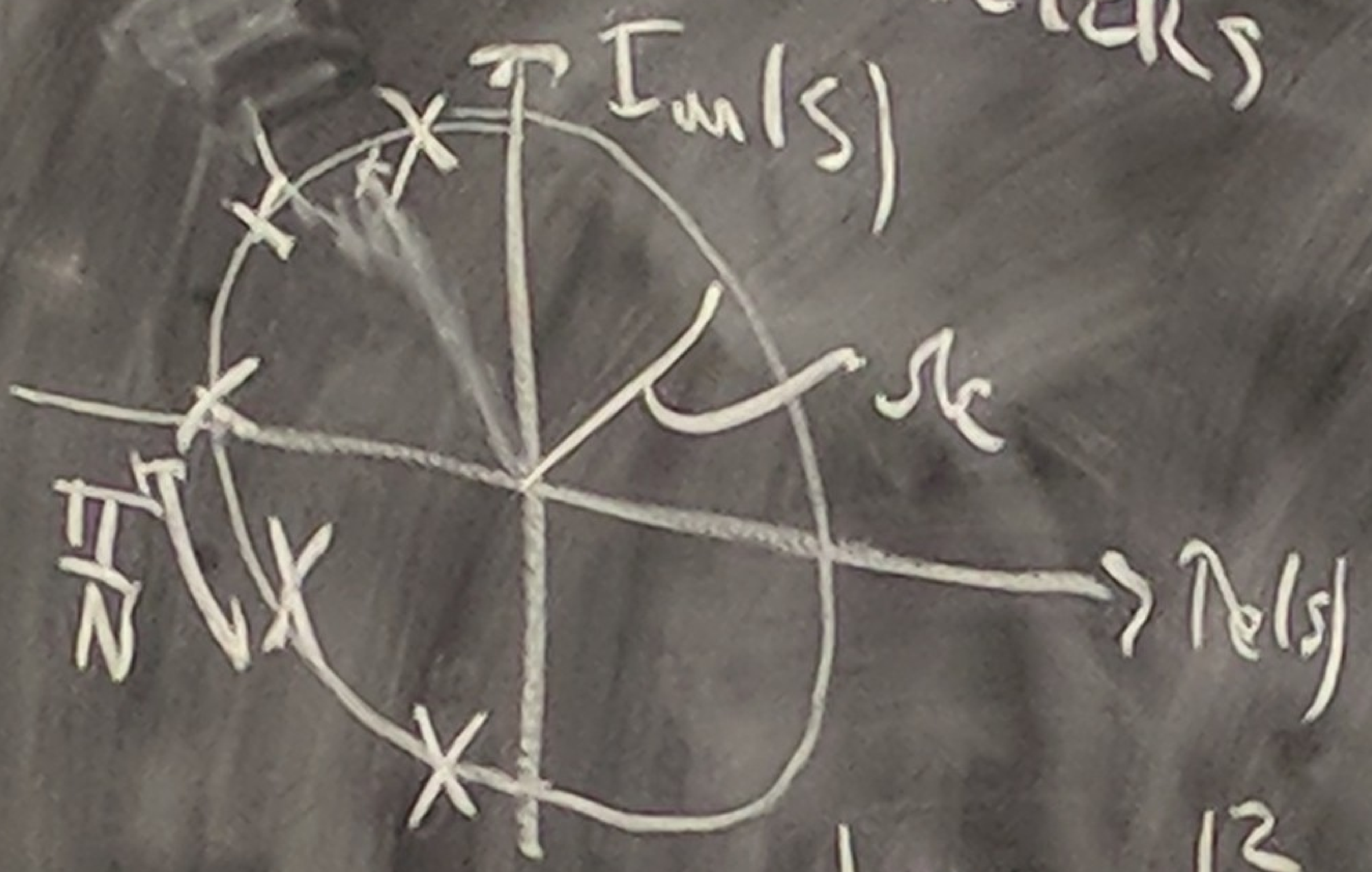
