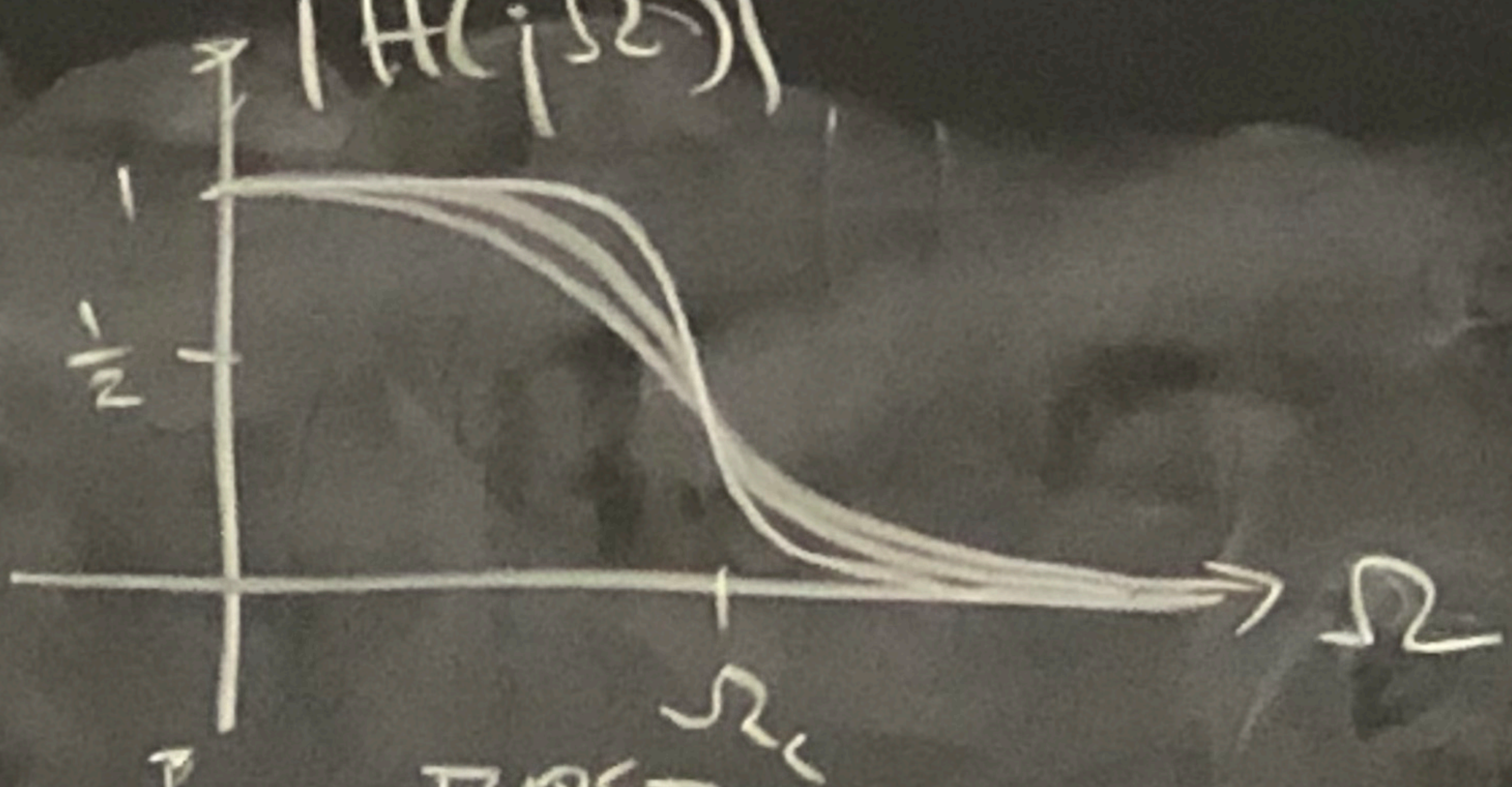
CONTINUOUS-TIME
(PROTOTYPES

BUTTERWORTH



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