_d(e^{j\omega}) - A_e(e^{j\omega})\right]$$





$$E(\omega) = W(\omega)\left[H_d(e^{j\omega}) - A_e(e^{j\omega})\right]$$



(9 ALTERNATIONS)

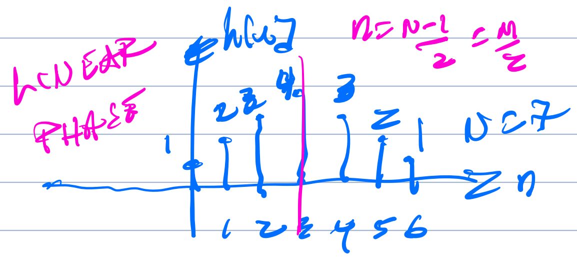
THE ALTERNATION THEOREM

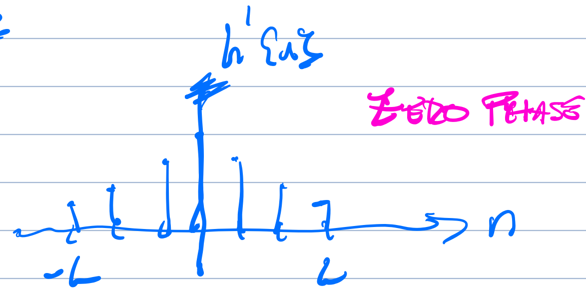
"IN ORDER FOR MAX $|E(\omega)|$ TO BE MINIMIZED, IT IS NECESSARY & SUFFICIENT FOR $E(\omega)$ TO EXHIBIT AT LEAST $L+2$ ALTERNATIONS"

$L = \frac{N-1}{2}$ FOR TYPE I, TYPE II FIR FILTERS

ALTERNATIONS ARE POINTS FOR WHICH $E(\omega) = \pm\delta$

CONSIDER TYPE I FILTER





$$H'(e^{j\omega}) = \sum_{n=-L}^{L} h'[n] e^{-j\omega n} = h'[0] + \sum_{\ell=1}^{L} h'[\ell] \left(\underbrace{e^{-j\omega \ell} + e^{j\omega \ell}}_{2\cos(\omega \ell)} \right)$$

$$H'(e^{j\omega}) = h'[0] + 2\sum_{\ell=1}^{L} h'[\ell] \cos(\ell\omega)$$

ORDINARY POLYNOMIALS

$$\sum_{\ell=0}^{L} a_\ell x^\ell$$

CHEBYSCHEV POLYNOMIALS

$$\sum_{\ell=0}^{L} a_\ell (\cos \omega)^\ell$$

CONSIDER:

1. $\cos(\omega)$
2. $(\cos(\omega))^2 = \frac{1}{2} + \frac{1}{2}\cos(2\omega)$
3. $(\cos(\omega))^3 = \left(\frac{1}{2} + \frac{1}{2}\cos(2\omega)\right)\cos(\omega)$

$$= \frac{1}{2}\cos(\omega) + \frac{1}{2}\cos(2\omega)\cos(\omega)$$

$$= \frac{1}{2}\cos(\omega) + \frac{1}{4}\cos(3\omega) + \frac{1}{4}\cos(\omega)$$

$$= \frac{3}{4}\cos(\omega) + \frac{1}{4}\cos(3\omega)$$

$\Rightarrow$ FOR TYPE I FILTERS

$$H'(e^{j\omega}) = h'[0] + 2\sum_{\ell=1}^{L} h'[\ell]\cos(\ell\omega) = \sum_{k=0}^{L} a_k (\cos(\omega))^k$$

For $H'(e^{j\omega}) = \sum_{k=0}^{L} a_k \left(\cos(\omega)\right)^k$

For how many ω will $H'(e^{j\omega})$ have zero slope?

$$\frac{\partial H'(e^{j\omega})}{d\omega} = \frac{d}{d\omega} \sum_{k=0}^{L} a_k (\cos\omega)^k$$

$$= -\sum_{k=0}^{L} a_k k (\cos(\omega))^{k-1} \underbrace{\sin(\omega)}_{\omega = 0, \pi} = 0$$

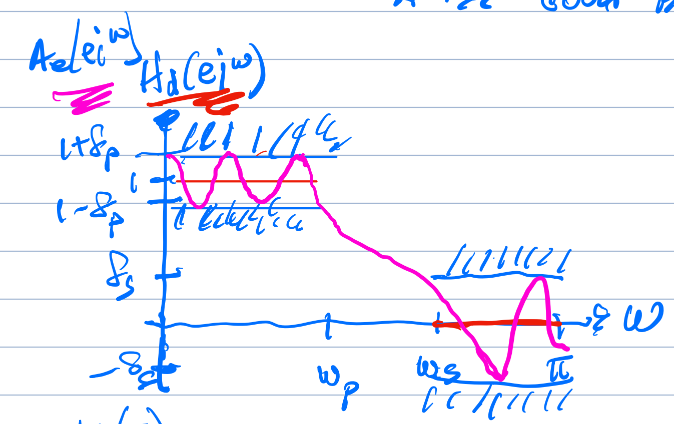
$$= -\sin(\omega) \underbrace{\sum_{k=1}^{L} a_k k \underbrace{\cos(\omega)^{k-1}}_{x}}_{L-1 \text{ pts of zero slope for } 0<\omega<\pi}$$

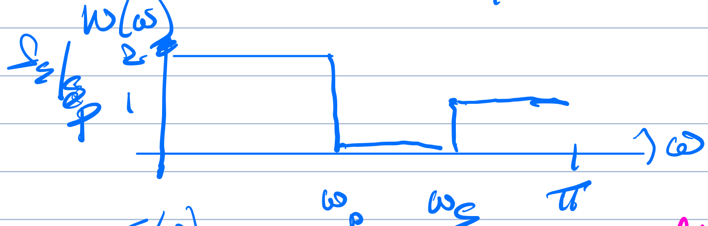
To satisfy alternation thm, need to have $L+2$ alternations

For filter of L, zero slope pts @ $0, \pi$, $L-1$ more

Also could be zero slope pts $+ \omega_s, \omega_p$

$$\Rightarrow \underbrace{L+1}_{\text{zero slope}} + \underbrace{2}_{\omega_p, \omega_s} = L+3$$





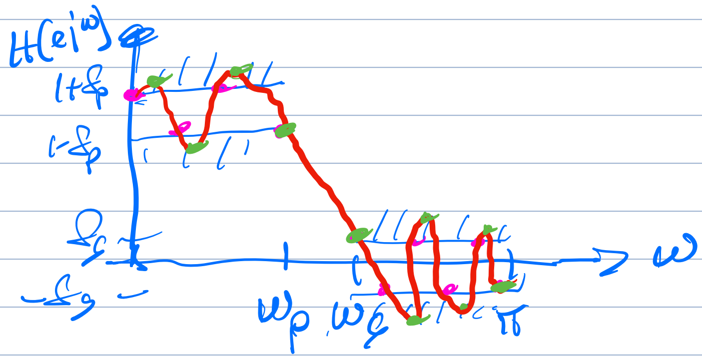
TO IMPLEMENT PARKS – McCLELLAN

1. CHOOSE ARBITRARY $L-1$ POINTS BET. $0<\omega<\pi$

PLUS ω_s, ω_p, EITHER 0 OR π

EX. SUPPOSE $N=21$, $L=10$, ASSUME $W(\omega)=1$, STOPBAND, PASSBAND

0, TRANSITION BAND



2. THREAD CHEBYSCHEV POLYNOMIAL THROUGH PTS.

3. "REMEZ EXCHANGE"