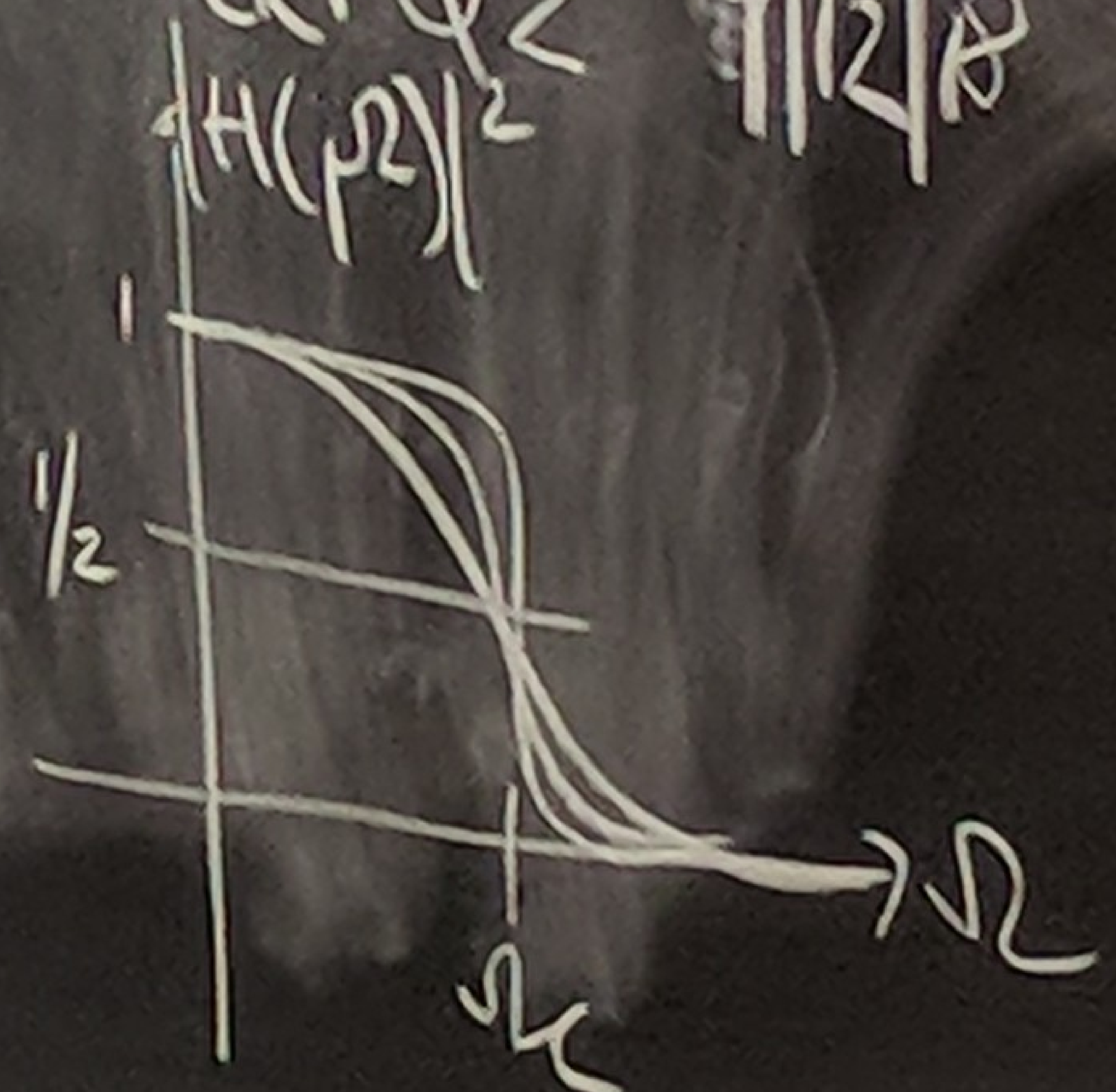
BUTTERWORTH FILTERS