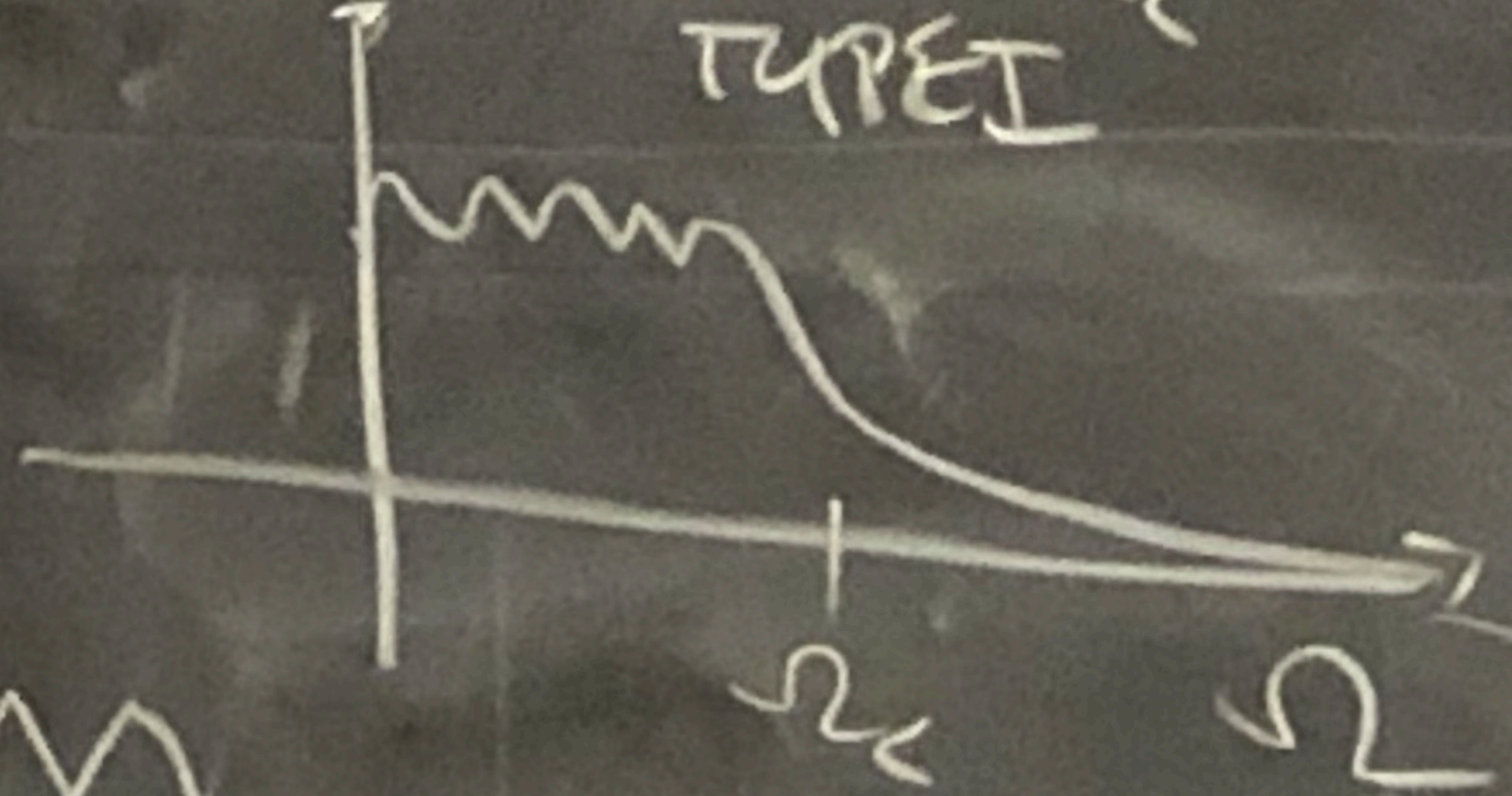
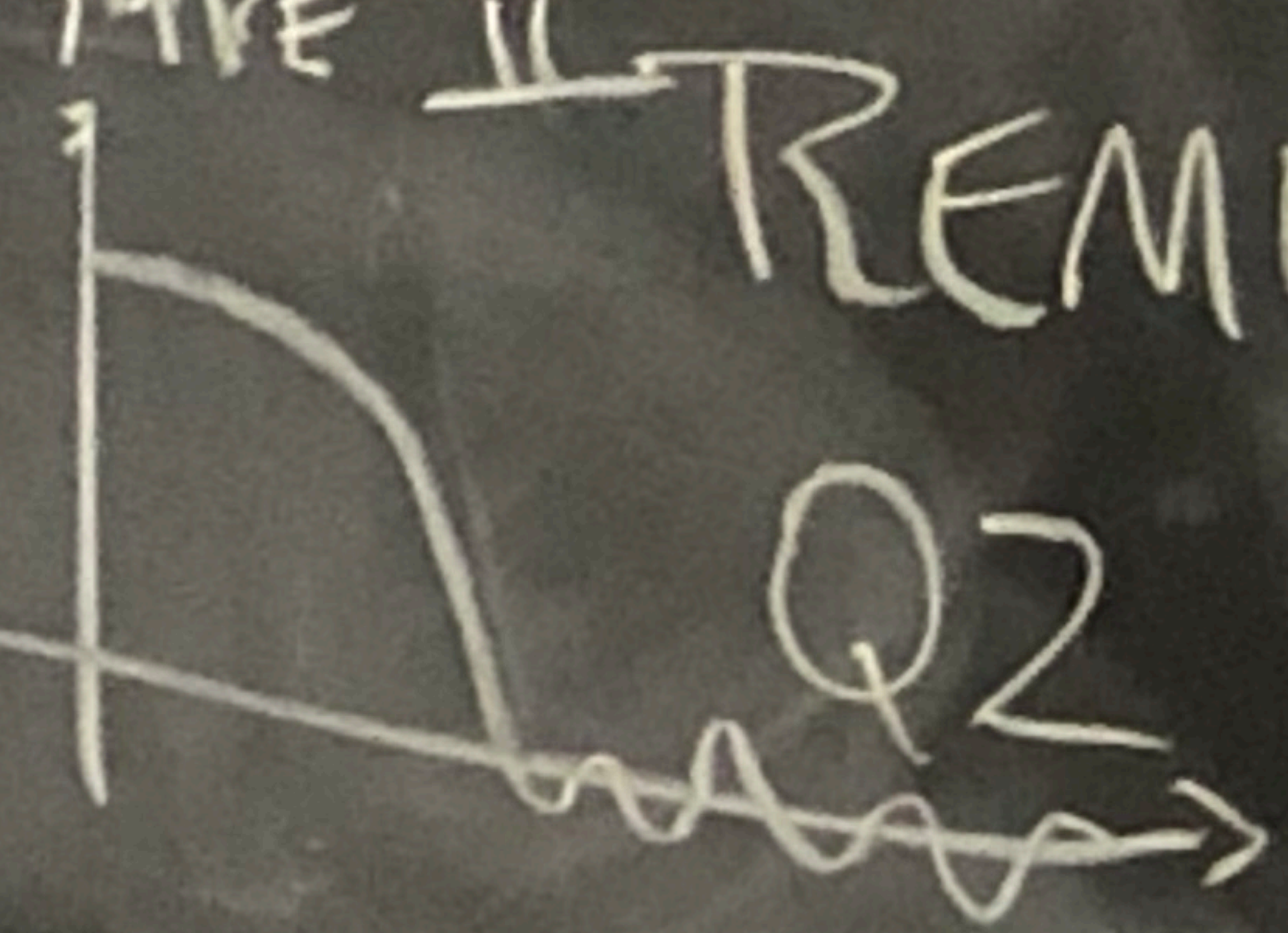
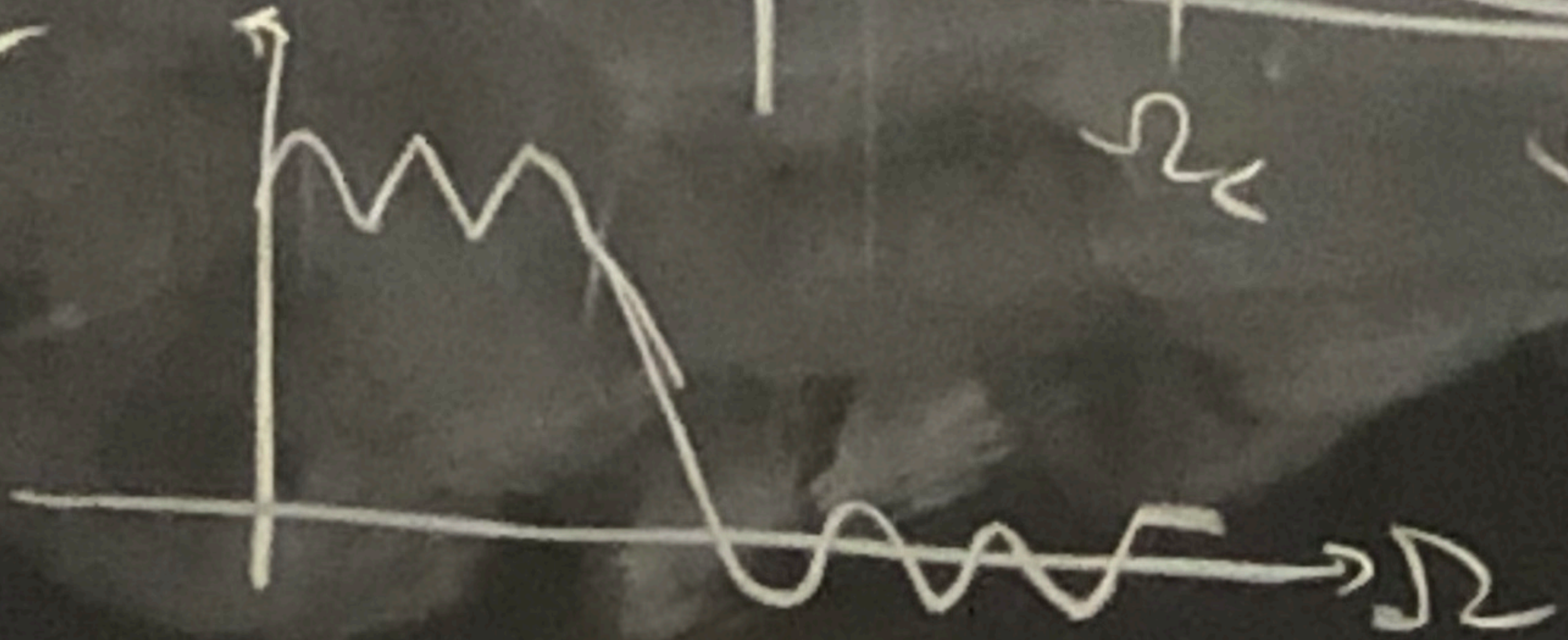




