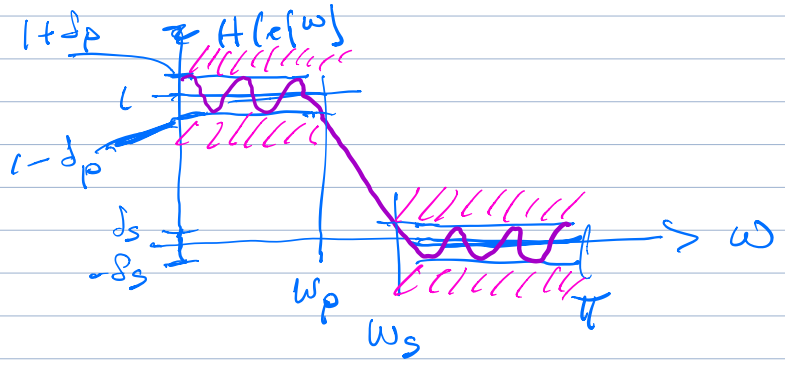
4/12/21 LECTURE 20C

CONCLUSION of IIR FILTER IMPLEMENTATION



REVIEW SO FAR:

SPECS: $\omega_p = .15\pi$

$\omega_s = .35\pi$

PASSBAND RIPPLE: $-3\,dB \le |H(e^{j\omega})| \le 0\,dB, \quad |\omega| \le \omega_p$

STOPBAND ATTENUATION: $|H(e^{j\omega})| \le -20\,dB, \quad \omega_s \le |\omega| \le \pi$

RELATING SPECS TO δ_p, δ_s:

$$1-\delta_p = 10^{-3/20} = .7079$$

$$\delta_s = 10^{-20/20} = .1$$

TWO DERIVED PARAMETERS:

$$\text{Let } k_1 \triangleq \frac{1}{(1-\delta_p)^2} - 1 = .9953$$

$$k_2 = \frac{1}{\delta_s^2} - 1 = 99$$

RELATING DT FREQS TO CT FREQS:

WITH IMPULSE INVARIANCE $\Omega = \omega/T$ SO

$\Omega_p = \omega_p/T = .47124/T$; $\Omega_s = \omega_s/T = 1.0996/T$

WITH THE BILINEAR TRANSFORM, $\Omega = \frac{2}{T}\tan(\omega/2)$

$\Rightarrow \Omega_p = \frac{2}{T}\tan(\omega_p/2) = .4802/T$; $\Omega_s = \frac{2}{T}\tan(\omega_s/2) = 1.226/T$

IN EITHER CASE,

$$N = \frac{1}{2}\frac{\log(k_2/k_1)}{\log(\Omega_s/\Omega_p)}$$

$N = 2.7144$ FOR IMPULSE INVARIANCE

$N = 2.4546$ USING THE BILINEAR TRANSFORM

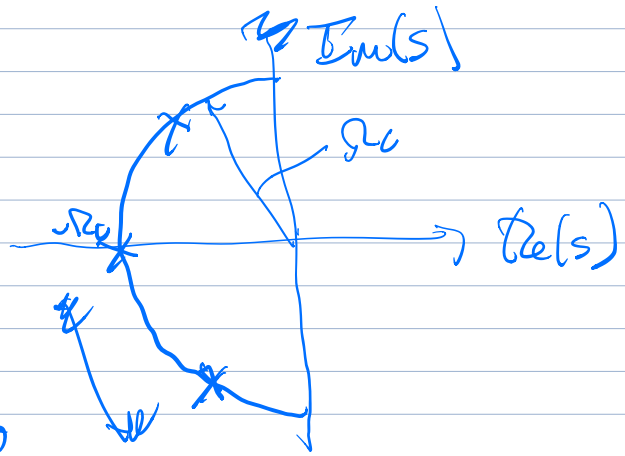
EITHER WAY, WE ROUND N UP TO 3

FOR IMPULSE INVARIANCE, $\Omega_c = \frac{.4716}{T}$, MATCHING SPECS AT THE PASSBAND EDGE

FOR DESIGN USING THE BLT,

$\Omega_c = \frac{.5698}{T}$, MATCHING SPECS AT THE STOPBAND EDGE

BUTTERWORTH FILTER IMPLEMENTATION



$\psi = \pi/N$

$$H(s) = \frac{\Omega_c^3}{(s-\Omega_c e^{j\pi})(s-\Omega_c e^{j(\pi-\pi/3)})(s-\Omega_c e^{j(\pi+\frac{\pi}{3})})}$$

IMPULSE INVARIANCE

$\Omega_c = \frac{.4716}{T}$

$$H(s) = \frac{(.4716/T)^3}{\left(s + \frac{.4716}{T}\right)\left(s - \left(\frac{-.236 + .408j}{T}\right)\right)\left(s - \left(\frac{-.236 - .408j}{T}\right)\right)}$$

$$H(s) = \sum_{k=1}^{N} \frac{A_k}{(s - s_k)}$$

$$h(t) = \sum_{k=1}^{N} A_k e^{s_k t} u(t)$$

$$h[n] = T\,h(t)\Big|_{t=nT} = T\sum_{k=1}^{N} A_k e^{s_k nT} u[n]$$

$$\Leftrightarrow H(z) = \sum_{k=1}^{N} \frac{TA_k}{1 - e^{s_k T} z^{-1}}$$

$$H(z) = \frac{.4716}{1 - e^{-.4716} z^{-1}} + \frac{-.2316 - .136j}{1 - e^{-.236 + .408j} z^{-1}} + \frac{-.2316 + .136j}{1 - e^{-.236 - .408j} z^{-1}}$$

BILINEAR TRANSFORMATION

$N = 3, \quad \Omega_c = \frac{.5698}{T}$

$$H(s) = \frac{\Omega_c^3}{\left(s - \Omega_c e^{j\pi}\right)\left(s - \Omega_c e^{j(\pi + \pi/3)}\right)\left(s - \Omega_c e^{-j\left(\frac{\pi}{3} + \frac{\pi}{2}\right)}\right)}$$

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

$$H(s) = \frac{.1850\,T^3}{s^3 + \frac{1.1397\,s^2}{T} + \frac{.649\,s}{T^2} + \frac{.1850}{T^3}}$$

$$= \frac{.1850\,T^3}{\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^3 + \frac{1.1397}{T}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \frac{.649}{T^2}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right) + \frac{.1850}{T^3}}$$

$$H(z) = \frac{.0132\left(1+3z^{-1}+3z^{-2}+z^{-3}\right)}{1 - 1.2807\,z^{-1} + 1.3315\,z^{-2} - .3244\,z^{-3}}$$