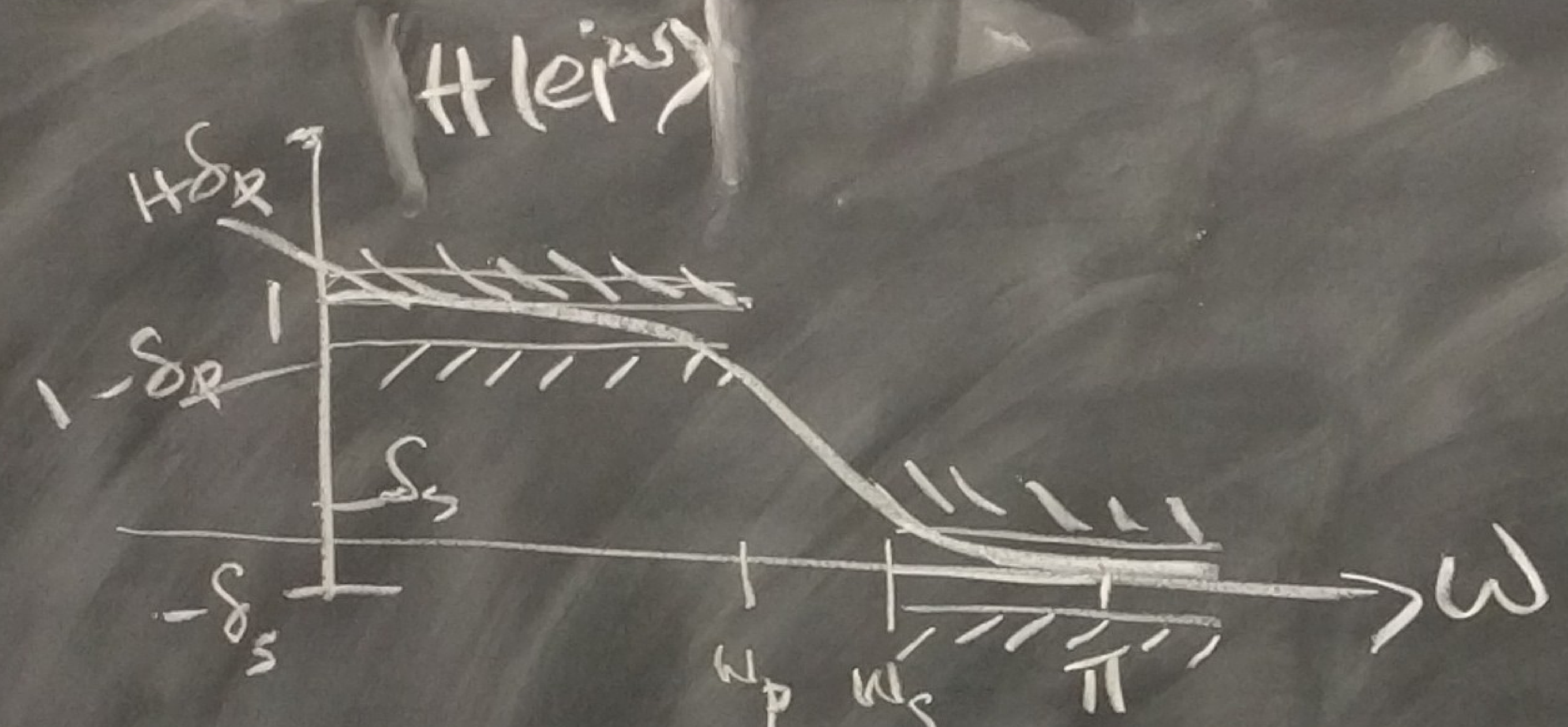


$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

IIR BUTTERWORTH FILTER DESIGN EXAMPLES

REMINDER: Q2 4/12/18





SPECS OF

DT LP FILTERS

LET

$\omega_p = .15\pi$

$\omega_s = .35\pi$

$\frac{dB}{20} = \log_{10} X$

$10^{dB/20} = X$

@ ω_p $1-\delta_p \geq 10^{-3/20}$ $.7079$ PASSBAND RIPPLE $-3dB \leq |H(e^{j\omega})| \leq 0dB$ $|\omega| \leq \omega_p$

@ ω_s $\delta_s \leq 10^{-20/20} = .1$ STOPBAND ATT $|H(e^{j\omega})| \leq -20dB$, $\omega_s \leq |\omega| \leq \pi$