ELLIPTICAL

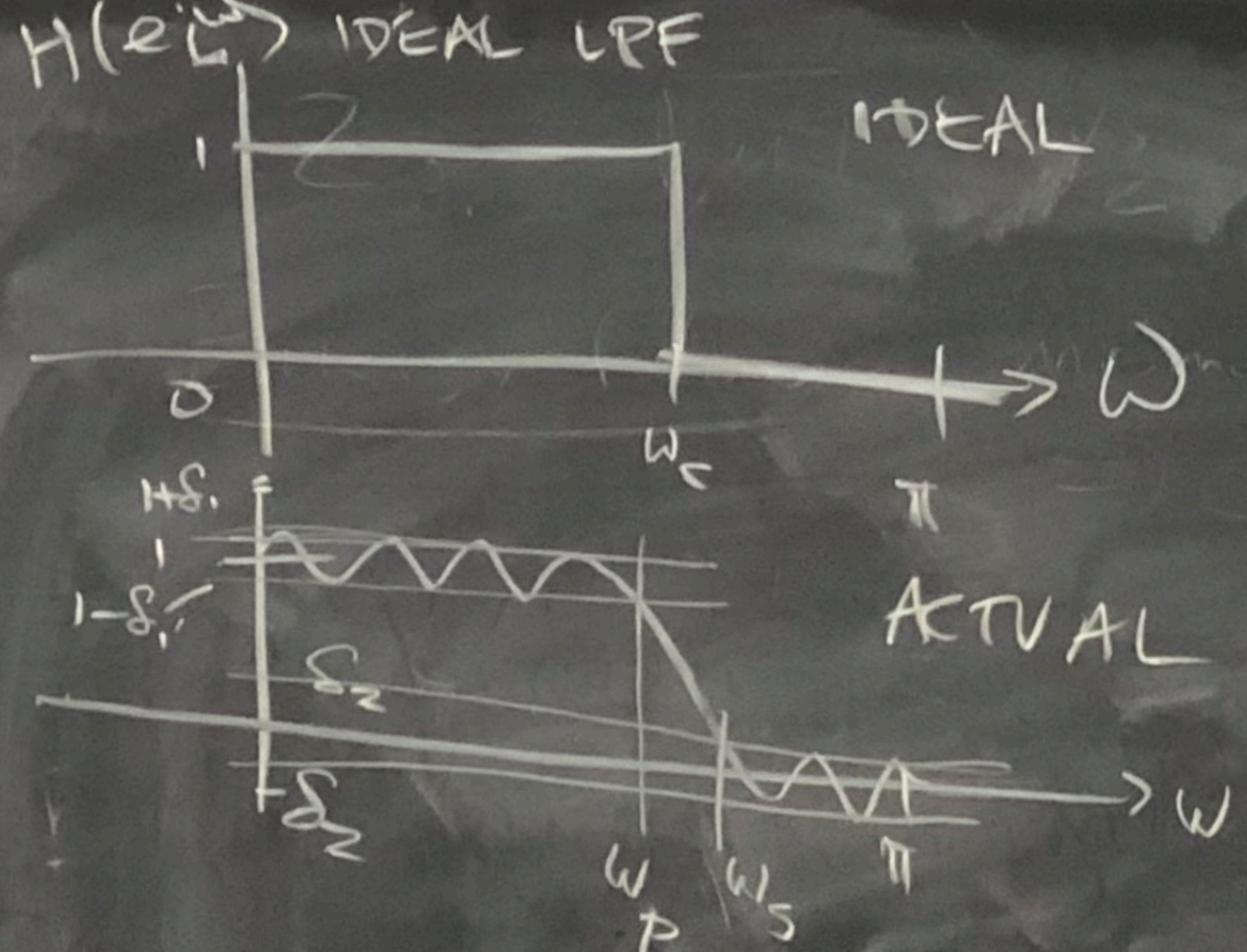


INTRO TO FILTER DESIGN

(O&SP 7.0-7.3)

REMINDER:

Q2 HERE WED.



POSSIBLE WAYS OF MAPPING FROM CT to DT ...

- CONVERT $h(t) \rightarrow h[n]$
- CONVERT $s(t) \rightarrow s[n]$
- CONVERT $H(j\Omega) \rightarrow H(e^{j\omega})$
- DIFF. EQ. $\rightarrow$ DIFF. EQ.

THINGS TO THINK ABOUT:

★ DOES $j\Omega$-AXIS IN S-PLANE MAP TO U.C. IN Z-PLANE?

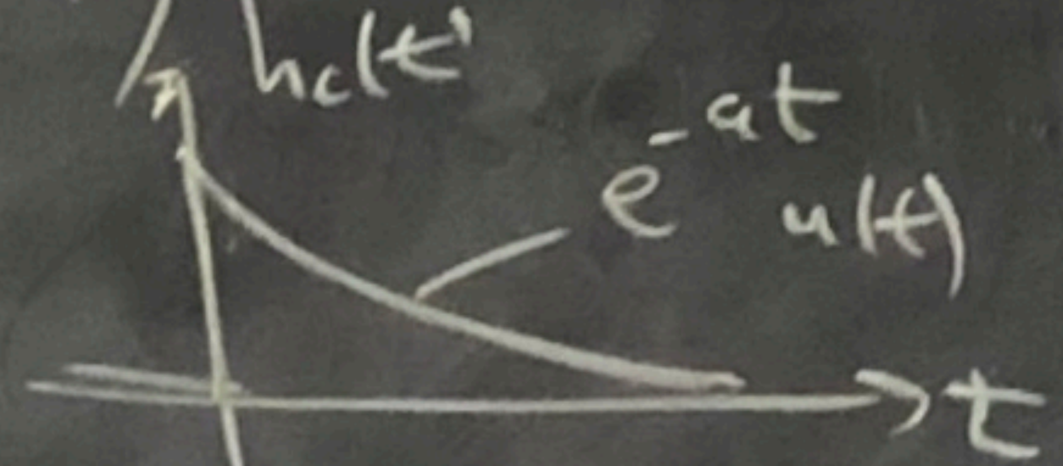
★ IF $H(s)$ IS STABLE IN CT IS $H(z)$ STABLE IN DT?

