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FIR FILTER DESIGN USING WINDOW FUNCTIONS

(OS&P 7.5–7.6)

FIR DESIGN TECHNIQUES

- WINDOW DESIGN
- FREQUENCY SAMPLING
- OPTIMUM EQUIRIPPLE DESIGN (PARKS-McCLELLAN ALGORITHM)

WINDOW DESIGN PROCEDURES

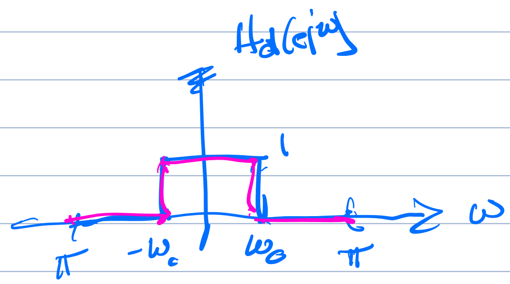
Let $h[n] = h_d[n] \cdot w[n]$

$h_d[n]$ ← IDEAL LPF, HPF, BPF, ETC.

$w[n]$ ← FINITE-DURATION WINDOW

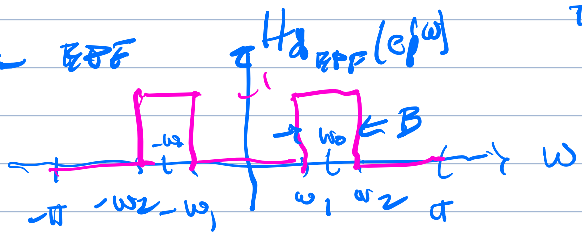
IDEAL FILTERS

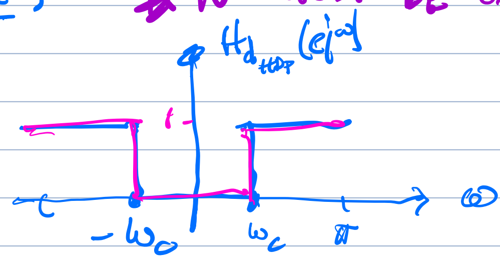
IDEAL LPF $h_d[n] = \dfrac{\sin(\omega_c n)}{\pi n}$



IDEAL HPF $h_d[n] = \delta[n] - \dfrac{\sin(\omega_c n)}{\pi n}$ ★ N MUST BE ODD!!

IDEAL BPF



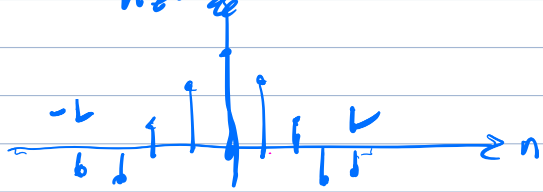


$$h_d[n] = \frac{\sin(\omega_2 n)}{\pi n} - \frac{\sin(\omega_1 n)}{\pi n}$$

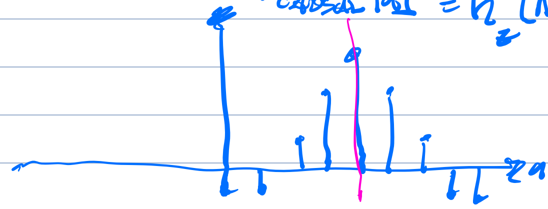
Let $B = \omega_2 - \omega_1$

$\omega_0 = \dfrac{\omega_1 + \omega_2}{2}$ $\quad h_d[n] = 2\dfrac{\sin\left(\frac{B}{2} n\right)}{\pi n} \cos(\omega_0 n)$

$h_z[n]$ ZERO-PHASE



$h_{causal}[n] = h_z[n-L]$



$n = 4$

$= \frac{N-1}{2} = \frac{M}{2}$

LENGTH $N = 2L+1$

$L = \frac{N-1}{2}$

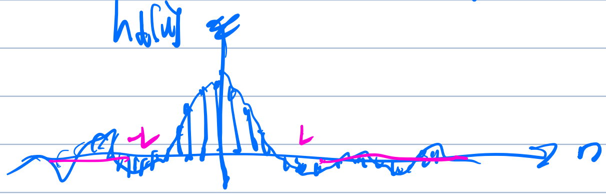
Let $M = N-1$

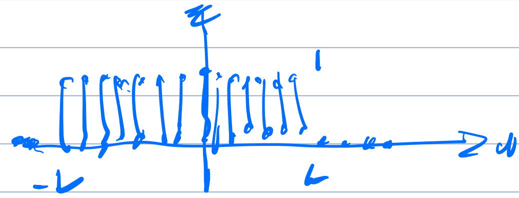
$h_{causal}[n] = h_z[n - \frac{M}{2}]$

APPLYING WINDOWS to $h_d[n]$ $\quad w[n] = 1, \; -L \le n \le L$

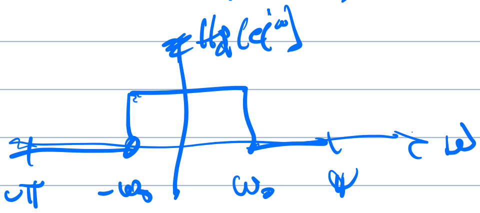
Ex USING ZERO-PHASE

$h_d[n] = \frac{\sin(\omega_c n)}{\pi n}, \; \forall n \quad . \; w[n] = 1, \; -L \le n \le L$