LET $K_1 = \frac{1}{(1-\delta_p)^2} - 1 = .9953$

$dB = 20 \log_{10}\left(\frac{X}{X_{REF}}\right)$ X AMPLITUDE

$= 10 \log_{10}\left(\frac{I}{I_{REF}}\right)$

$K_2 = \frac{1}{\delta_s^2} - 1 = 99$

DESIGN W IMPULSE INVARIANCE

$h[n] = h_c(nT)$

$\omega = \Omega T$

$\Omega_p = \frac{\omega_p}{T} = \frac{.47124}{T}$

$\Omega_s = \frac{\omega_s}{T} = \frac{1.0996}{T}$

AT Ω_p

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}} = (1-\delta_p)^2$$

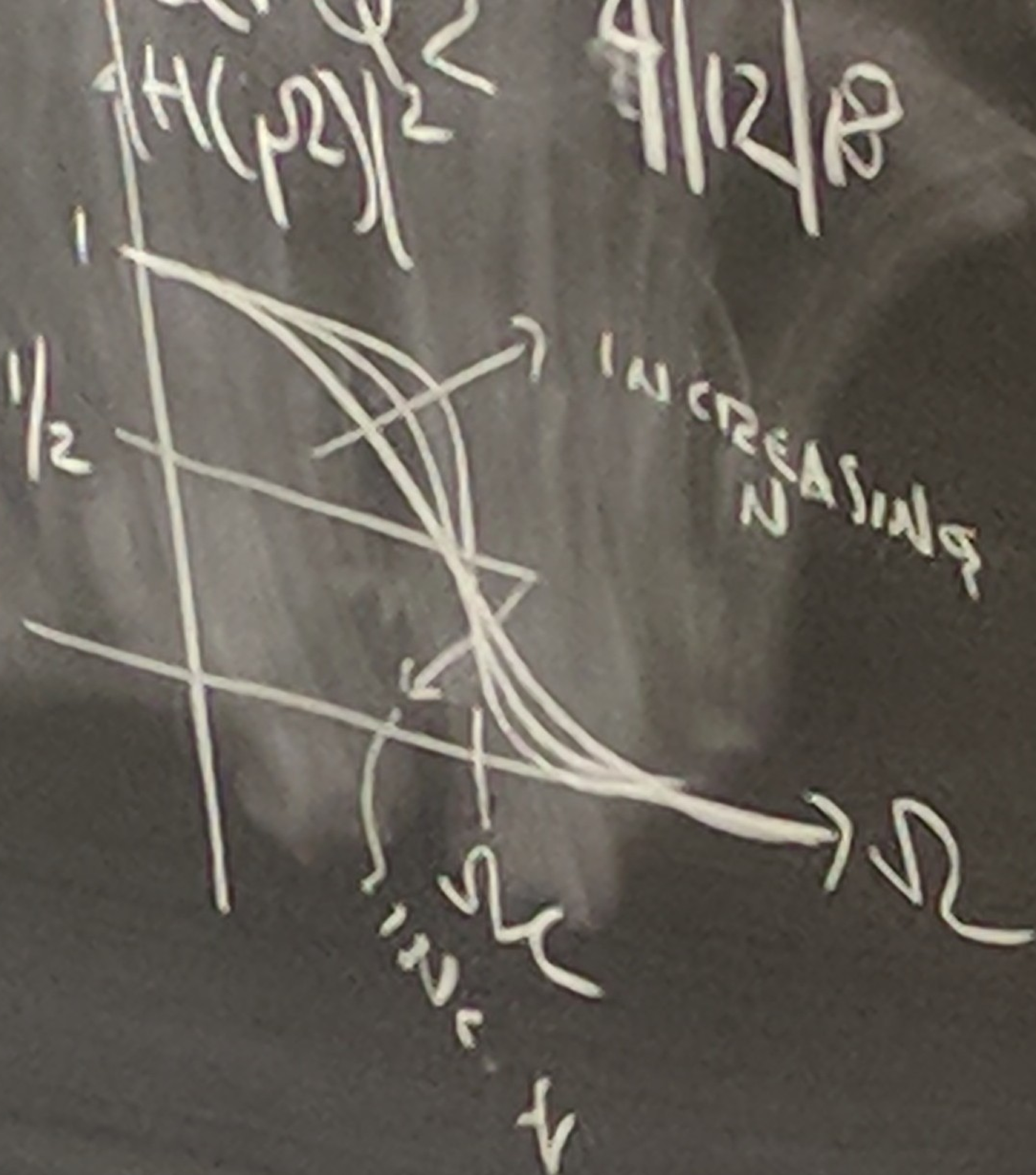
$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \frac{1}{(1-\delta_p)^2} - 1 = K_1$$