METHOD I: IMPULSE INVARIANCE

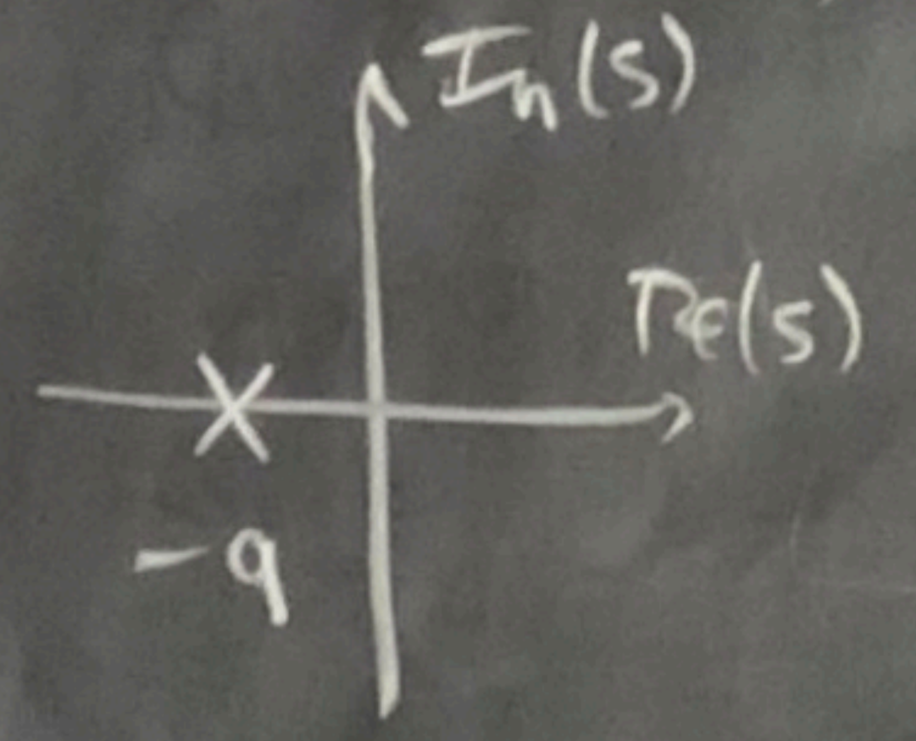
Given $h_c(t)$, Let $h[n] = T h_c(nT)$

$\omega = \Omega T$

Ex. $h_c(t) = e^{-at}u(t)$

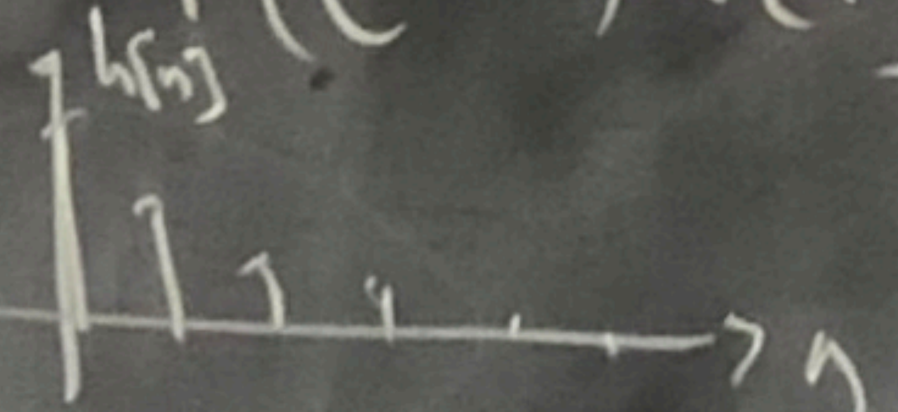


$h[n] = T e^{-anT}u[n] = T(e^{-aT})^n u[n]$



$H(s) = \frac{1}{s-s_0}$

$s_0 = \sigma_0 + j\Omega_0$



$H(z) = \frac{T}{1-e^{-aT}z^{-1}} = \frac{Tz}{z-e^{-aT}}$

Pole @ $z = e^{-aT}$

Zero @ $z = 0$

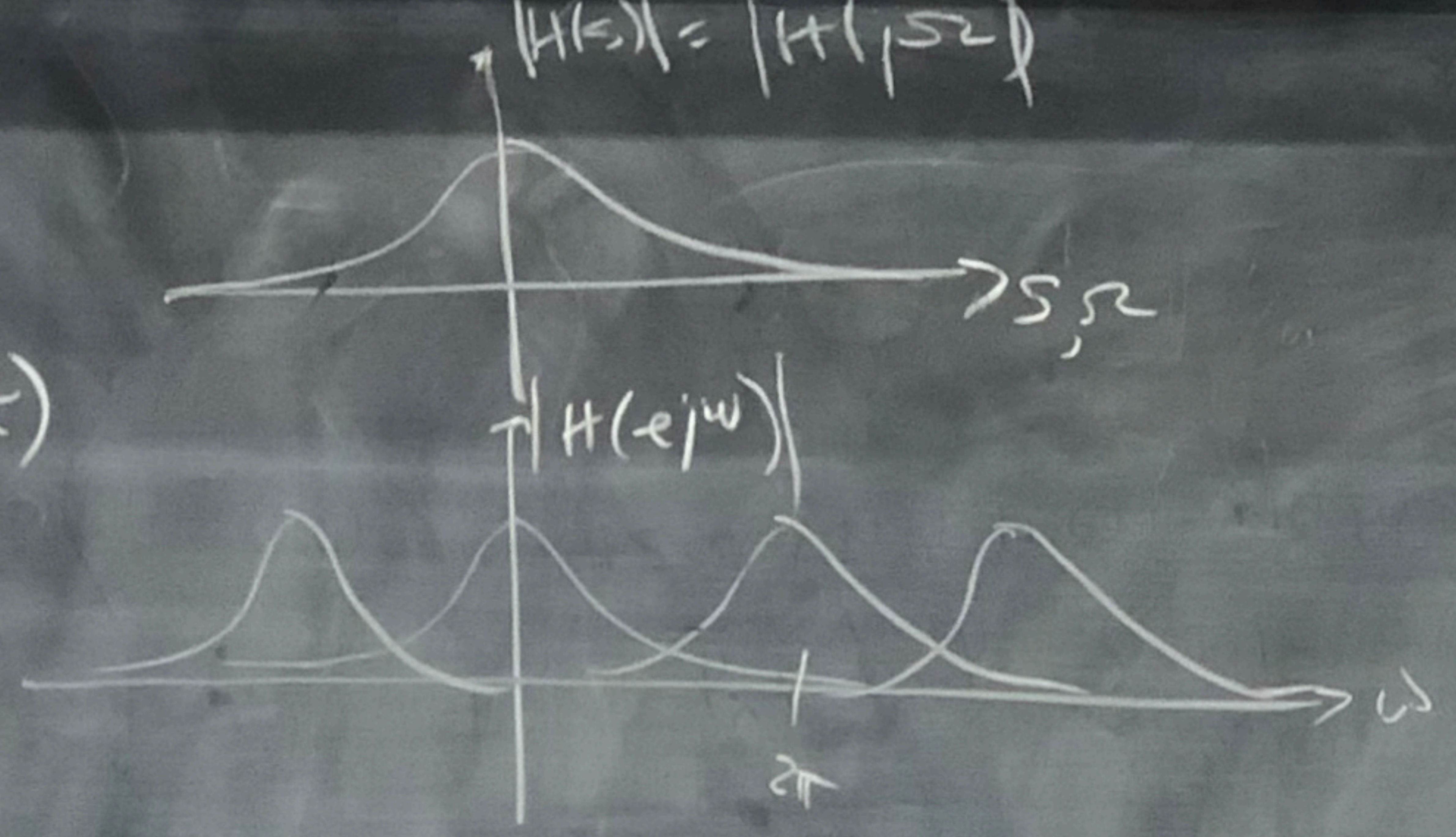
$H(z) = \frac{Tz}{z-e^{-(\sigma_0+j\Omega_0)T}} = \frac{Tz}{z-e^{-\sigma_0 T}\cdot e^{-j\Omega_0 T}}$