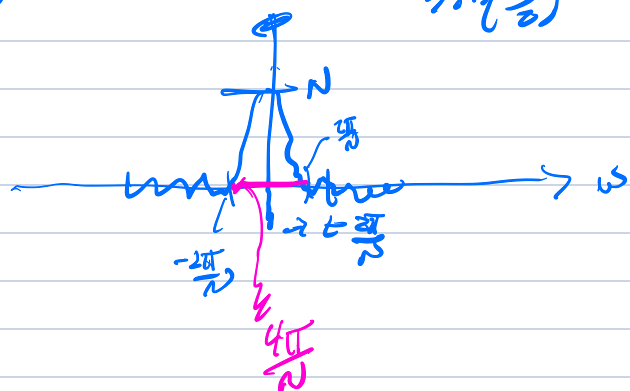




$h[n] = h_d[n] \cdot w[n] \iff H(e^{j\omega}) = \frac{1}{2\pi} H_d(e^{j\omega}) \circledast W(e^{j\omega})$



$W(e^{j\omega}) = \frac{\sin(\frac{N\omega}{2})}{\sin(\frac{\omega}{2})}$

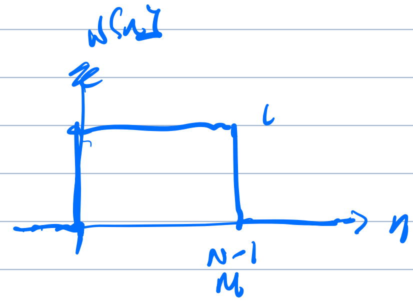


$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta}) \, W(e^{j(\omega-\theta)}) \, d\theta$$

FIVE "CLASSIC" WINDOWS

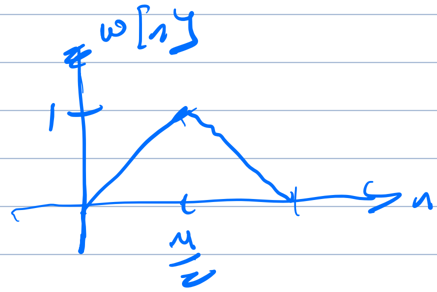
1. RECTANGULAR WINDOW

$$w[n] = \begin{cases} 1, & 0 \le n \le N-1 = M \\ 0, & \text{ELSE} \end{cases}$$



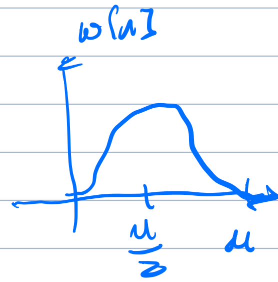
2. BARTLETT WINDOW

$$w[n] = \begin{cases} \frac{2n}{M}, & 0 \le n \le \frac{M}{2} \\ 2\left(1 - \frac{n}{M}\right), & \frac{M}{2} \le n \le M \\ 0, & \text{ELSE} \end{cases}$$



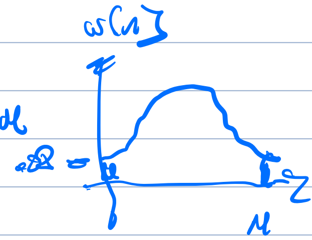
3. VON HANN, "HANNING" WINDOW

$$w[n] = \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{M}\right)\right), & 0 \le n \le M \\ 0, & \text{ELSE} \end{cases}$$



4. HAMMING WINDOW

$$w[n] = \begin{cases} .54 - .46\cos\left(\frac{2\pi n}{M}\right), & 0 \le n \le M \\ 0, & \text{ELSE} \end{cases}$$



5. $$w[n] = \begin{cases} .42 - .5\cos\left(\frac{2\pi n}{M}\right) + .08\cos\left(\frac{4\pi n}{M}\right), & 0 \le n \le M \\ 0, & \text{ELSE} \end{cases}$$

FIR DESIGN PROCEDURE USING WINDOWS:

1. CHOOSE <u>SHAPE</u> of WINDOWS BASED ON STOP BAND ATTENUATION

2. CHOOSE <u>LENGTH</u> of WINDOWS BASED ON TRANSITION BW

EX:

$\omega_p = .55\pi$ PASSBAND RIPPLE < 1 dB

$\omega_s = .6\pi$ STOPBAND RIPPLE ≤ -60dB

BLACKMAN WINDOW

$$\frac{12\pi}{M} = .05\pi \Rightarrow M = \frac{12}{.05} = 240$$

$$\omega_c = .575\pi$$

$$h[n] = \begin{cases} \dfrac{\sin\left(.575\pi\left(n - \frac{M}{2}\right)\right)}{\pi\left(n - \frac{M}{2}\right)} \cdot \left(.42 - .5\cos\left(\frac{2\pi n}{M}\right) + .08\left(\frac{4\pi n}{M}\right)\right) & 0 \le n \le M, \quad M = 240 \\ 0, & \text{ELSE} \end{cases}$$