AT Ω_s

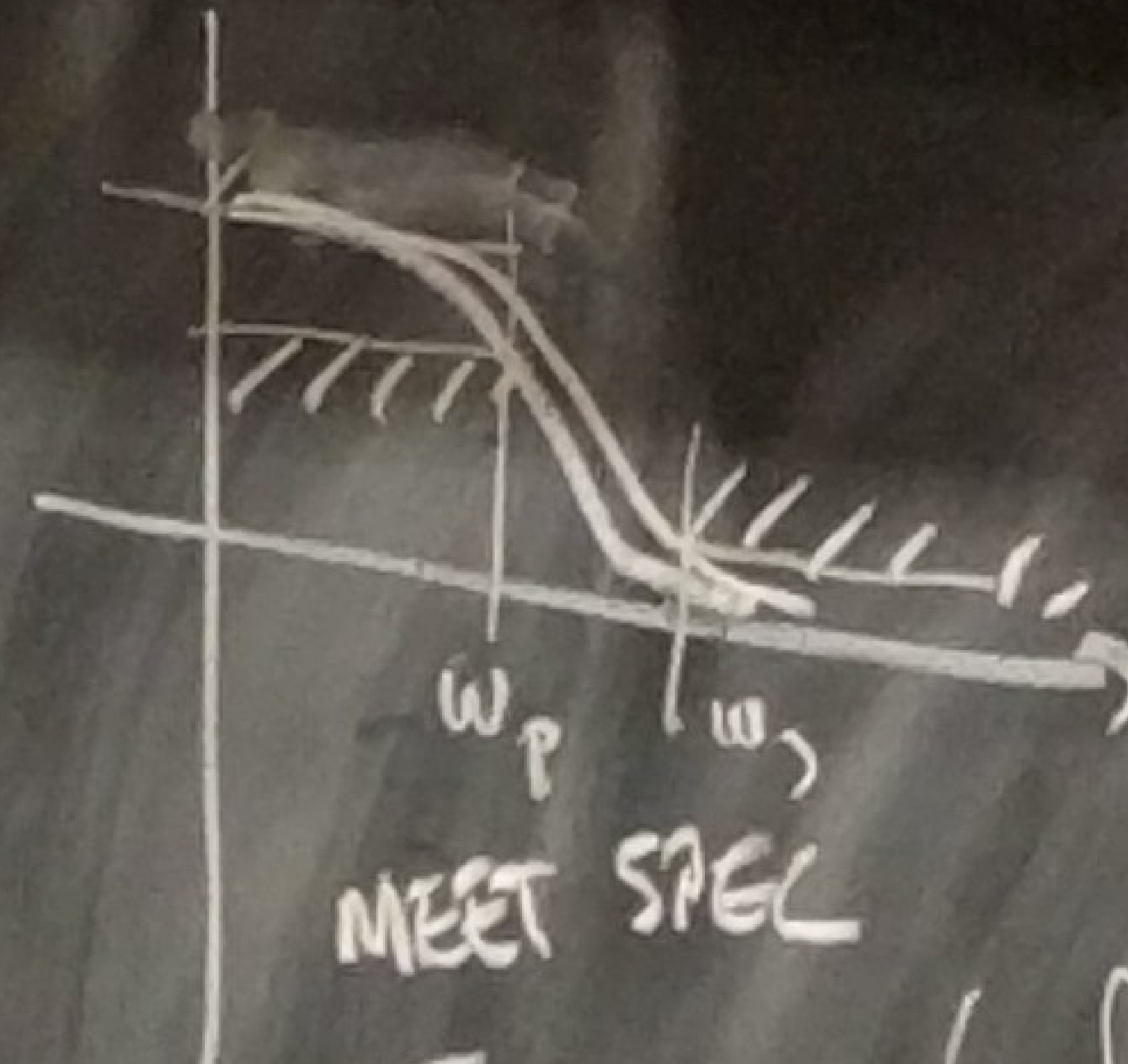
$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}} = \delta_s^2$$

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = \frac{1}{\delta_s^2} - 1 = K_2$$