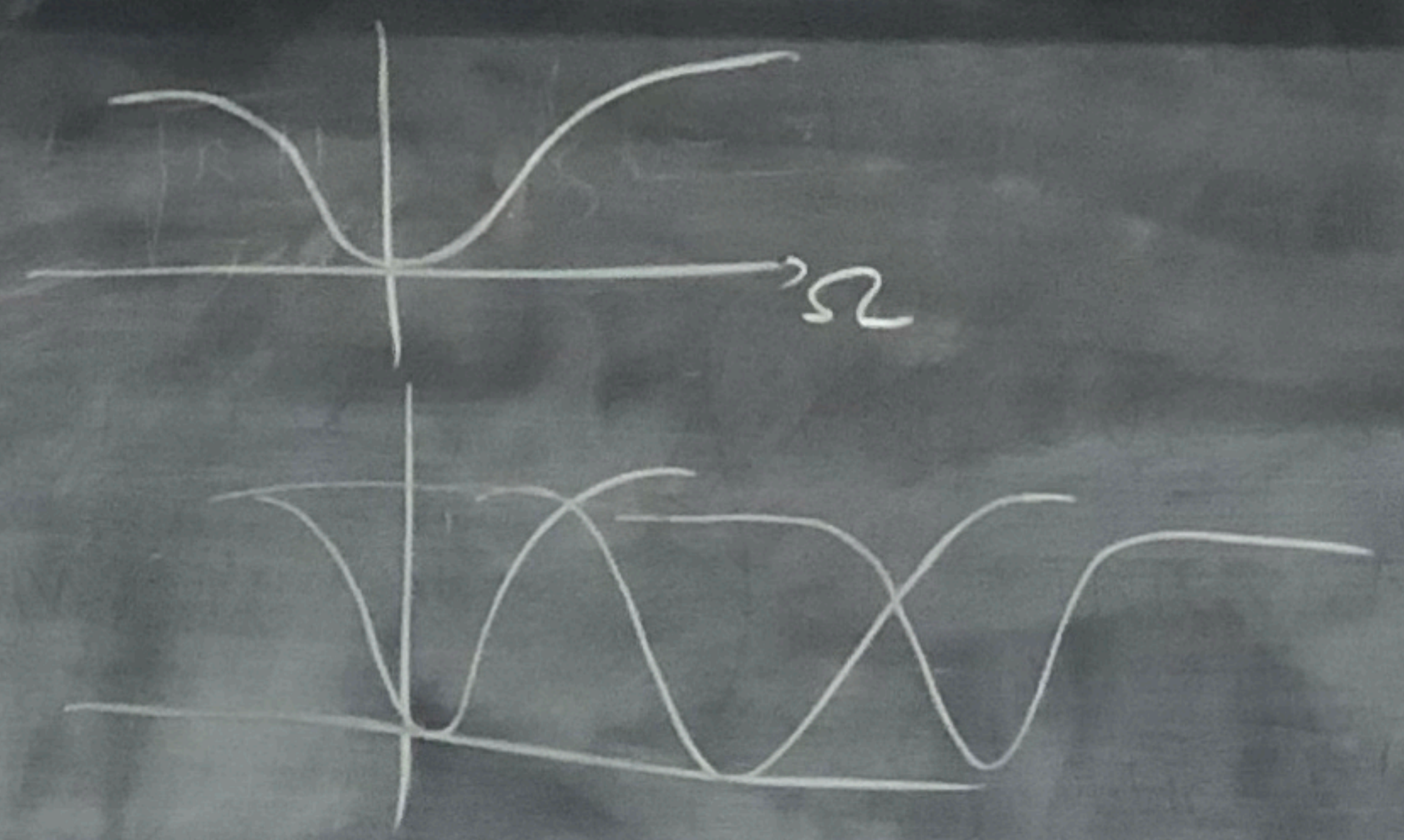
More complicated filters

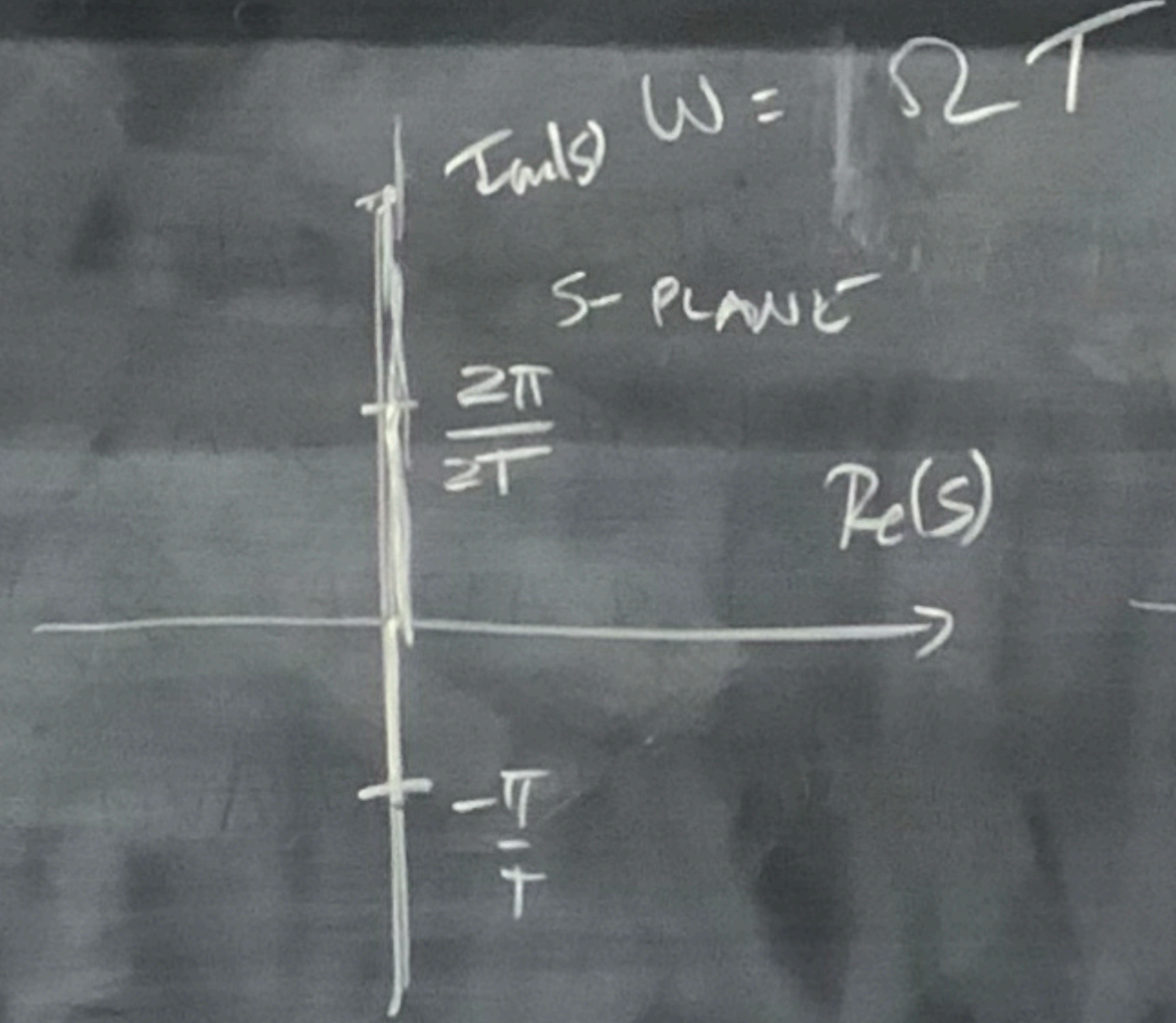
$H(s) = \sum_{k=1}^{N} \frac{A_k}{s-s_k} \Rightarrow H(z) = \sum_{k=1}^{N} A_k \frac{Tz}{z-e^{-s_k T}}$

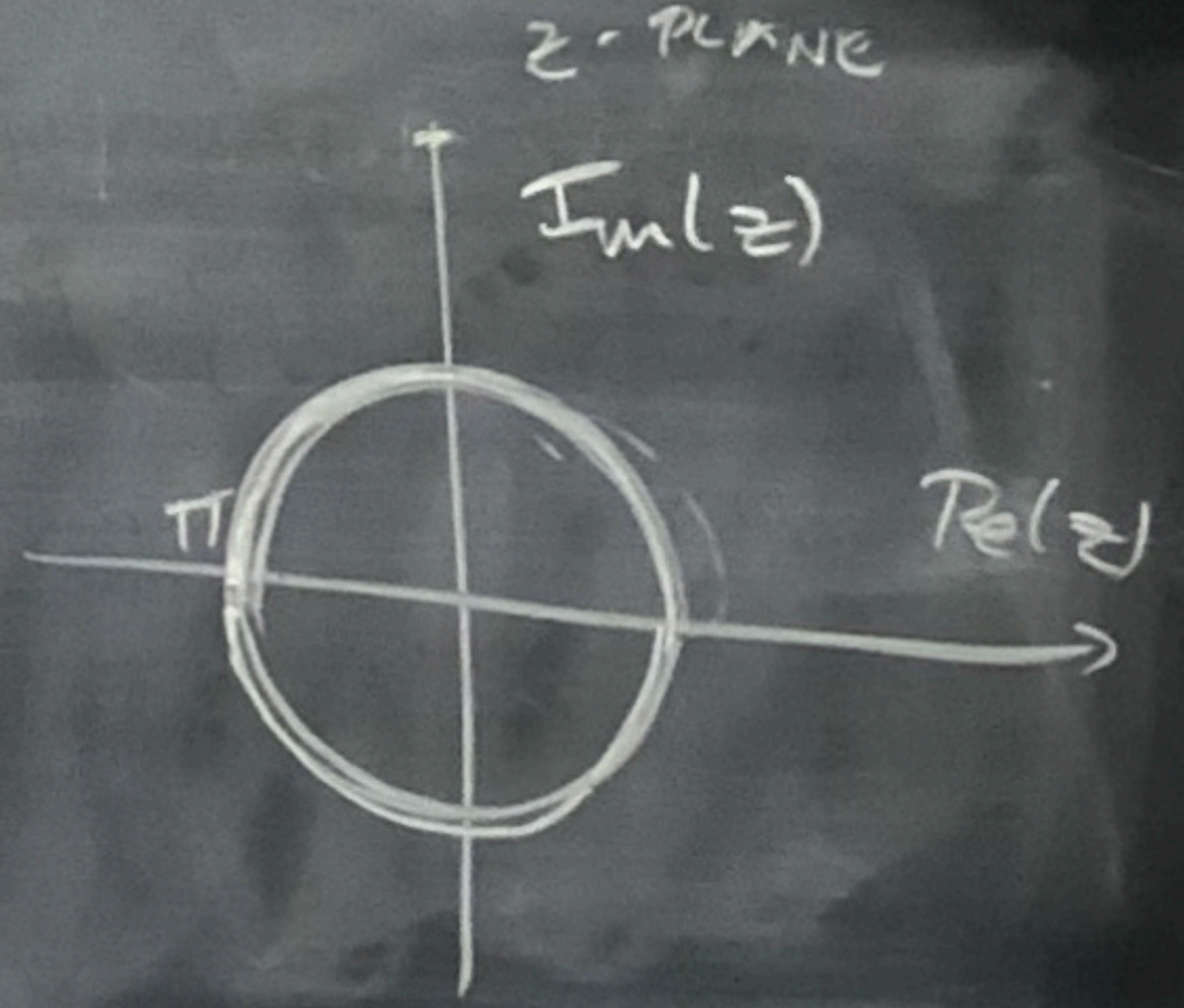
$$H(s) = \frac{1}{s+a}$$

$$h(t) = e^{-at} u(t)$$









ANOTHER POSSIBLE MAPPING: THE BILINEAR TRANSFORM

$$S = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

$$H(s) = \frac{1}{s+a} \Rightarrow H(z) = \frac{1}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + a}$$

$$S = \frac{2}{T}\,\frac{(z-1)}{(z+1)} \Rightarrow$$

$$STz + ST = 2z - 2$$

$$-STz + 2z = +2 + ST$$

$$z(2 - ST) = 2 + ST$$

$$z = \frac{2+ST}{2-ST} =$$

$$z = \frac{1 + \frac{ST}{2}}{1 - \frac{ST}{2}}$$

EAR TRANSFORM

Let $s = j\Omega$

Let $s = \sigma + j\Omega$

Let $s = j\Omega$

$z = e^{j\omega}$

$$|z| = \frac{1 + \frac{sT}{2}}{1 - \frac{sT}{2}} = \left| \frac{1 + j\frac{\Omega}{2}}{1 - j\frac{\Omega}{2}} \right| ,$$

$$z = \frac{1 + \frac{(\sigma + j\Omega)T}{2}}{1 - \frac{(\sigma + j\Omega)T}{2}} = \frac{\left(1 + \frac{\sigma T}{2}\right) + j\frac{\Omega T}{2}}{\left(1 - \frac{\sigma T}{2}\right) - j\frac{\Omega T}{2}}$$

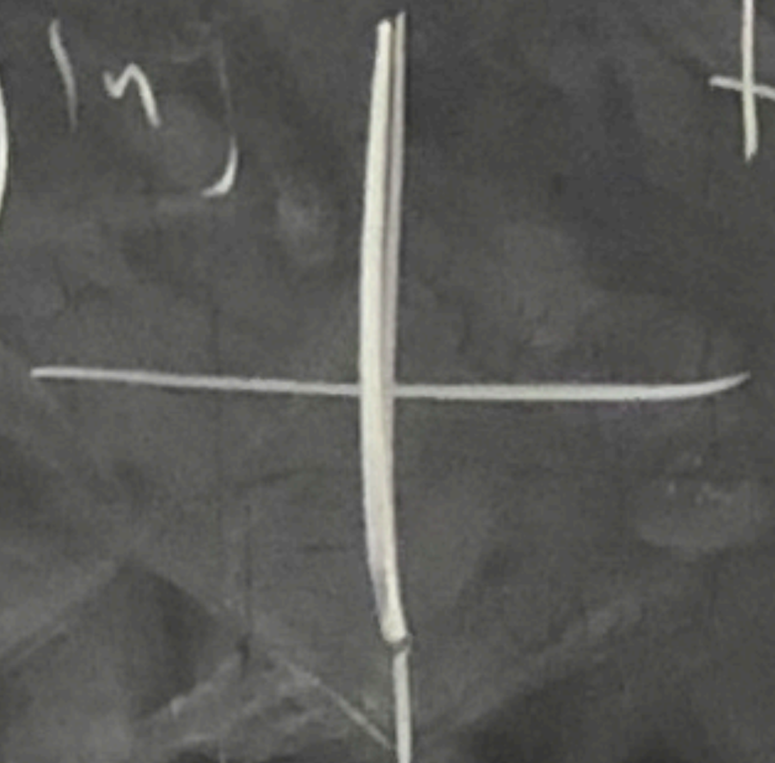
$\sigma < 0 \Rightarrow |z| < 1$

$\sigma > 0 \Rightarrow |z| > 1$

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \qquad s = j\Omega \qquad z = e^{jw}$$

$$j\Omega = \frac{2}{T}\,\frac{\left(1-e^{-jw}\right)}{\left(1+e^{-jw}\right)} = \frac{2}{T}\,\frac{\cancel{e^{-j\frac{w}{2}}}\left(e^{j\frac{w}{2}}-e^{-j\frac{w}{2}}\right)}{\cancel{e^{-j\frac{w}{2}}}\left(e^{j\frac{w}{2}}+e^{-j\frac{w}{2}}\right)}$$