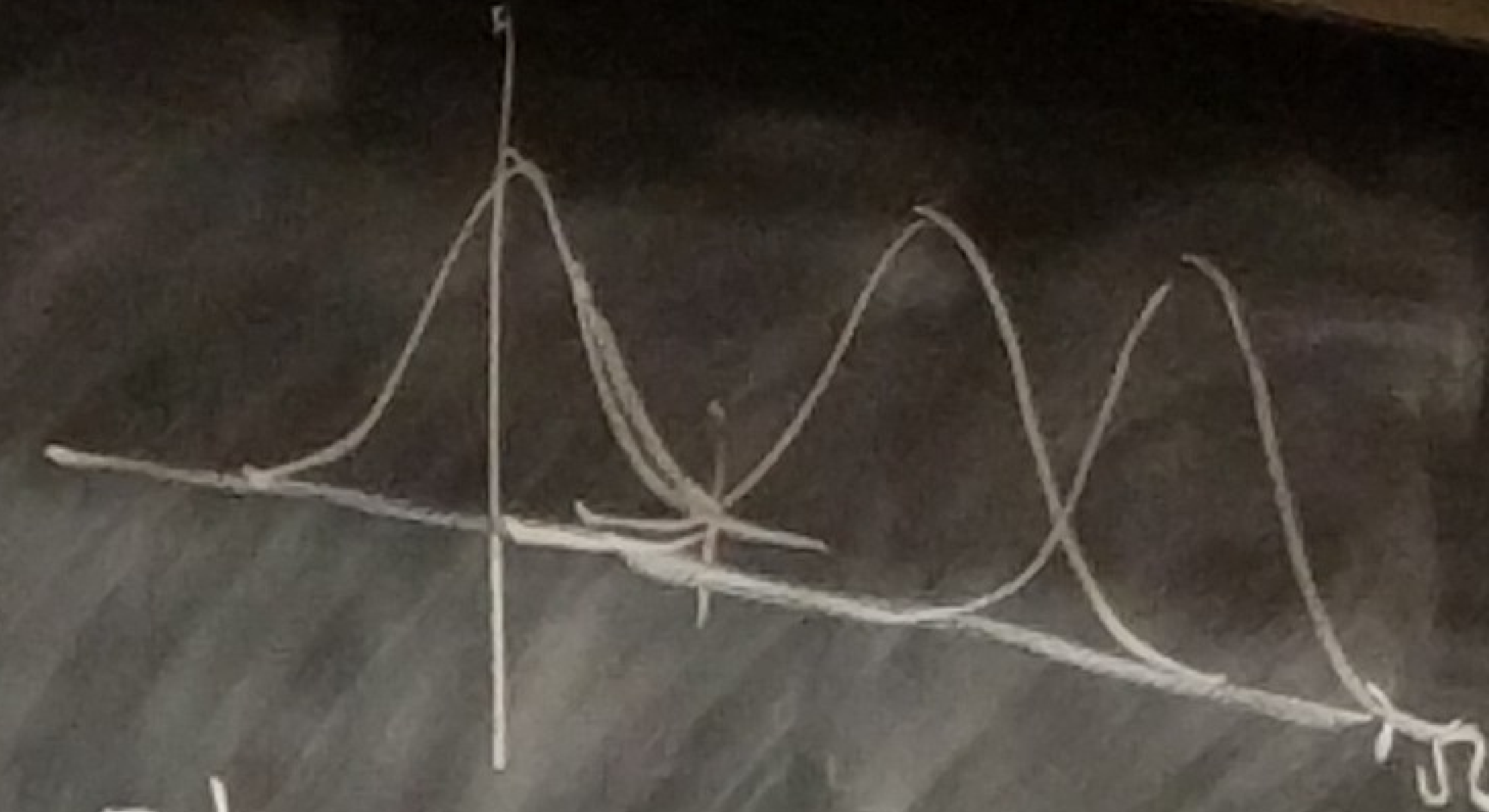
4/12/18



$$\left(\frac{\Omega_p}{\Omega_s}\right)^{2N} = \frac{k_1}{k_2}$$



MEET SPEC



$$2N \log(\Omega_p/\Omega_s) = \log(k_1/k_2)$$

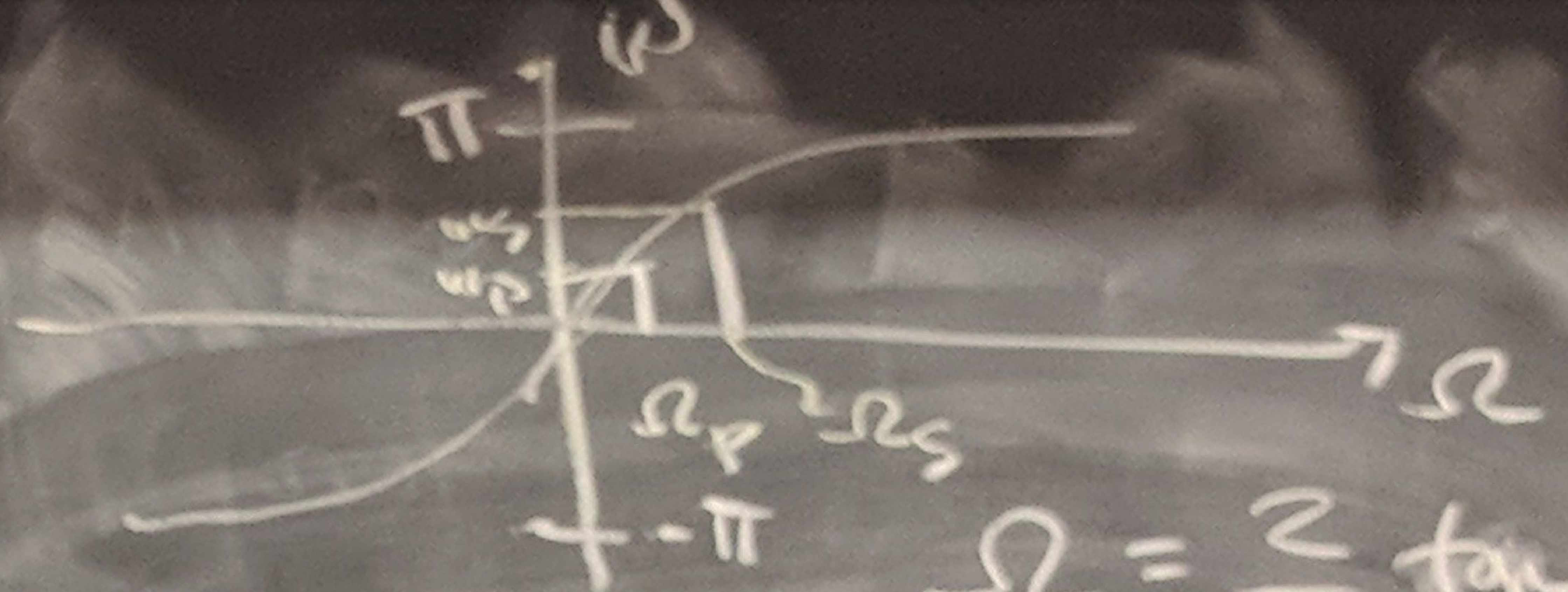
$$N = \frac{\log(k_2/k_1)}{2\log(\Omega_s/\Omega_p)} = 2.7144 \Rightarrow 3$$

AT Ω_p: $\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = k_1$; $\Omega_c = \frac{\Omega_p}{k_1^{1/2N}} = \frac{.4716}{T}$ ★

MEET @ Ω_s: $\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = k_2$; $\Omega_c = \frac{\Omega_s}{k_2^{1/2N}} = \frac{.5112}{T}$

DESIGN w

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$



$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = .4802/T$$

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = 1.226/T$$

$$N = \frac{\log\left(k_2/k_1\right)}{\log\left(\omega_s/p_p\right)} = 2.4546$$

$$H(s) = \frac{\Omega_c^3}{\left(s-\Omega_c e^{j\pi}\right)\left(s-\Omega_c e^{j(\pi+\pi/3)}\right)\left(s-\Omega_c e^{-j(\pi+\pi/3)}\right)}$$

$$H(s) = \frac{\Omega_c^3}{s^3 + .9612 s^2 + .4620 s + .110}$$

$$H(z) = \frac{.11/T^3}{\frac{2}{T}\left(\frac{(1-z^{-1})}{(1+z^{-1})}\right)^3 + .9612\left(\frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}\right)^2 + \frac{.462}{T^2}\left(\frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}\right) + \frac{.11}{T^3}}$$

$$\frac{.9612}{T} + \frac{.4620}{T^2} + \frac{.110}{T^3}$$

$$H(z) = \frac{.00854\left(1+3z^{-1}+3z^{-2}+z^{-3}\right)}{1-2.1z^{-1}+1.5z^{-2}-.39z^{-3}}$$