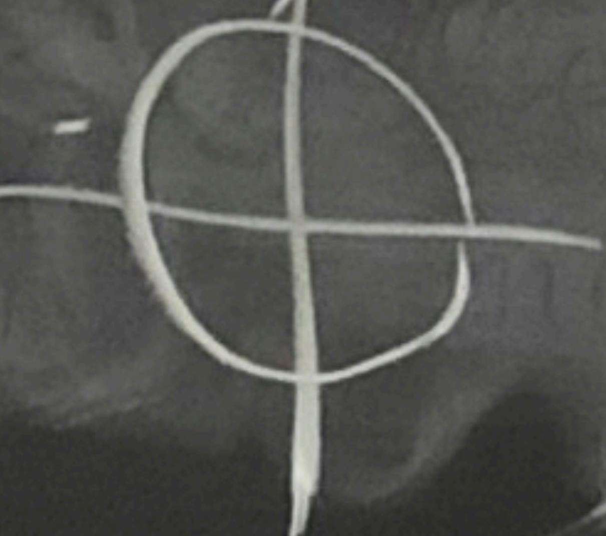
$$\cancel{j}\Omega = \frac{2}{T}\,\frac{\cancel{2}\cancel{j}\sin\left(\frac{w}{2}\right)}{\cancel{2}\cos\left(\frac{w}{2}\right)}$$



$$\Omega = \frac{2}{T}\tan\left(\frac{w}{2}\right)$$

$$\frac{\Omega T}{2} = \tan\left(\frac{w}{2}\right)$$

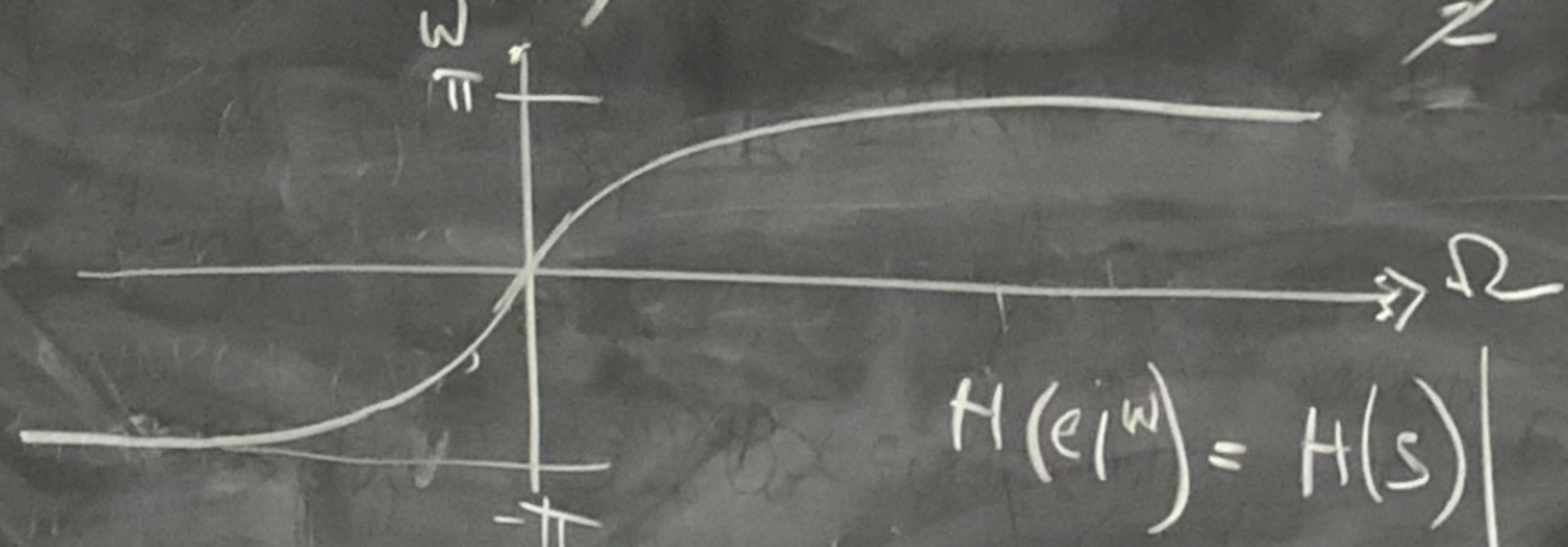
$$\tan^{-1}\left(\frac{\Omega T}{2}\right) = \frac{w}{2}$$



$$w = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$

SMALL Ω

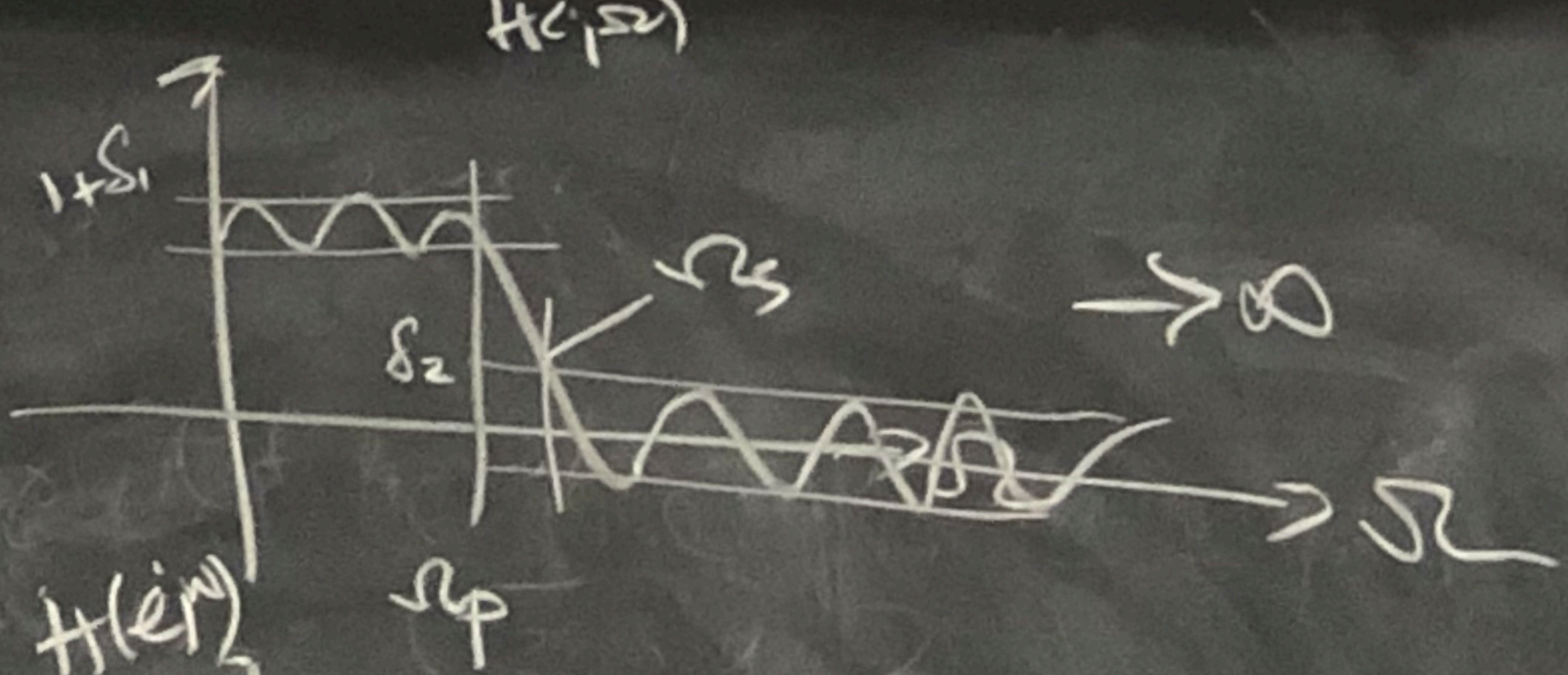
$$w = \cancel{2}\,\frac{\Omega T}{\cancel{2}}$$

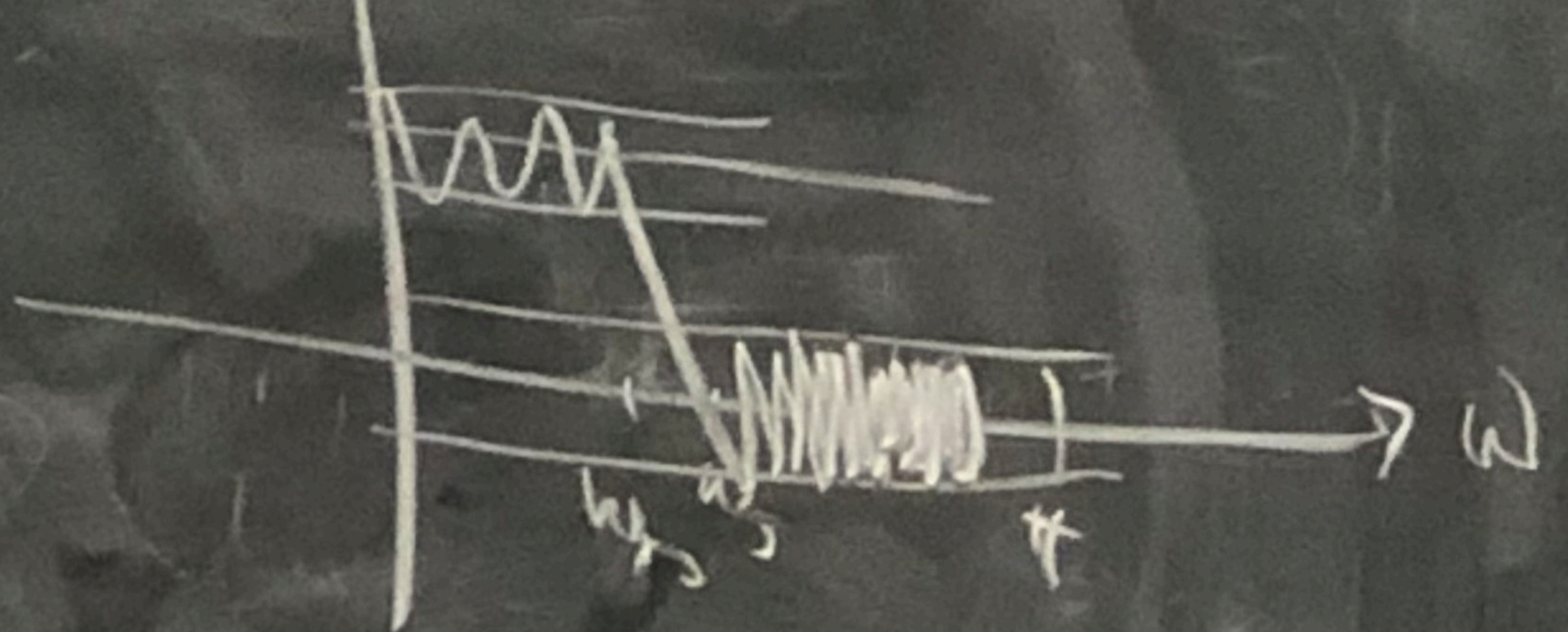


$$H\left(e^{jw}\right) = H(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} = H(j\Omega)\Big|_{\Omega=\frac{2}{T}\tan\left(\frac{w}{2}\right)}$$

let $s = j\Omega$

$$\left. \frac{j\frac{\Omega}{2}}{-j\frac{\Omega}{2}} \right|$$

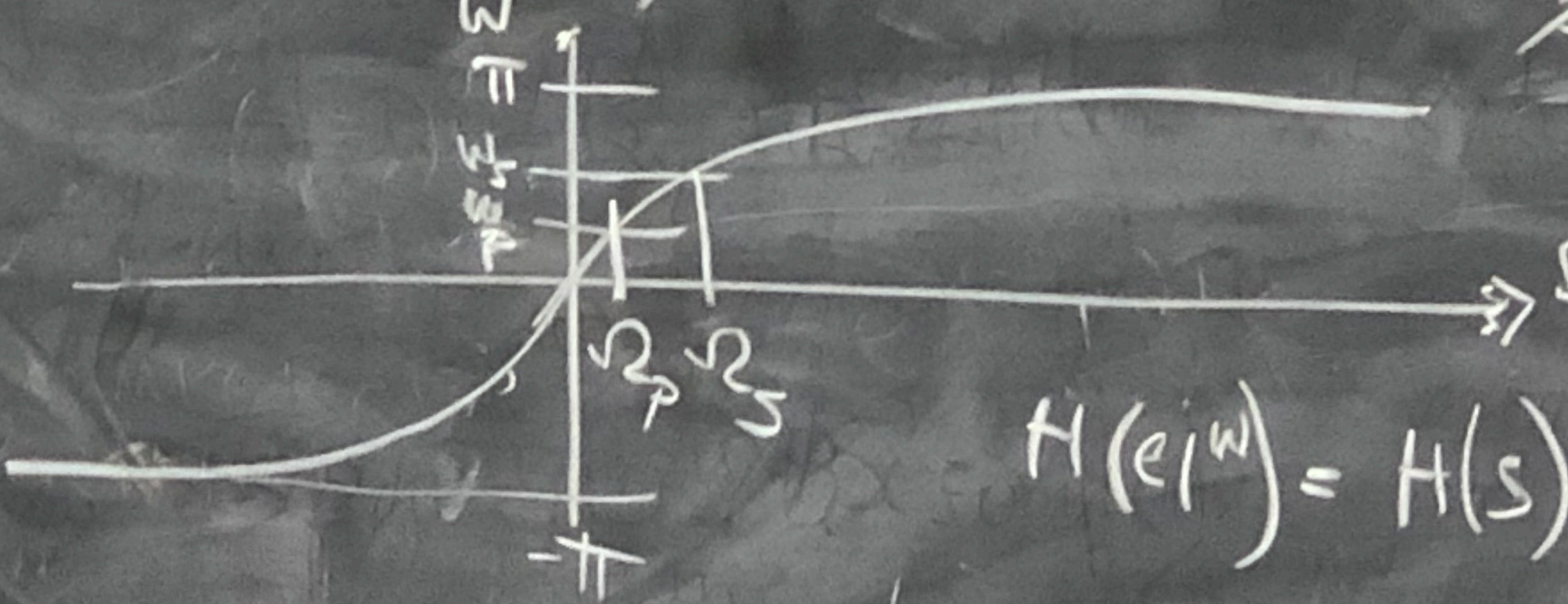




$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$

SMALL Ω

$$\omega = \cancel{2}\,\frac{\Omega T}{\cancel{2}}$$



$$H(e^{j\omega}) = H(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} = H(j\Omega)\Big|_{\Omega=\frac{2}{T}\tan\left(\frac{\omega}{2}\right)}$$